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Physics. — *"An optical method for determining the ratio between the mean and maximal velocities in the turbulent motion of fluids in a cylindrical tube. Contribution to the experiment of* FIZEAU". By Prof. P. ZEEMAN.

(Communicated in the meeting of January 29, 1915).

1. In my experiments on the FRESNEL-coefficient for different colours[1]) the mean velocity of the water-current was found by determining the total volume that had passed through the tubes. Thence the maximal velocity near the axis of the tubes was derived by division of this mean velocity by 0,84. This numerical coefficient has been calculated from numerous and careful observations of American engineers. It seems however not quite satisfactory, that this coefficient has not been measured under the same circumstances that occur in the experiment of FIZEAU. This is the reason I have made some measurements myself. The particularity of the method used was that no measuring apparatus had to be brought into the tube in which we are going to investigate the motion.

If POISEUILLE's law were valid for the tubes used in FIZEAU's experiment, the maximal velocity v_m might directly be derived from the mean velocity v_0, v_m being equal to $2v_0$. In FIZEAU's experiment however only velocities occur far above the limit of the critical velocity given by the criterion of OSBORNE REYNOLDS. If POISEUILLE's law did hold, the distribution of the velocities in the tube would be represented by a parabola, so that near the axis the velocity would change rather rapidly. In the case of the great velocities occurring in our experiments the distribution of the velocities is much more uniform and the interference fringes are much sharper than else might have been expected. As the velocity is not everywhere the same, a curving of the wavefronts must occur, from which a broadening of the interference fringes will result. At the inspection of the interference fringes we are struck by the fact, that the turbulence of the motion leaves the interference fringes so sharp. This is also shown by the sharpness of the photos added to my communication of May 25 1915.

2. In FIZEAU's experiment the water is led into the tubes through which the interfering beams pass, by symmetrical connecting-pieces. In the experiments to be described in this communication the water supply has been arranged in the same way. To the wall of the

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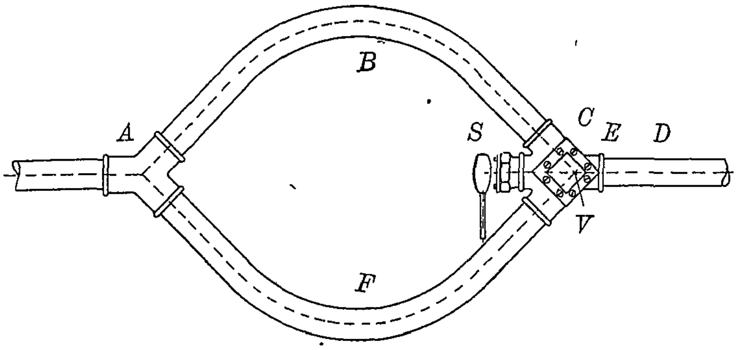
[1]) These proceedings 445, **17**. 1914; 398, **18**. 1915.



laboratory the ironpiece $A\,B\,C\,F$ is fixed which passes into the glass tube $E\,D$. By means of the mirror S a strong light-beam can be thrown along the axis of the tube (diameter 2 cm.). The meaning of the window V will be given below (§ 10). Before the water reaches A it is mixed with small gas-bubbles. The paths of these bubbles beautifully contrast with the dark back-ground, which enables us to pursue particulars of the motion of the fluid[1].

3. The illumination is arranged in such a way that a vertical plane through the axis of the tube ED is strongly illuminated.

On the horizontal axis of a small continuous-current motor, the speed of which can be regulated, a vertical disk of card-board with a number of holes in it is fixed; through these we look at the tube. At a proper velocity of the disk the confuse image of the many entangled stream-lines is decomposed into simple elements. At E we then discern straight line-elements, of which many are horizontal, while others show an inclined direction. It is by the latter that the radial motion becomes visible, which has been discovered by Osborne Reynolds. The further we go from E to D i.e. in the direction of the streaming water, the simpler the image becomes, until at a distance of 20 cm. from E it does not change any longer. There are however

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[1] The gas-bubbles can be introduced into the fluid by pressing compressed air through a fine opening. Better results are reached however with an electrolytic developer as Zenneck uses in his method for demonstrating the stream-lines in the inner part of the fluid. (Berichte deutsch. phys. Ges. p. 695. 1914). In my experiments the electric current entered through two coal plates. The behaviour of different coal plates is not always the same. The gas-bubbles had a diameter of 0,1 to 0,3 mm. When of one of the plates a piece had been broken off we took into use a new pair. Then the gas-bubbles proved to have become far too small, so that we took again the old plates. If the bubbles are too small the stream lines cannot be observed very well, at least not in the mirror of § 4. In the experiments of Zenneck the bubbles might be very small,

still elements that are inclined with respect to the axis of the tube, but the inclination has decreased considerably.

4. Now we pass to a second experiment. A strong light beam, which over a length of a few centimeters has a diameter of 4 mm., can easily be thrown along the axis of the tube *ED*. The velocity of the small gas-bubbles illuminated by this light-beam was measured by means of a rotating mirror the axis of which was parallel to that of the glass tube.

If the mirror is at rest we see in a telescope, pointing to the mirror, the streamlines principally in horizontal direction. From the inclination which the apparent stream-lines assume when the mirror rotates with a known velocity we can derive the value of the velocity in the axis of the tube i.e. the maximal velocity. It is evident, that this determination is most accurate when the velocity of the mirror and the distance from the glass tube are so chosen that the angle of inclination α of the stream-lines with the horizontal direction becomes nearly 45°. By reversing the current in the electromotor, by which the mirror is rotated we can directly read the angle $2\,\alpha$. The velocity of the mirror was 1,052 rotations per second. The "effective length" of the distance of the axis of rotation of the mirror to the axis of the tube could be determined within a fraction of a millimeter, attention being paid of course to the passage of the light through water and through glass.

5. In a direction perpendicular to the tube the small gasbubbles scatter less light than in a direction which makes a smaller angle with the water-current.

In some experiments I have observed in an *inclined* direction, the greater intensity in which is advantageous for the accuracy of the adjustments. But then the observations must be reduced to the values that would be found in a plane perpendicular to the axis of rotation of the mirror. Instead of calculating this reduction from the angles that determine the deviation I have preferred to determine experimentally by a separate experiment the corrections for the observed values of α.

For this purpose I used a series of luminous points, which were moving with the velocity of the water-current, but exhibiting a greater intensity and a particular regularity. Thirty-six small steel balls were fixed near the circumference of a copper disk of about 20 cm. diameter. In this additional experiment the plane of the disk could be placed perpendicularly to the direction in which in the flow-experiments the light from the tube fell on the mirror. At the same time care must however be taken that in the lower part of their path the balls have the same direction as the water current in the middle of the tube. The distance between the copper disk and the glass tube was made very small. A strong light-beam is concentrated on the lower part of the disk, so that the clear luminous points of the moving balls can be observed in the rotating mirror. The velocity of the copper disk is thus regulated that it is approximately equal to the velocity of the water in the tube. The observations on the disk with the balls are so much more accurate than those on the water-current that the correction by which observations in an inclined direction must be reduced to a plane perpendicular to the mirror becomes exactly known. This correction approximately amounts to $1^{\circ}/_{0}$ of the angle.

6. The result of the experiment for a part of the tube at a distance of about 23 cm from the point denoted by V is given in the following table.

V	T	v_0	α'	α	v_m	$\varphi = v_0/v_m$
3001	2623	330.3	44°.9	44°.3	389.9	0.847
1051	985	308.1	46.2	45.6	372.6	0.827
1609	1479	314.1	46.2	45.6	372.6	0.843
1286	1175	316.0	45.6	45.0	380.5	0.830
1617	1501	311.0	46.0	45.4	375.2	0.829
						0.835

Under V is given the water-volume, expressed in liters, which has flowed through the tube in T seconds. This amount has been corrected for the error ($-1{,}6^{\circ}/_{0}$) of the "Ster"meter, as it was given to me by the Direction of the municipal water-works.

Then v_0 follows (in cm./sec.) from the transverse section of the glass tube. The angle α' is half the directly read angle, α the value corrected for the inclination. $v_m = v/tang\,\alpha$ gives the maximal velocity in cm./sec.; $v = \frac{4\pi l}{1\,052} = \frac{4\pi \times 31{,}85}{1{,}052} = 380{,}5 \frac{cm}{sec}$ is the velocity which by means of the rotating mirror is added to the velocity of the water, where l is the "effective" distance from the axis of the mirror to that of the tube.

7. A second series of observations has been made for points of the tube at distances between 20 and 30 cm from the point V. By using a stronger source of light, I could now make observations in a plane *perpendicular* to the axis of rotation of the mirror so that the correction of α' to α can be omitted. The "effective" length l was 31.3 cm and v was 374.0.

In the same way as above the measurements have been collected in this table.

V	T	v_0	α	v_m	$\varphi = v_0/v_m$
1165	1067	315.2	45.4	368.8	0.855
1757	1625	312.2	45 4	368.8	0.846
1075	979	317.0	45.2	371.4	0.854
1497	1383	312.5	45.3	370.1	0.844
886	814	314.3	45.2	371.4	0.846
762	696	316.1	45.3	370.1	0.854
					0.850

8. Finally, a few observations have been made in the initial part of the tube, in the neighbourhood of E.

With the effective length $l = 31.8$ cm and $v = 380{,}0$ cm, the measurements gave the following result:

V	T	v_0	α	v_m	$\varphi = v_0/v_m$
1346	1267	306.7	45.8	369 5	0.830
1038	956	313.5	44.6	385.3	0.814
1358	1261	310.9	45.5	373.4	0.833
					0.826

9. As is shown by the experiment of § 3 the motion near E is more complicated than that further away in the tube, so that the result of § 8 cannot directly be compared with those of §§ 6 and 7. The image observed in the rotating mirror is also less simple in the experiments of § 8 than in the other series of observations.

In the latter experiments a distinct principal direction of the stream-lines can be indicated; in the experiments of § 8 this is only the case to a much less degree. Paths of particles which have high radial velocities and therefore exhibiting extraordinary inclinations are shooting again and again through the field of view of the telescope. In the experiments of § 6 and § 7 the accuracy of the adjustment is not so much lowered by this phenomenon, though even here such deviating paths are visible from time to time. Summarising the results of § 6 and § 7 by taking the mean of the values of φ we obtain 0,843. [1])

If we assume that over a length of 10 cm. a smaller value of φ holds, this has no perceptible influence on the final result, as in FIZEAU's experiment the whole length of the tube is 300 cm. For an estimate of the accuracy of the determinations I give here as an example those referring to the last observation of § 7.

$V = 762\ L$ $T = 696$ seconds.

Reading of the divided circle.

Direction of rotation of the mirror:

to the right	to the left
308.2	218.1
307.0	217.5
310.5	218.2
309.9	218.0
309.1	215.4
308.2	219.5
309.8	218.8
307.0	218.9
307.8	219.0
311.3	219.6
308 8	218.3

Difference $= 2\alpha = 90°5$, $\alpha = 45°,3$.

It is difficult to estimate the accuracy of the determination $\varphi = 0,843$, but we shall not be far from the truth, if we call a deviation of more than 1 °/₀ very improbable. An effort to increase the accuracy by an accumulation of observations would only then have sense if we were quite sure that the influence of the strongly deviating paths on the optical phenomena might be neglected, which is very probable, and if the measurements could be made with the same tube that was used in FIZEAU's experiment. This is not well possible. It is however not necessary to justify an important conclusion concerning

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[1]) The mean of all observations of § 6—§ 8 gives 0.839.

the *absolute* values of the differences in phase, observed in FIZEAU's experiment.

From the table in my second communication on the experiment of FIZEAU (These Proceedings Vol. **18**, 403. 1915), I take these data:

$p = 2{,}14\ K.G/cm^2$ $\quad v_0 = 465$ *cm./sec.* $\quad v_{max} = 553.6$ *cm./sec.*

λ in A°.E.	Δ_{Fr}	Δ_L	Δ_{exp}
4500	0.786	0.825	0.826 ± 0.007
4580	0.771	0.808	0.808 ± 0.005
5461	0.637	0.660	0.656 ± 0.005

Under Δ_{Fr} en Δ_L the shifts of the interference fringes are given here without and with the dispersion term. Under Δ_{exp} are to be found the observed shifts together with the probable error in the final determination.

Now the values under Δ_L and Δ_{Fr} have been calculated with the value $\varphi = 0.840$. For other values of φ the results are given in the following table:

$\lambda = 4500$

Δ_{Fr}	Δ_L	φ	Δ_{exp}
0.771	0.810	0.856	
0.779	0.817	0.848	
0.786	0.825	0.840	0.826 ± 0.007
0.794	0.833	0.832	
0.802	0.841	0.824	

From this table it is evident that no plausible value of φ can be indicated for which the values under Δ_{Fr} would agree with the results of the experiment. With the theory of LORENTZ, however, for the neigbourhood of $\varphi = 0.843$ the measured differences in phase are in extremely good agreement as to their *absolute* values. Already in my above cited communication I came to the conclusion of the necessity of LORENTZ's dispersion term. This was based upon the ratio of the measured shifts independently as well of the value of φ as of the length of the whole tube. We may say however that now the validity of that term has been proved with still greater certainty as to its *absolute* value. Especially the measurements in the blue part of the spectrum have here the greatest convincing power.

Finally I wish to express my thanks to Miss C. M. PEEREBOOM assistant at the Physical Laboratory, Amsterdam, for her assistance in part of the experiments and of the calculations and to Mr. J. VAN DER ZWAAL, mechanician of the Laboratory for his assistance in making the apparatus.

10. In one respect the experiments can teach still something on the question whether the length of the moving water-column with which the calculations have been carried out, has been fairly well chosen.

According to these Proceedings (Vol. **18**, 401, 1915) I took for this length the distance between corresponding points of intersection of the axes of the O-shaped supplying-tubes with the axis of the apparatus. With the apparatus described in § 2 I have been able to prove that this was the right length. Through the window V stream-lines can be observed, if a vertical plane through the axis of the tube is illuminated. With an accuracy of some millimeters we can indicate in which point the stream-lines become rather suddenly parallel to the axis of the tube ED, while on the left of that point the fluid is nearly at rest. There cannot exist any doubt whether the motion must be reckoned from the point V indicated in the Figure. On the whole length of FIZEAU's tube i.e. 302 cm. the inaccuracy of some millimeters in the determination of the place of V is of no consequence.

Physics. — "*The specific heat at low temperatures.* III. *Measurements of the specific heat of solid nitrogen between* 14° K. *and the triple point and of liquid nitrogen between the triple point and the boiling point.*" By W. H. KEESOM and H. KAMERLINGH ONNES. Communication N°. 149*a* from the Physical Laboratory at Leiden. (Communicated by Prof. H. KAMERLINGH ONNES).

(Communicated in the meeting of January 29, 1916).

§ 1. The investigation of the specific heat of condensed monatomic and di-atomic gases appears of special interest with a view to the conclusions, which may be drawn about the crystal structure from the comparison of the specific heat in the solid state for these two groups of substances. In particular the question arises whether for the last mentioned group of substances their di-atomic nature does or does not show itself in the solid state also.

§ 2. We began with the investigation of the specific heat of