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into ammonia, is also in harmony with the structure of the weakest base (II). For, when the OH-group is attached to nitrogen it may be reduced more readily than when attached to carbon. Hence, the stronger base (I) ought, on reduction, to yield methylamine. This, in fact, has proved to be the case when the methyl derivative is reduced, anyhow to a large extent.

These matters will be further explained in a more extended article in the *Rec. tr. Chim. d. Pays-Bas* in which the analytical figures and the titration and rotation figures of the velocity measurements will be also given.

Oximes being amphoteric electrolytes, I have tried to determine also the dissociation constant of the acid. From the dried ethereal solution of formaldoxime a small quantity of a solid sodium salt is precipitated by sodium methoxide. This however, is so unstable that I have not been able to sufficiently purify it to execute trustworthy velocity measurements with it.

's-Hertogenbosch, Nov. 1915. *Lab. H. B. S.* 5 *j. c.*

Astronomy. — *"On the orbital planes of Jupiter's Satellites, as derived from measurements made at Berlin.* By Prof. W. De Sitter.

(Communicated in the meeting of December 18, 1915).

In the years 1906 to 1909, with the Repsold micrometer attached to the nine inch telescope by Fraunhofer of the Observatory at Berlin, Dr. P. Guthnick made three valuable series of observations of the four old satellites of Jupiter, from which he derived the position of the orbital planes of these bodies.[1]) The observations were compared with Sampson's tables. As Dr. Guthnick remarks, his results, especially those for the satellites II and IV, agree better with my theory of 1908.[2]) than with Sampson's. The comparison with my theory made by Guthnick was however of a preliminary nature, and it appeared desirable to carry it out more in detail.

The immediate results from Guthnick's discussions are the corrections to the adopted elements, which are given on the pages 121 and 122 of his paper, together with the resulting final inclinations and nodes referred to the adopted fundamental plane. This funda-

[1]) Veröffentlichungen der Kön. Sternwarte zu Berlin—Babelsberg. Band I. Heft 3.

[2]) These Proceedings, March 1908; Vol. X, pages 653—673 and 740—729.

mental plane is defined by its inclination and node referred to the fixed orbit of Jupiter of 1900. 0. These are

$$\Phi = 3^\circ.10350$$
$$\Psi = 316^\circ.0510 \quad -0^\circ.0000023\, t$$

The longitude of the node is counted from the fixed equinox of 1900. 0 along the ecliptic of 1900. 0 to the node of Jupiter's orbit, and hence along this orbit. The time t is counted in days from 1900 Jan. 0 mean Greenwich time. The inclination and node of Jupiter's orbit are[1])

$$I = 1^\circ.3098 \qquad \Omega_0 = 99^\circ.4244$$

From these we derive the inclination and node of the fundamental plane on the fixed ecliptic of 1900. 0, and counted from the fixed equinox of 1900. 0:

$$I' = 2^\circ\ 11'\ 45.''3$$
$$N' = 336^\circ\ 52'\ 44'' \ -3.''99\ T,$$

where now the time T is counted in tropical years. The longitudes are counted from a point O', which differs 180° from the node Ψ, and whose distance from the node N' is thus

$$O'N' = 200^\circ\ 50'\ 57'' - 0''.97\ T.$$

The corresponding values of my theory are

$$I = \quad 2^\circ 12'\ 8''.7$$
$$N = 336\ 24\ 24 \quad -1''\ 34\ T$$
$$O'N' = 200\ 37\ 40 \quad -1\ .24\ T.$$

Let now i and Ω be the inclination and node of a satellite's orbit, and put

$$p = -i \sin \Omega,$$
$$q = \quad i \cos \Omega\ ,$$

and the same with accents, when referred to GUTHNICK's fundamental plane. The longitude of the node is counted from the point O [or O'] and the inclination i [or i'] is expressed in degrees. We then find the following formulas of transformation[2])

$$\left.\begin{aligned} p &= p' - aq' + P \\ q &= q' + ap' + Q \end{aligned}\right\} \quad \dots\dots\dots \quad (1)$$

where

[1]) GUTHNICK, l. c. page 17.

[2]) GUTHNICK, in transforming from my fundamental plane to his, has only used the inclinations and nodes on Jupiter's orbit, thus taking no account of the fact that his fundamental plane is referred to HILL's *fixed* orbit of Jupiter, and mine to LEVERRIER's moving orbit.

$$a = \quad 0\ .00438 - 0\ .0000142\ T$$
$$P = + 0^\circ.01465 - 0^\circ.0000266\ T$$
$$Q = + 0\ .01256 - 0\ .0000101\ T\ .$$

The values of p' and q' can be derived at once from GUTHNICK's final results. I have reduced the results for the different satellites to one and the same epoch for each of the three series, for which the epoch of satellite II was chosen. The corrections to be applied to the other satellites on this account are very small and only in one or two cases affected the fourth decimal place. I thus found:

	p_1'	q_1'	p_2'	q_2'	p_3'	q_3'	p_4'	q_4'
1907.183	$+0^\circ.0451$	$-0^\circ.0086$	$-0^\circ.4478$	$+0^\circ.1260$	$-0^\circ.0058$	$-0^\circ.1286$	$+0^\circ.1846$	$+0^\circ.2943$
1908.235	$+$ 0149	$-$ 0203	$-$ 4065	$+$ 2152	$-$ 0278	$-$ 1308	$+$ 1884	$+$ 2897
1909.265	$-$ 0157	$-$ 0433	$-$ 3644	$+$ 3038	$-$ 0428	$-$ 1300	$+$ 1828	$+$ 2924

These must now be transformed to my fundamental plane by the equations (1). We find:

	p_1	q_1	$p.$	q_2	p_3	q_3	p_4	q_4
1907	$+0^\circ.0596$	$+0^\circ.0041$	$-0^\circ.4338$	$+0^\circ.1366$	$+0^\circ.0093$	$-0^\circ.1161$	$+0^\circ.1978$	$+0^\circ.3076$
1908	$+$ 0294	$-$ 0077	$-$ 3930	$+$ 2260	$-$ 0128	$-$ 1184	$+$ 2016	$+$ 3030
1909	$-$ 0011	$-$ 0309	$-$ 3513	$+$ 3148	$-$ 0278	$-$ 1177	$+$ 1960	$+$ 3057

These are thus mean elements as derived from the observations for the respective mean epochs. In comparing them with the theory we must therefore leave out of account those theoretical terms, of which the period is short compared with the periods covered by the observations of each series. These latter periods are 191, 197 and 105 days in the three cases. Such terms, however, do not exist. Beyond the so called "secular" terms, there are terms with periods in the neighbourhood of 6 years, and some small terms with a period of about 248 days. All these have been taken into account [1]). I then find the following theoretical values:

[1]) GUTHNICK only takes account of the "secular" terms and rejects the others, which he calls "kurzperiodisch".

	p_{10}	q_{10}	p_{20}	q_{20}	p_{30}	q_{30}	p_{40}	q_{40}
1907	+0°.0163	+0°.0116	−0°.4514	+0°.1303	−0°.0073	−0°.1175	+0°.2032	+0°.3009
1908	+ 0132	− 0097	− 4104	+ 2213	− 0160	− 1194	+ 2054	+ 2954
1909	− 0058	− 0203	− 3522	+ 3045	− 0245	− 1201	+ 2083	+ 2938

The differences between the observations and the theory are thus :

	Δp_1	Δq_1	Δp_2	Δq_2	Δp_3	Δq_3	Δp_4	Δq_4
1907	+0°.0433	−0°.0075	+0°.0176	+0°.0063	+0°.0166	+0°.0014	−0°.0054	+0°.0067
1908	+ 162	+ 20	+ 174	+ 47	+ 32	+ 10	− 38	+ 76
1909	+ 47	− 106	+ 9	+ 103	− 33	+ 24	− 123	+ 119

For the sake of comparison I add the differences with SAMPSON's theory expressed in the same units. It will be seen that, with a few exceptions, the Δp_i and Δq_i are smaller than the $\Delta p_i'$ and $\Delta q_i'$.

	$\Delta p_1'$	$\Delta q_1'$	$\Delta p_2'$	$\Delta q_2'$	$\Delta p_3'$	$\Delta q_3'$	$\Delta p_4'$	$\Delta q_4'$
1907	+0°.0274	−0°.0109	−0°.0106	−0°.0341	−0°.0034	−0°.0124	−0°.0182	+0°.0266
1908	+ 81	− 38	− 177	− 343	− 172	− 147	− 168	+ 216
1909	− 9	− 250	− 386	− 265	− 218	− 153	− 212	+ 234

The mean errors, expressed in the same unit, are

	p_1 and q_1	p_2 and q_2	p_3 and q_3	p_4 and q_4
1907	± 0.0095	± 0.0060	± 0.0040	± 0.0027
1908 and 1909	± 73	± 46	± 29	± 17

From the residuals Δp_i and Δq_i I have derived the quantities Δx_i and Δy_i, which I also used in my discussion of the Cape observations[1]), by the formulas

$$\Delta x_i = \sum_j \sigma'_{ij} \Delta p_j$$

$$\Delta y_i = \sum_j \sigma'_{ij} \Delta q_j ,$$

[1]) Annals of the Cape Observatory, Volume XII, Parts 3 and 5.

If for σ'_{ij} we take the values corresponding to the masses derived by me, and which are given in *Cape XII*, 5, p. 14, Table X, we find

	Δx_1	Δy_1	Δx_2	Δy_2	Δx_3	Δy_3	Δx_4	Δy_4
1907	+0.0430	−0.0076	+0.0158	+0.0060	+0.0174	+0.0008	−0.0023	+0.0069
1908	+ 158	+ 19	+ 171	+ 45	+ 41	+ 2	− 30	+ 76
1909	+ 47	− 108	+ 14	+ 98	− 18	+ 11	− 125	+ 122

We can now determine the quantities g_i and G_i by the equations [1])

$$g_i \sin G_i = x_{io} + \Delta x_i$$
$$g_i \cos G_i = y_{io} + \Delta y_i,$$

where x_{io} and y_{io} are defined by

$$x_{io} = \gamma_i \sin \Gamma_i$$
$$y_{io} = \gamma_i \cos \Gamma_i .$$

Here γ_i and Γ_i are the "own" inclinations and nodes, and are taken from my theory of 1908. We then find

	g_1	G_1	g_2	G_2	g_3	G_3	g_4	G_4
1907	0°.0690	89°.6	0°.4542	288°.90	0°.1797	188°.84	0°.2475	127°.67
1908	0405	109.8	4559	301.54	1825	195.61	2465	128.36
1909	0385	170.4	4740	313.17	1834	200.06	2362	129.65

If now we neglect a possible correction to the position of the equatorial plane, i.e if we suppose the fundamental plane of my theory to coincide exactly with the mean equator of Jupiter, then g_i and G_i must be considered as observed values of γ_i and Γ_i, and consequently we find corrections to the theory $\Delta\gamma_i = g_i - \gamma_i$ and $\Delta\Gamma_i = G_i - \Gamma_i$. These corrections are given below. I have also carried out the same computations for the values of p_i and q_i resulting from the five Cape series, which are given in *Cape XII* 5, page 17, Table XII. These were first reduced from the tabular fundamental plane used in the reduction of the observations to the final fundamental plane of my theory. Then the differences Δp_i, Δq_i were formed exactly as has been explained above for the Berlin observations; from these Δx_i and Δy_i; were computed and finally g_i and G_i. We thus find:

[1]) These are the formulae (11) of Cape XII, 3, p. 102, if we take $x_{00} = y_{00} = 0$.

	$\Delta\gamma_1$ $\Delta\Gamma_1$	$\Delta\gamma_2$ $\Delta\Gamma_2$	$\Delta\gamma_3$ $\Delta\Gamma_3$	$\Delta\gamma_4$ $\Delta\Gamma_4$
1891.75	−0.0028+12°.3	+0.0005 0°.00	+0.0010−0°.97	+0.0036−0°.26
1901.61	+ 55+ 1.0	− 9+1.12	− 105− .77	− 67+ .24
1902.62	+ 50− 1.0	− 19− .03	− 36− .63	− 19− .64
1903.72	− 57−16.1	+ 23− .30	+ 20+2.59	− 34+ .90
1904.89	+ 22−12.3	+ 4− .89	+ 63+ .15	+ 26− .32
1907.183	+ 408+17.4	− 141+1.35	− 42−5.32	− 61− .92
1908.235	+ 133−15.2	− 124+1.57	− 14−1.20	− 71− .92
1909.265	+ 113− 6.0	+ 57+ .99	− 5+ .65	− 174− .32

These residuals, or rather the values Δx_i and Δy_i from which they were derived, might now be used as the basis of a discussion similar to the one carried out by me in *Cape XII,* 3 and 5, for the derivation of new corrections to the inclinations and nodes. The case of satellite I is worthy of notice. Each of the series 1891—1904 and 1907—1909 considered separately seems to point to a negative correction to the adopted motion of the own node Γ_1. The two series taken together, however, confirm the theoretical value. For a thorough discussion of all the orbital planes, however, the difference of epoch between the observations of Berlin and the Cape is still too short. We shall have to wait till about 1920.[1]) This discussion must necessarily be supplemented by a new investigation of the inequalities in the longitudes for the determination of the masses. The observations which are now being made at the Cape and at Johannesburg will furnish a valuable material for such an investigation.

[1]) See *Cape XII,* 3 p. 121 and "Elements and masses" p. 720. [Proceedings Amsterdam, March 1908, Vol X]. A summary of the present state of the theory and of the investigations which are still desirable, is given in *History and Description of the Cape Observatory,* pages xcvii to ci.