*Citation:*

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nutrition and the period of incubation is produced by raising the protein- and the fat-content of the diet that evolves polyneuritis.

3. On the ground of the first conclusion the hypothesis of a neuritic poison in the diet, or a toxin, originating from the food in the intestinal canal, as probable causative factors of feeding-polyneuritis, must be abandoned, while probably no endogenous neuritic poison is in operation.

4. According as the food is composed it plays the principal part in preventing, but an inferior part in producing the disease.

5. Feeding, most likely causes an increased consumption of antineuritic substances. It seems hardly permissible to conclude with BRADDON and COOPER and with FUNK that carbohydrates promote the katabolism much more than, for instance, proteins do.

6. The digestion of polished rice in fowls is aided through the admixture of cellulose to the food, not however, through the addition of the antineuritic extract from ricepolishings. The admixture of cellulose may exert a favourable influence upon the condition of nutrition; it would seem, on the other hand, that the outbreak of the disease is accelerated by it rather than retarded.

7. The conclusions regarding the causative or the prophylactic agency of certain factors in polyneuritis gallinarum, are, in so far as they are based upon a shortening or a lengthening of the period of incubation, more or less doubtful on account of the considerable individual differences, exhibited by the animals experimented upon, even with a uniform treatment.

*Utrecht,* January 1916. *The Institute of Hygiene of the Utrecht University.*

**Physics.** — *On the Brownian Movement in Gases".* By Miss A. SNETHLAGE. (Communicated by Prof. J. D. VAN DER WAALS).

(Communicated in the meeting of February 26, 1916).

Among the different derivations of the deviation which a particle suspended in a gas or liquid obtains in a time $t$, there is one of VON SMOLUCHOWSKI [1]), in which only kinetic considerations are made use of. According to VON SMOLUCHOWSKI this derivation will only hold for gases, and that only when the dimensions of the particle are small with respect to the mean free path of the surrounding molecules. As the writer himself observes, there is still something

[1]) VON SMOLUCHOWSKI. Ann. der Phys. 21 p. 769 (1906).

wanting in the accuracy of his calculation. Among other things he assigns equal velocity to all the molecules. I will, therefore, derive by a somewhat different course an expression for the mean deviation, which only differs from the above mentioned one in numerical coefficient, but in which Maxwell's distribution of velocity is taken into account.

Let us suppose that a spherical "Brownian particle" with mass $M$ executes elastic collisions with a number of spherical molecules with mass $m$, the MAXWELLIAN distribution of velocity prevailing. We will call the particle shortly $M$.

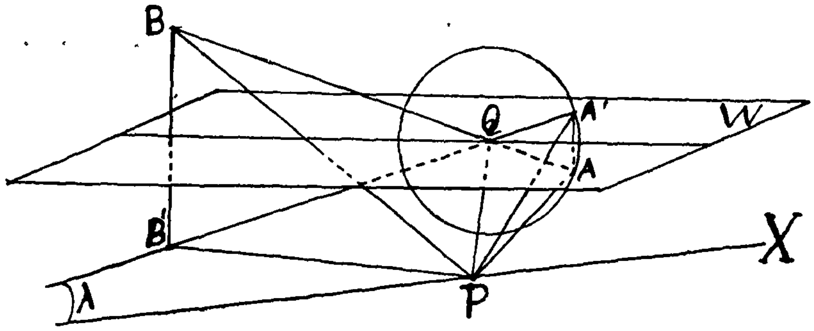


Fig. 1.

If we represent by $PA$ and $PB$ (fig. 1) the velocities of the particle and the colliding molecule before the collision, and bring a plane $W$ through $Q$, the end point of the velocity of the centre of gravity, parallel to the plane of collision, we shall find the velocities $PA'$ and $PB'$ after the collision by reflecting the velocities relative to the centre of gravity with respect to $W$ and by adding them to the velocity of the centre of gravity. When $W$ changes its position, $A'$ moves on a sphere round $Q$ with radius $QA$. MAXWELL [1]) showed that all the points of this sphere have equal probability for $A'$, so that the mean value of $QA'$ for all the collisions of $M$ in every direction is 0. The velocity of the centre of gravity, however, has on an average a projection in the direction of $PA$. From this JEANS [2]) derives that a molecule on collision with other similar ones on an average retains a part $\theta s$ of its original velocity $s$. For collisions between particles of different mass $\theta$ assumes a value which I have calculated, and which has already been communicated by Prof. KUENEN [3]).

1) J. C. MAXWELL. Phil. Mag. 4 19. p. 19 (1860).
2) J. H. JEANS. Phil. Mag. (6) 8 p. 700. (1904).
3) J. P. KUENEN. These Proc. XVII, p. 1068.

For a particle $M$, in statistical equilibrium with molecules $m$, this factor amounts to:

$$\theta_M = \frac{1}{2}\frac{M}{M+m} + \frac{1}{4}\frac{M^2}{m(M+m)}\sqrt{\frac{m}{M+m}}\,log\left(1+2\frac{m}{M}+2\frac{m}{M}\sqrt{\frac{M+m}{m}}\right)$$

If $\frac{M}{m}$ is large (and in the case under consideration this is so) we may develop $\theta$ to the first power of $\frac{m}{M}$, which causes the above expression to become simplified to

$$\theta_M = 1 - \frac{4}{3}\frac{m}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let us now call the velocity of the particle at a given moment $s_1\,(u_1\,v_1\,w_1)$. When during a time $\tau_1$ it has travelled with this velocity, it collides with a molecule with velocity $s'_1\,(u'_1\,v'_1\,w'_1)$, and gets therefore itself a velocity $s_2$. This again consists of 2 parts:

1. The velocity of the centre of gravity, on an average $= \theta_M s_1$.
2. A component that can have all possible directions, is 0 on an average, and whose value $= \frac{m}{M+m} V_1$, when $V_1$ represents the relative velocity of the particle and the molecule before the collision ($AB$ of fig. 1).

For $u_2$ we can now on an average draw up the relation:

$$u_2 = \theta_M u_1 + \frac{m}{M+m} V_1 \cos\lambda_1 . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$\lambda_1$ is the angle between the $x$-axis and the relative movement after the collision.

In $\tau_1$ a distance along the $x$-axis is passed over:

$$s_1 = u_1 \tau_1$$

in $\tau_2$

$$s_2 = u_2 \tau_2 .$$

In a time $t = \sum_1^{\nu} \tau_k$ this distance amounts to:

$$\Delta = \sum_1^{\nu} s_k = \sum_1^{\nu} u_k \tau_k .$$

If we now suppose that always a time $\tau$ passes between two collisions, we can bring this quantity outside the $\Sigma$ sign, and write:

$$\Delta = \tau \sum_1^{\nu} u_k .$$

We further follow the method applied by EINSTEIN and HOPF [1]) to another problem, by Mrs. DE HAAS—LORENTZ [2]) to the Brownian movement, but with a slight abbreviation. We substitute $R_1$ for $\frac{m}{M+m} V_1 \cos \lambda_1$ and write equation (2) in the form:

$$u_2 - \theta_M u_1 = R_1$$

$$u_3 - \theta_M u_2 = R_2.$$

This gives on summation over $\nu$ intervals $\tau$, when $t = \nu\tau$:

$$\sum_2^{\nu+1} u_k - \theta_M \sum_1^{\nu} u_k = \sum_1^{\nu} R_k$$

or

$$\frac{\Delta}{\tau}(1-\theta_M) + u_{\nu+1} - u_1 = \sum_1^{\nu} R_k.$$

If we increase $t$, the absolute value of the first term of the first member increases in general, $u_{\nu+1} - u$, however, does not. If $t$ is sufficiently long we may neglect these terms and write:

$$\frac{\Delta}{\tau} = \frac{\sum_1^{\nu} R_k}{1-\theta_M}.$$

If we raise both members to their square and take the mean for a great number of $t$'s, this becomes:

$$\frac{\overline{\Delta^2}}{\tau^2} = \frac{\overline{(\Sigma R)^2}}{(1-\theta_M)^2} \quad \ldots \ldots \ldots \quad (3)$$

The $R$'s are independent of one another, hence:

$$\overline{(\Sigma R)^2} = \nu \overline{R^2}$$

$\overline{R^2}$ is still unknown, but can be calculated in the following way.

We raise both members of equation (2) to their square, after we have first written it in the form:

$$u_k = \theta_M u_{k-1} + R_{k-1}.$$

If we now also take the mean for a great number of times $\tau$, we have:

$$\overline{u^2} = \theta^2_M \overline{u^2} + 2\,\theta_M \overline{u_{k-1} R_{k-1}} + \overline{R^2}$$

$u_{k-1}$ and $R_{k-1}$ are mutually independent, and equally often positive as negative, hence

$$\overline{u_{k-1} R_{k-1}} = 0$$

and

[1]) EINSTEIN and HOPF. Ann. der Phys. 33 p. 1105 (1910).

[2]) G. L. DE HAAS—LORENTZ. Die Brownsche Bewegung. (1913) p. 51 et seq.

$$\overline{R^2} = \overline{u^2}\,(1 - \theta^2{}_M) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This value introduced into equation (3) gives:

$$\overline{\Delta^2} = \nu\tau^2 \frac{1-\theta^2{}_M}{(1-\theta_M)^2}\,\overline{u^2} = \frac{1+\theta_M}{1-\theta_M}\,\overline{u^2}\,\tau\,t\,.$$

If we now substitute the value (1) for $\theta$ and put the value 2 for $1+\theta$, we get finally:

$$\overline{\Delta^2} = \tfrac{3}{2}\,\frac{M\overline{u^2}}{m}\,\tau\,t = \tfrac{3}{2}\,\frac{RT}{Nm}\,\tau\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This formula differs from that of VON SMOLUCHOWSKI only in numerical coefficient. Where we find $\frac{3}{2}$, v. S. puts $\frac{64}{27}$ or with a somewhat different derivation 3.

Another calculation [1]), which was not founded on purely kinetic considerations led to quite the same expression as equation (5). It was asserted there that there was no reason to confine the validity of this formula to those cases in which the dimensions of $M$ are small with respect to the mean free path of the molecules.

If with MAXWELL [2]) we now substitute the value

$$\tau = \frac{1}{2\sqrt{2\pi}}\,\frac{1}{\sigma^2 n}\sqrt{\frac{Mm}{M+m}}\sqrt{\frac{N}{RT}}$$

for $\tau$, in which $\sigma$ represents the sum of the rays of the particle and a molecule, and therefore may be replaced by $a$, the radius of the particle, while $n$ denotes the number of molecules per ccm., (5) reduces to:

$$\overline{\Delta^2} = \frac{3}{4\sqrt{2\pi}}\,\frac{1}{\varrho a^2}\sqrt{\frac{RTm}{N}}\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which $\varrho = n$, $m =$ the density of gas.

$\Delta$ is the projection of the path passed over in an arbitrary direction. To find the mean square of the total path passed over we must multiply (6) by 3.

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1) J. D. V. D. WAALS JR. and A. SNETHLAGE. These Proc. XVIII, p. 1322.

2) J. C. MAXWELL. l. c.