Citation:

Haas, W.J. de, Further experiments on the moment of momentum existing in a magnet, in:
KNAW, Proceedings, 18 II, 1916, Amsterdam, 1916, pp. 1281-1299

points, that is to say there are three φ^3 which have a six-point contact with a conic in B.

From this it ensues that the locus σ of the *sextactic points* S is intersected by any ray s from A in 3 points, and has triple points in the five base-points B. Between the plane pencil (s) and the pencil (φ^3) exists therefore a (3,3); for any φ^3 bears three points S.

Consequently the *"sextactic" curve* σ is of order *twelve.* It has a *nonuple* point in A, and *triple* points in the five single base-points. Symbol $\sigma^{12}(A^9,5B^3)$.

Other singular points it cannot have for in genus it must answer to ι^7; for to each point of inflexion I belongs *one* point S and the reverse.

§ **6.** A conic touching φ^3 in B in five points, intersects it moreover in a point R, which I shall call the *residual point* of B. The locus ϱ of the points R has a *quintuple* point in B; the tangents fall along the tangents of the three conics, which have a six-point contact in B, and along the two stationary tangents of the two φ^3, which have their point of contact in B.

The straight line BR intersects the corresponding curve φ^3 in the second tangential point B'' [1]. Now the curve τ^9_2 has in B a node, in C a triple point; on BC therefore lie moreover four points B'', so that ϱ must pass four times through C. As B'' further comes thrice in C, three points R lie on BC; but ϱ is then a curve of order *twelve.* Of its intersections with an arbitrary φ^3 5 lie in B, 16 in the points C, one in the residual point of B; we conclude from this that ϱ has a *septuple* point in A.

We can represent it therefore by the symbol $\varrho^{12}(A^7,B^5,4C^4)$ [2]. It may serve as confirmation that the curve must be rational.

Physics. — *"Further experiments on the moment of momentum existing in a magnet".* By Dr. W. J. DE HAAS. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of September 25, 1915).

Introduction. In a former paper on this subject [3] it has been remarked that the attempt to determine the sign of the effect had proved a failure. New experiments were therefore desirable. But

[1] Cf. e.g. SALMON-FIEDLER, ibid.

[2] In my paper referred to above (*Arch. Teyler*) I have considered the curve ρ for a general pencil (φ^m); any conic associates then to a point of contact B $(2m—5)$ residual points R It has the symbol $\rho^{10m+2}(B^{15},C^{10})$

For the sextactic curve the symbol $\sigma^{24m-27}(B^{12})$ was found there.

[3] A. EINSTEIN and W. J. DE HAAS, These Proceedings, XVIII p. 696, 1915.

besides this the importance of the observed phenomenon for the theory of electrons and for our knowledge of the nature of magnetism sufficiently justified a repetition of the investigation. In the Berlin experiments the phenomena were so much complicated, that I tried to find a more elegant method, by which part of the sources of error could be eliminated, so that it might surpass the former method in clearness and convincing power.

I have succeeded in rigorously separating the effect which I should like to call the EINSTEIN-effect, from the secondary phenomena. This time again I have not had in view an accurate quantitative determination; yet it may be mentioned that the quantitative agreement between experiment and theory is quite satisfactory. At the same time a way is opened for a later accurate determination of $\frac{e}{m}$.

§ 1. *Principle of the new method.*

In the former method a cylinder that could perform torsional vibrations, was brought into an alternating magnetic field of about the same frequency as that of the free oscillations of the cylinder.

From the amplitude of the observed vibrations of the cylinder the existing moment of momentum could be calculated. As we have seen the disturbing effects consisted in:

1. the action of the static terrestrial field on the alternating horizontal components of the magnetism in the cylinder;

2. the action of the alternating field of the coil on the horizontal components of the static magnetism in the cylinder;

3. the action of the horizontal components of the alternating field of the coil on the alternating magnetization of the cylinder.

As we remarked already, we need not fear any disturbance by effect 3, as it has double the frequency of the EINSTEIN-effect. We must keep in mind however, that disturbances of this kind will appear as soon as the dynamo gives e. g. an asymmetrical sinusoidal current. A very small asymmetry already may reach the value of the remaining part of the terrestrial field (which is never completely compensated). An asymmetry of the current may give rise to a disturbing effect having the period of the free vibrations of the cylinder.

The characteristics of the new method are the following:

1st a very slow resonance, which enables us to investigate the difference in phase between the effect and its cause;

2nd a complete elimination of the sources of error 2 and 3. For this purpose the iron cylinder is not placed in the magnetic field of a fixed coil, but the wire is wound on the cylinder itself. As under these circumstances action and reaction are acting within one and the same solid body, the source of error 2 and eventually 3 are wholly eliminated. We may therefore concentrate our attention on the actions of the magnetic field of the earth.

§ 2. *Description of the apparatus.*

Fig. 1 shows the apparatus schematically. The proportions have not been indicated rightly. S is the soft-iron cylinder. Length 23 cm, thickness 3 mm. Over a length of 21.5 cm it is wound with enamelled copper wire of the A. E. G. 0.08 mm thick. The whole resistance of this wire and the two wires p, q through which the current is led, is 320 Ω. p and q are silver hair wires of HARTMANN and BRAUN, thickness 0.015 mm. Preliminary experiments proved that they did not influence the torsion of the suspending wire r. The wire r is fixed to a turning disk k, which makes it possible to turn the cylinder about its vertical axis through an angle of 360°. Beside the cylinder the figure shows a pendulum, consisting in an ivory ball I charged with lead and suspended by a steel wire dr; length a little below 1 m. and thickness $^1/_6$ mm. Point of suspension g.

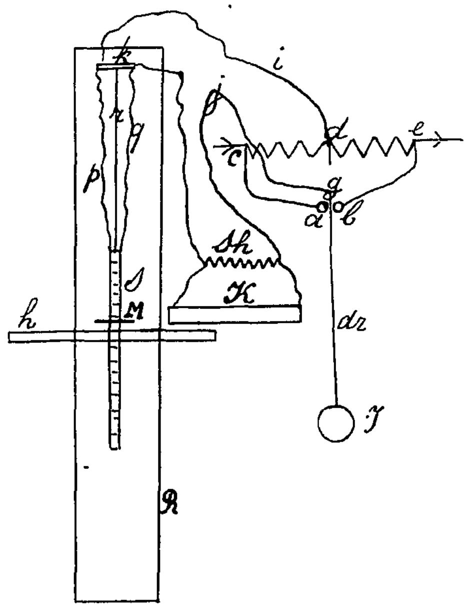


Fig. 1.

If I oscillates, the steel wire alternately makes contact with a and b, two copper wires represented in transverse section and placed about 12 cm below g. The distance between a and b is about $^1/_3$ mm. Further cd and de are two resistances of 67.5 Ω each, through which the current of a battery of accumulators flows from c to e. The further connections are evident from the figure.

When *dr* makes contact with *a*, the current flows along *c,a,g,j,q*, coil, *p, i, d* to *e*; when, on the contrary, *dr* touches *b*, the current takes the way *d,i,p*,coil, *q,j,g,b* to *e*, circulating in opposite direction about the cylinder *S*, so that, the pendulum being in motion, the cylinder *S* is alternately magnetized in one direction and the opposite one, synchronically with the pendulum. The potential difference between *c* and *d*, *d* and *e* has been chosen in such a way, that the cylinder is magnetized to a sufficient degree.

The torsional motion of the cylinder was observed with telescope and scale.

§ 3. *Compensation of the terrestrial field.*

The following considerations show of how much importance this is. If the direction of the magnetization deviates 3′ or 4′ from the vertical, the horizontal component amounts to one 1000^{th} of the vertical magnetization. Now a deviation of this amount may easily occur as the suspension is not accurately centred and the windings are not laid quite regularly[1]).

If the field of the earth were not compensated and the horizontal component of the magnetization were perpendicular to the magnetic meridian, the vibrations resulting from the above mentioned cause would have an amplitude equal to 130 times that which we wish to observe, (for the period used in the experiments).

We shall now describe in what way a rather satisfactory compensation of the field can be reached. The compensation problem is the same as in the case of a sensitive galvanometer, with this difference, that here we have to compensate over the whole length of the cylinder with that of the wires *p* and *q* in addition, so that the terrestrial field has to be compensated in a space of breadth 5 to 7 mm and length 35 to 40 mm.

In order to make an estimate as to how far this may be done, I constructed an apparatus, which made it possible to lower or to draw up by a spider-thread a light galvanometer mirror, to which two small strong magnets from a spring of Wolfram steel (they were about 5 mm long) had been fixed; by this means the mirror could be placed at any point of the field that had to be compensated. The spider-thread was so thin, that the mirror was suspended practically without torsion, so that only the terrestrial magnetic couple was acting. From the period of oscillation it is easy to deduce, in how far the field of the earth is compensated. Starting from a

[1]) In a repetition of the experiments more attention than was possible now would have to be paid to an accurate construction in these two respects.

period of 0,08 second in the uncompensated field we were able to reach over the whole length of the cylinder and the leads periods ranging from 10 to 20 seconds. From this it follows that a compensation from 1 to $^1/_4$ % has been reached.

As to the arrangement used for the compensation a brief description will suffice. In fig. 1 R and h are resp. a frame and a hoop wound with copper wire through which a current from a battery of accumulators was made to flow. The regulation of the intensity of the current was effected by means of resistance boxes with shunts for the finer regulation.

When in this way the greater part of the earth-field had been got rid of, the remaining part was compensated in the following way:

At a distance of about 3 m. a rather large permanent magnet was placed at the height of the middle of the cylinder S. This magnet was turned in the horizontal plane in which it lay till the above described apparatus had reached a period of 20 seconds, so that the compensation near the middle of the cylinder could be considered as sufficient. The compensation at the extremities of the cylinder, however, was not reached at the same time and I had therefore further to effect it without disturbing that in the middle. For this purpose small permanent magnets of equal moment were placed in horizontal direction vertically above and beneath the cylinder (not shown in the figure). These magnets could be turned about a vertical axis and shifted in vertical direction. To ensure the equality of their magnetic moments, they were made of pieces of a hardened and magnetized knitting-needle, from which a bundle was formed. By taking equal numbers of pieces of equal length equivalent magnets were obtained.

§ 4. *Compensating alternating field.*

The compensation which was reached in this way and which we shall denote by I was not good enough however for sufficiently eliminating the source of error to which attention has been drawn and for making the weak effect appear that was sought for.

In order to neutralize the "residual field" as it will be called, the following artifice was used (II). To the iron cylinder S a small magnet M (magnetized piece of a watch-spring) was fixed in horizontal direction and a horizontal coil (the "compensating coil") was placed with its axis along a line perpendicular to the magnet and passing through it. This coil could be shifted in the direction of its length. The same current that passed through the windings was also made to flow through this coil. It is evident that now, with

83*

proper connections and a rightly chosen distance of the compensating coil, the alternating couple with which it acts on the transverse magnet will always neutralize the forces, with which the "residual field" acts on the alternating horizontal magnetization of the cylinder. To reach this the coil could be shifted and besides there was a shunt *Sh*, the resistance of which could be regulated, so that only a larger or a smaller part of the whole current passed through the compensating coil.

Because of the small changes in the terrestrial field, which gave rise to rather large changes of the residual field, the compensation II had to be changed continually. It could however be kept constant long enough to allow the observation of the EINSTEIN-effect.

In the last experiment it was found necessary to put an iron core in the compensating coil, as the latter could not be brought near enough to the cylinder.

§ 5. *The different effects.*

We shall call the effect we are seeking for the *first* effect and the effect caused by the action of the residual field on the horizontal magnetization of the cylinder, combined with that of the compensating coil on the transverse magnet the *second*.

There is however still a *third* effect, which like the first consists in alternately directed impulses, acting on the cylinder at each inversion of the current. Indeed, when the pendulum-wire leaves one of the two copper wires a and b there elapses a certain time before it makes contact with the other one. During that interval τ there is no current in the apparatus. The suspended cylinder has its remanent magnetism, which like the magnetization existing before the breaking of the current has a horizontal component and this component is acted on by the residual field. This action is not neutralized in the way described in *II*, as during the time τ the compensating coil is without current. We shall call this the *third* effect a.

Finally there is a *third* effect b, the direction of which it is difficult to indicate without further examination. It may as well have the sign of the effect $3a$ as the opposite one.

In order to understand the nature of this effect, we must keep in mind that, at each change of the direction of the current the magnetization of the iron passes through a hysteresis-cycle, as is roughly shown in fig. 2, where the abscissae represent the intensity of the current i and the ordinates the magnetization. OA or OD corresponds to the remanent magnetism, which remains every time after the breaking of the current.

(If the figure refers to the whole magnetic moment, the horizontal one, on which the residual field acts may be derived from it by multiplication by a certain constant factor).

The magnetization passes through ABC or DEF in a certain

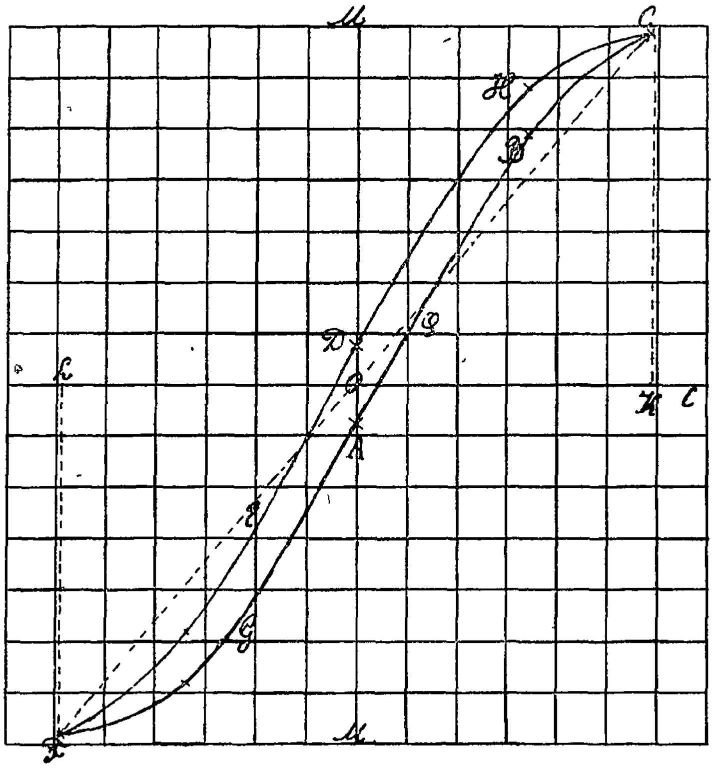


Fig. 2.

time τ'.[1]) Now it is evident that, in the case of a perfect compensation Π, the forces exerted by the residual field would be wholly neutralized, if the magnetization followed the line OC. For then it would increase proportionally to i and so the action of the residual field on the magnetic moment of the cylinder and that of the current in the compensating coil on the transverse magnet would also change proportionally to each other. They would neutralize each

[1]) We may imagine the breaking of the current and therefore the change represented by FGA or CHD to take place in an extremely short time, so that the forces acting on the iron during this time cannot have an appreciable influence on its motion. The interval in question may be neglected with respect to the time τ' necessary for the changes ABC or DEF, which take place after the above considered interval τ during which the remanent magnetization OA or OD has existed. On account of the self-induction it lasts an observable time τ' after the closing of the circuit before the current has reached its full intensity i_1 (OK or OL) and the magnetic moment the corresponding value M_1 (KC or LF).

other continually, because they do so at the end, when the values i_1 and M_1 are reached.

In reality however during the time τ' it is not the magnetization M' indicated by the straight line OC that exists, but the magnetization M represented by the curve ABC. Consequently the action of the residual field on the magnetization $M—M'$ remains. This action gives rise to an impulse proportional to $(\overline{M—M'})\ \tau$, if we denote by $\overline{M—M'}$ the mean value of the difference during the interval τ'. If the negative values of $M—M'$ found between O and Q, surpass the positive ones between Q and C, the mean difference $M—M'$ will have the same direction as the remanent magnetization OA, which existed a moment before and which gave rise to effect $3a$. In this case the effects $3a$ and $3b$ have the same sign. In the case that has been supposed in the figure the positive values $M—M'$ are of more importance than the negative ones, so that here the two effects $3a$ and $3b$ have opposite signs [1] [2].

As it is desirable to make the third effect as small as possible, the experiment was arranged in such a manner, that the time τ was short. This is why I used the pendulum. In order roughly to measure the time τ, the wires a and b were now connected, so that one and the same conductor was closed by the contact of the steel wire with a and by that with b. The conductor in question was one of

[1]) Here we may remark, that effect $3a$ would be preceded by an effect $3b$ of the same direction as $3a$ itself, if the disappearing of the current took an appreciable time This follows from the circumstance that over its whole length FGA lies beneath the straight line OF.

[2]) $(\overline{M—M'})$ or $\int (M—M')\,dt$ over the time τ' might be calculated, if we knew, not only M as a function of i (i. e. the exact form of the curve in figure 2) but also for each value of i, the number of lines of induction N enclosed by the windings, a number that might be represented by a curve similar to that in the figure Indeed, if r is the resistance of the circuit and E the electromotoric force

$$ri = E - \frac{dN}{dt},$$

or, since

$$ri_1 = E,$$

$$r(i_1 - i) = \frac{dN}{dt}.$$

so that we may write for $\int (M—M')\,dt$

$$\frac{1}{r}\int \frac{M-M'}{i_1-i}\,dN.$$

The value of this expression might be derived from sufficiently accurate graphical representations.

the branches of a WHEATSTONE bridge. If now the resistances in the latter had been regulated and the pendulum passed from one extreme position to the other, the breaking of the contact caused a current to flow through the galvanometer during the time τ. From the sudden deflexion the length of the time τ may be deduced. The result was about $\frac{1}{80}$ sec.

§ 6. *The phase of the effects.*

Now the *phase* of the different effects will be considered in details. Fig. 3 may be of use for this purpose. It represents as seen from above the iron cylinder C, the small magnet AB fixed to it, the mirror E, the compensating coil K, the telescope L and the scale S. We shall call the direction of rotation shown by the arrow the positive one. To this rotation corresponds a shift of the cross-wire over the scale to the side marked with $+$. We shall speak of a positive reversal of the current, when it gives rise to a north pole at the upper end of the cylinder. When all has been arranged it is easy to determine the direction in which the pendulum must move if such a positive reversal shall take place.

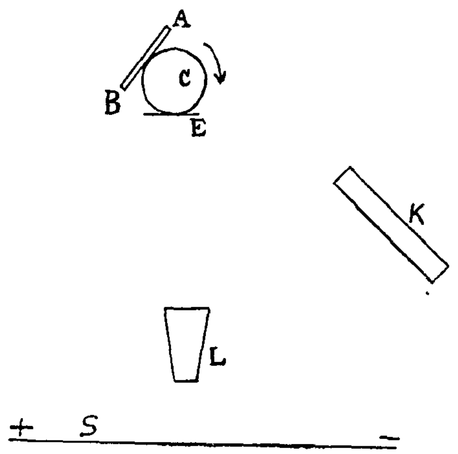


Fig. 3.

The molecular currents existing after a positive reversal have a direction opposite to that of the arrow. If these currents consist in circulating negative electrons, the moment of momentum to which the reversal gives rise has the direction of the arrow. The couple acting on the cylinder must be opposite to this moment and we may conclude: At a positive reversal the first effect consists in an impulse in the negative direction. Of course it would be in the positive direction if we had to do with positive electrons.

We may remark here, that strictly speaking the effect we are seeking for consists in two equal impulses, the first of which (breaking of the current) takes place at the beginning of the time τ while the second (closing of the current) begins at the end of this interval and lasts during the period τ'. On account of the shortness of τ and τ' we may say however, that the two impulses coincide. It ought to be observed that, with a view to the appreciable duration τ' of effect $3b$, an attempt still further to shorten the time τ would be useless.

Counting the time from the moment of a positive reversal we may represent the couple acting on the cylinder in virtue of the first effect by a line as shown in fig. 4. The width of each of the peaks will be very small compared with the half period ($\frac{1}{2}T$)

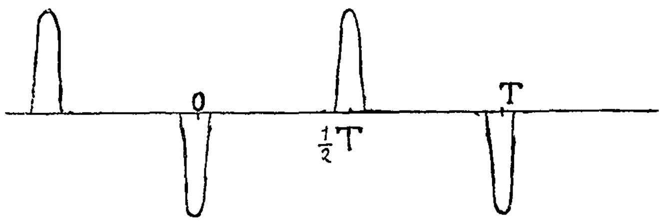


Fig.. 4.

of the pendulum. Now in developing the value of the couple in a FOURIER-series we may confine ourselves to the first term

$$-4\frac{S}{T}\cos 2\pi\frac{t}{T},$$

as it represents the only part of the couple whose action is intensified by resonance, where S denotes the impulse of rotation produced by *one* reversal.

The third effect ($3a$ and $3b$ taken together) is also represented by a term with $\cos 2\pi\frac{t}{T}$, the amplitude having the same or the opposite sign as that in the above expression, and we may therefore also use an expression of the form

$$p\cos 2\pi\frac{t}{T} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

for the effects (1,3) combined.

As to the second effect, we may represent the couple acting on the cylinder (the couple exerted on the magnet by the compensating coil being included) by fig. 5, where h may be as well

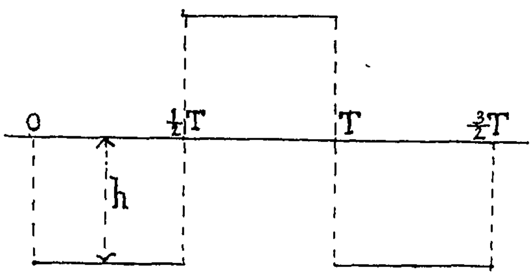


Fig. 5.

positive as negative. Developing in a series we are led to an expression of the form

$$q \sin 2\pi \frac{t}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\beta)$$

with positive or negative amplitude q.

It is seen by this, that the second effect is always a quarter of a period in phase behind effect (1, 3).

If all the three effects exist, we may replace (α) and (β) by a single alternating couple, which determines the motion of the cylinder. Its amplitude is $\sqrt{p^2+q^2}$, while its phase lies between those of (1, 3) and 2.

We may use this to separate the effects (1, 3) and 2. When, in effecting the compensation II, we come nearer and nearer to the right position of the coil K, the amplitude q and at the same time $\sqrt{p^2+q^2}$ will decrease. After the coil has passed through the position in question, $\sqrt{p^2+q^2}$ will increase again (the resonance in the 2nd effect being then caused by the action of the compensating coil on the small magnet, which now exceeds that of the residual field on the cylinder). If we succeed in reaching a minimum of resonance, the effect (1,3) will be free from 2 and this will be confirmed by the phase of the effect. Since the beginning when the second effect had the greatest influence till the minimum, when it has disappeared, the phase must have changed by a quarter of a period.

It remains to separate the effects 1 and 3 (a, b). For this purpose we may change by 180° the azimuth of the suspension arrangement of the cylinder. For the same state of magnetization the couple with which the residual field acts on it will then change its sign and the action of the compensating coil on the magnet fixed to the cylinder, if we leave the current in the coil unchanged, will likewise be reversed. So the compensation, when it has once been reached, will remain. But it is evident that both the effects $3a$ and $3b$ will be reversed. Both the remanent magnetism (OA in fig. 2) and the difference $M-M'$ (that is to say their horizontal components) are turned through 180°, while the remaining field has not changed.

It is clear that the change of azimuth does not influence the first effect. Thus we are able to make the effects 1 and $3(a, b)$ act in the same or in the opposite sense[1]).

[1]) In reality the turning by 180°, by which among other things the distance between the magnet AB and the coil K was altered more or less, required so many changes in the apparatus that after it a new compensation was necessary. This however alters nothing in the above considerations on the change of sign of effect $3(a, b)$, with which we are here concerned.

As to 1 and $3a$ it is easy to find out which of these two cases occurs. We need only consider the direction of the deflexion that is obtained, when the current caused by a positive reversal is sent through the windings of the compensating coil only and not through those of the cylinder.

Suppose e.g. the deflexion to be to the positive side. Then before the positive reversal, the current flowing through the coil has given rise to a couple in the negative direction acting on the magnet AB by the compensating coil. But that couple served to neutralize the action of the residual field on the magnetism of the cylinder. This action therefore had the positive direction and the same may be said of the action of the residual field on the remanent magnetism after a positive reversal. Thus effect $3a$ has a direction opposite to that of effect 1.

§ 7. *Description of the experiments.* First different alterations of the apparatus were tried in preliminary experiments, but though these gave clear indications of the effect we were seeking for, we did not obtain perfectly satisfactory results. Then however we succeeded in making a series of experiments, by which the existence of the effect became quite evident. At least I should think it difficult to interpret in another way the result I am going to describe. I regret that these observations could not be repeated, as they were made on the last day that was at my disposal for this work.

The period of the pendulum was found to be 1,856 sec., that of the cylinder 1,912 sec. The damping of the cylinder was so great that the amplitude diminished to half its original value in 39 half oscillations, while by means of an added known moment of inertia I found 0,814 gr. cm^2 for that of the cylinder.

The equation of motion of the cylinder for free vibrations is

$$\frac{d^2x}{dt^2} = -n_0^2 x - g\frac{dx}{dt},$$

where x is the angle of deflexion, and g the coefficient of resistance divided by the moment of inertia, while n_0 denotes the frequency of the free vibrations as it would be without damping. A solution of this equation is:

$$x = C\, e^{-\frac{1}{2}gt} \cos 2\pi \frac{t}{T_0},$$

if

$$T_0 = \frac{2\pi}{\sqrt{n_0^2 - \frac{1}{4}g^2}}$$

Let us now introduce an external force which produces an acceleration $-a\cos nt$. Then the forced vibrations are determined by the equation

$$\frac{d^2x}{dt^2} = -n_0^2\,x - g\,\frac{dx}{dt} - a\cos nt\ ^{1)}\,.$$

Its solution is

$$x = \frac{a}{q}\cos(nt+\varphi)\,, \qquad (\gamma)$$

if

$$n^2 - n_0^2 = q\cos\varphi\,, \quad ng = q\sin\varphi$$

From this one finds

$$\varphi = 11^\circ$$

and

$$\frac{1}{q} = \frac{1}{\sqrt{(n^2-n_0^2)^2+n^2g^2}} = 1.48.$$

If therefore there is a cause, which produces an acceleration alternating with the frequency n, the amplitude of this acceleration has to be multiplied by 1,48 in order to obtain the amplitude of the forced vibration [2]).

If now the acceleration is represented by the curve of fig. 4, so that the area of each of the peaks is a measure for the velocity S arising from one impulse, then on development in a FOURIER series the amplitude a of the first term is equal to $\frac{4S}{T}$ and that of the forced vibrations becomes

$$1{,}48\,.\,\frac{4S}{T} = 3\,19\ S \qquad (\delta)$$

We shall use this to determine the multiplication by resonance. If the cylinder when at rest suddenly receives a velocity S at the time 0, the first deviation is (here we may neglect the damping)

$$\frac{T_0}{2\pi}S = \frac{1{,}912}{2\pi}S = 0{,}30\ S$$

The effect is therefore multiplied somewhat more than 10 times.

In my experiments the period of the external force was smaller than that of the free vibration. As to the phase of the impressed vibrations, it might be supposed to be what it would have been without damping, or if this exists, at a large distance from the equality

---

1) In this equation x, $\frac{dx}{dt}$ etc. may be regarded as angle of rotation, angular velocity etc. but also as deflexion, velocity etc. on the scale.

2) If we wholly neglect the damping, we find

$$\frac{1}{q} = \frac{1}{n^2-n_0^2} = \frac{1}{4\pi\left(\frac{1}{T^2}-\frac{1}{T_0^2}\right)} = 1.51.$$

Thus the damping has an influence of a few percent on the amplitude-factor.

of frequency. Indeed, we can infer from the above value of φ, that the error made on this assumption is about 11°, i. e. less than $\frac{1}{30}T$.

The method, by which the phase was determined, was however not so accurate, that a difference of this amount could be observed. While I followed the oscillations of the cross-wire over the scale, a second person looked at the pendulum and gave a sign every time when it passed through the position of equilibrium in such a way that a positive reversal of the current took place. Now I observed, where the wire stood at these moments and in what direction it was moving. If we had to do with effect 1, the wire should have its greatest positive deviation at the moment of a sign. For we have seen, that at the instant of a positive reversal, the first effect produces an impulse in negative direction. We then found for the term in the FOURIER-series that is of importance here

$$- a \cos \frac{2\pi}{T} t$$

($a = \frac{4S}{T}$ being positive). From this and (γ) we infer, that, if φ is neglected, x has its greatest positive value for $t = 0$.

This result is confirmed by a simple reasoning. If external forces shall cause a body to oscillate more rapidly than it would under the action of the elastic force alone, they must drive it towards the position of equilibrium. Now at the instant of a positive reversal the effect we want to observe consists in an impulse in negative direction. If therefore the cylinder shall be driven towards the position of equilibrium, it must be on the positive side of it.

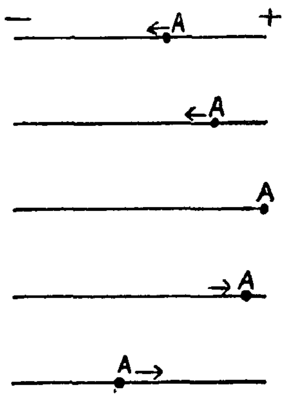
As has been said already effect 3 (a, b) has the same or the opposite phase as 1; if in the latter case it exceeded the first effect, we should see the cross-wire just at the negative end of its oscillation at the moment of a sign.

As to effect 2, this differs a quarter of a period in phase from 1; if it exists alone or preponderates, as was always the case before the application of compensation II, the wire will be in its position of equilibrium at the moment we consider, the direction of its motion depending on circumstances.

If all the effects exist at the same time an intermediate phase will be observed.

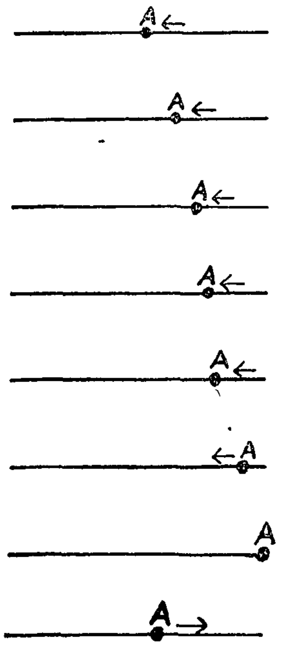
§ 8. *Description of the experiments of September* 4th. Intensity of the current in the coil S 29,5 milliampères. Every time before the determination of the deviation and the phase I waited during 80 to 100 full periods in order that all free oscillations might be extinct and the cylinder might perform its forced vibrations only.

At first the effects 1 and $3a$ had the same direction. Before the compensating coil K was put in action the amplitude amounted to several meters; but by means of compensation II this could be reduced to a few centimeters. Continually changing the compensation I made several determinations, some of which are given below. The straight lines represent the distance over which the cross-wire moves to and fro on the scale, the *right* hand side being the *positive* one. Beneath each line the length in cm is given. A always indicates the position of the wire at the moment of a sign and the arrow shows the direction in which it is then moving.



	Deviation
I	5 cm.
II	3 „
III	2 à 2.5 „
IV	2.2 „
V	4 „

Azimuth of the cylinder changed by 180°. Now the effects 1 and $3a$ are opposite to each other.



VI	15 cm.
VII	10.5 „
VIII	9 „
IX	6.5 „
X	4 „
XI	3 „
XII	0.7 „
XIII	5 „

§ 9. *Conclusions and remarks.*

1. The way, in which, both in the figures I—V and VI—XIII, the phases succeed each other agrees with the foregoing considerations. Simultaneously with the minimum of the amplitude I observed the phase in which, at the moment considered the wire is in its extreme position. The further we are away from the minimum, the more the phase approaches that of effect 2.

2. The change in phase that is observed at the passage through the minimum (II and IV, XI and XIII) is caused by our having gone too far with the compensation II. After having been neutralized, effect 2 reappears with opposite phase. It consisted initially in the action of the residual field on the horizontal component of the magnetization, and finally in the oppositely directed action of the compensating coil on the small magnet fixed to the cylinder.

3. In the experiments III and XII a minimum is reached while at the same time the phase is that of effect 1. Now in one of these cases effect 3 must have had the opposite phase, so that we may conclude to the existence of the effect we were seeking for. As in XII the amplitude (0,7 cm) was smaller than in III, this latter experiment must have been the one in which $3a, b$ and 1 had the same phase. It is seen by this that the total effect $3a, b$ has had the same phase as $3a$ alone.

4. For the minimum amplitude of the oscillation of the wire we may take 2 cm before the change in azimuth (see experiments III and IV) and afterwards 0,7 cm. The first value corresponds to the sum of effects 1 and 3, the last to the difference of these two effects. Therefore we may estimate the first effect at half the sum of the measured distances viz. 1,4 cm and the third effect at 0,6 or 0,7 cm. Of course the result 1,4 does not give much more than the order of magnitude.

5. Taking this value for the deflexion on the scale caused by effect 1 we may make an estimate of the magnetic moment of the cylinder.

According to (δ) half the distance over which the wire is seen to oscillate, is given by

$$3 \, . \, 19 S,$$

if S is the velocity produced by one sudden impulse. Hence

$$S = \frac{0.7}{3.19} = 0.22 \ {}^{\text{cm}}/_{\text{sec}},$$

which must be corrected to

$$S = 0.21 \ {}^{\text{cm}}/_{\text{sec}}.$$

on account of the inclined position of the scale. As the distance from the mirror to the scale is 315 cm, we find for the angular velocity due to one impulse

$$\frac{0.21}{2 \times 315} = 3.3 \times 10^{-4} \, {}^{1}/_{\text{sec}},$$

and (using the value 0,815 gr.cm² of the moment of inertia)

$$3.3 \times 10^{-4} \times 0.815 = 2.7 \times 10^{-4}$$

for the corresponding moment of momentum.

As this is due to the change of the magnetization from $-S$ into $+S$, the moment of momentum corresponding to the magnetization S is

$$1.35 \times 10^{-4}.$$

This [1]) must be equal to 1,13 $10^{-7} \times$ the magnetic moment of the cylinder, so that we find for this moment

$$\frac{1.35 \times 10^{-4}}{1.13 \times 10^{-7}} = 1200 \,.$$ [2])

A rough experimental determination gave the value

$$1400 \,.$$

6. Approximate evaluation of effect $3a$.

We observed that a deflexion of 9,2 cm on the scale was caused by a current that did not pass through the windings of the iron cylinder, but only through the compensating coil and its shunt and the total intensity (current in the coil + current in the shunt) of which was 124,5 m.A. The resistance of the shunt was 30 Ω, that of the coil 27 Ω. The night before the coil had been used for the compensation II in the same position and with the same shunt, but then the intensity of the current had been only 29,7 m.A. If therefore *that* current had acted on the cylinder, it would have produced the permanent deflexion

$$\frac{29.7}{124.5} \times 9.2 = 2.2 \text{ cm}.$$

The value of the remanent magnetism follows from the curve of fig. 2, where $OD : OK = 0.14$.

Thus, the remanent magnetism is 0,14 time CK.

The couple, with which the residual field acts on the (horizontal) magnetization has the same magnitude as that which gives rise to

1) A. Einstein and W. J. de Haas. loc. cit. p. 3.

2) The volume of the cylinder is 150 cm³, so that this calculation gives for the magnetization per unit of volume 800.

the deflexion of 2,2 cm. We may therefore say that there would have been a permanent deviation of $0{,}14 \times 2{,}2 = 0{,}31$ cm, if the couple with which the residual field acts on the remanent magnetism, had existed permanently.

The angular acceleration which this couple may produce is found by multiplication of the permanent deflexion by n_0^2 [1]). Thus we find for the acceleration on the scale

$$n_0^2 \times 0.31 = \frac{4\pi^2}{T_0^2} \times 0.31 = 3.3 \ {}^{cm}/_{sec.^2}.$$

Now the couple in question gives rise to effect $3a$, when it acts during $\frac{1}{80}$ sec. It therefore causes a sudden change of velocity of

$$\frac{3{,}3}{80} = 0.04 \ {}^{cm}/_{sec.}$$

As at each impulse the first effect gives rise to a velocity $0{,}21 \ {}^{cm}/_{sec}$, effect $3a$ has been about 5 times weaker than effect 1.

7. It must be remarked that our conclusions are drawn from the following phenomenon. By continually regulating compensation II the amplitude of the oscillation of the cylinder is made to pass through a minimum, while the phase is reversed. If it is 0 at first, it finally becomes $\frac{1}{2}\,T$, passing through $\frac{1}{4}\,T$ when the amplitude is at its minimum.

Now it might be thought, that this could be caused by effect 2 alone without the intervention of 1 or 3. Let us suppose, that the current in the compensating coil is first 0 and then increases to a value far above that which is required for the compensation II. Then it might be said, that there is initially a vibration λ, caused by the action of the residual field on the horizontal magnetism of the cylinder and that gradually a second vibration μ of opposite direction, caused by the action of the compensating coil on the magnet fixed to the cylinder, is added to λ.

Now if it might be supposed that the vibration μ had not exactly the opposite phase as λ, but a slightly different one, the value 0, would not be reached, but only a certain minimum and at that minimum the phase would differ $\frac{1}{4}\,T$ from that of λ or μ.

To this we may object that in reality, whatever be the position of the magnet and the axis of the compensating coil with respect to the horizontal magnetism of the cylinder and the residual field, the actions, which cause the vibrations λ and μ have exactly *opposite*

---

[1]) Because if the deflection is x, the elastic couple causes an acceleration $n_0^2 x$ (see the equation of motion).

phases. The observed phenomena can only be understood by assuming that besides λ and μ there is another vibration (1 and 3), which appears when λ and μ neutralize each other.

(As, on account of the self-induction, the actions giving rise to λ and μ do not begin quite simultaneously, *this* is a cause for a small difference in phase. But this is just what we have called 3*b*, so that it has been taken into account.)

These experiments were carried out in the physical laboratory of the TEYLER-Institute at Haarlem. I readily express my thanks to the Directors of the Institute and to the Curator of the Laboratory for their so kindly putting at my disposal the necessary apparatus.

Physics. — *"On the influence of alternating currents of decreasing intensity on the magnetisation of iron."* By Dr. G. J. ELIAS. (Communicated by Prof. Dr. H. A. LORENTZ).

(Communicated in the meeting of Jan. 29, 1916.)

1. It is generally supposed that magnetized iron is quite demagnetized when placed in a magnetic field of alternating direction, the intensity of which decreases to zero. It is however desirable to investigate this process more closely. As we must then necessarily use a definite representation about the magnetic state, we shall suppose, referring to the considerations of WEISS, that iron consists of a very great number of "elementary crystals". Of each crystal we shall suppose that its magnetic moment can be directed along one definite line only either in positive or in negative sense. We shall neglect the mutual actions of the neighbouring crystals. Further we shall idealise the case by supposing that there exists "complete" hysteresis, so that all magnetic moments keep their direction, when the magnetizing force has vanished.

In unmagnetized state the magnetic moments are equally distributed over all directions, so that the resulting moment of the iron is zero. Let us now suppose that an external magnetic force H is working and that a magnetic force h in a sense opposite to that of the magnetic moment of an elementary crystal is needed to let this moment change its direction. Then all those magnetic moments that are lying within a cone with a top equal to $2\, arc\cos \frac{h}{H}$ will do so and the corresponding elementary crystals contribute to the magnetisation of the iron.