rapidly, on the other hand very slowly in glacial acetic acid or chloroform[1]); the reaction may, however be accelerated under the influence of daylight.

Maleic acid behaves in quite an analogous manner; the velocity of absorption, however, (as might be expected from the constitution) is *greatest with glutaconic acid.*

From the material communicated in this treatise I believe it may be safely concluded that the symmetric formula of THORPE is indeed a fairly proper interpretation of the properties of glutaconic acid.

In the following communication, I hope to elucidate this formula with a model.

*Delft*, February 15, 1916.

**Chemistry**. — *"In-, mono- and divariant equilibria."* VIII. By Prof. F. A. H. SCHREINEMAKERS.

(Communicated in the meeting of March 25, 1916).

12. *Further consideration of the bivariant regions; the turning lines.*

The different properties of the curves and the regions, which we have deduced in the previous communications, are only true under some conditions, which we have up to now assumed silently. They are valid viz. not only in the immediate vicinity of the invariant point, but still also at some distance, viz. under the conditions:

1. the points under consideration must not be situated in the $P,T$-diagram too far from the invariant point; consequently the $P$ and $T$ of the equilibria under consideration must not differ too much from the $P$ and $T$ of the invariant point;

2. the compositions of the occurring phases must not differ too much from the compositions, which they have in the invariant point.

Further we shall indicate somewhat more exactly what is the meaning of "not too far" and "not too much" in these conditions.

As long as those conditions are satisfied, the deduced properties remain valid; when they are not satisfied, deviations may occur.

When all phases have a constant composition, the latter condition is always satisfied; this should be the case in fig. 1 (II) for instance when one of the phases represents watervapour and the others

[1]) The bromination in chloroform in sun-light is the best way of preparing the $\alpha\beta$-dibromoglutaric acid. There are always formed, however, small quantities of by products (probably higher brominated ones).

salts or their hydrates. However also phases with variable composition may occur e.g. a vapour, which contains two or more of the components, solutions or mixed crystals.

Let us take in fig. 1 (II) the simple case that the phases 1, 2, 3 and 4 have a constant composition, e.g. that they are salts; we take a solution for the phase 5.

Now we take the bivariant equilibrium 235 and we go in fig. 2 (II) from the invariant point towards a point of the region 235. As the $P$ and $T$ of the equilibrium 235 have changed now, the solution 5 will obtain, therefore, also another composition; consequently point 5 alters its place in fig. 1 (II).

Hence it is apparent that in each point of the region 235 the phase 5 has no more the composition, represented by point 5 in fig. 1 (II), but it has another composition; it appears also that this composition changes from point to point. Of course the same is also true for other phases with changeable composition. Hence it is apparent, therefore, that the composition of the changeable phases in fig. 2 (II) changes from point to point, generally so much the more in proportion as we remove further from the invariant point. Only in the invariant point itself, all phases have the same composition as is expressed in fig. 1 (II).

These changes in the compositions of the phases may also cause radical alterations in the partition of the regions.

Let us take again the case that in fig. 1 (II) only the phase 5 has a changeable composition. Now we may imagine that in fig. 1 (II) point 5 takes its place on the line 23 e.g. between 2 and 3; then between these phases the reaction $2 + 3 \rightleftarrows 5$ may occur.

On further change of $P$ and $T$ point 5 may come now within the triangle 234. This involves that the reaction between the phase changes in some of the monovariant equilibria.

Let us take as an example the equilibrium $(1) = 2 + 3 + 4 + 5$; as long as the point 5 is situated outside triangle 234, the phase-reaction in this equilibrium and the partition of the regions are:

$$\left.\begin{array}{c} 2 + 3 \rightleftarrows 4 + 5 \\ \begin{array}{c|c} 234 & 245 \\ 235 & 345 \end{array} \end{array}\right\} \quad \ldots\ldots\ldots \quad (1)$$

As soon as the point 5 comes however within triangle 234, we find:

$$\left.\begin{array}{c} 2 + 3 + 4 \rightleftarrows 5 \\ \begin{array}{c|c} 234 & \begin{array}{c} 235 \\ 245 \\ 345 \end{array} \end{array} \end{array}\right\} \quad \ldots\ldots\ldots \quad (2)$$

When in fig. 2 (II) the equilibrium $(1) = 2 + 3 + 4 + 5$ traces curve (1) starting from the invariant point, then consequently the partition of the regions is first indicated by reaction (1). Therefore, as is also drawn in fig. 2 (II), towards the one side the regions 234 and 235, towards the other side the regions 245 and 345 start from curve (1).

When on curve (1) we remove further from the invariant point, then instead of reaction (1) now reaction (2) may occur. The region 235 will no more go now from this part of the curve (1) towards the right as is drawn in fig. 2 (II), but it will go towards the left. Consequently this region will show a peculiarity, to which we shall refer later.

When the equilibrium (1) traces curve (1) in fig. 2 (II), then point 5 traces in the concentration-diagram a curve, which we shall call curve $5^{(1)}$; when the other phases have also a changeable composition, then each of them also follows a curve. The phases 2, 3, 4 and 5 of the equilibrium (1) follow in fig. 1 (II), therefore the curves $2^{(1)}$, $3^{(1)}$, $4^{(1)}$ and $5^{(1)}$. By this the quadrangle 2345 may be deformed in different ways, so that the reaction in the equilibrium (1) can change in many ways.

When the equilibrium $(2) = 1 + 3 + 4 + 5$ follows in the $P,T$-diagram curve (2), then each of the points 1, 3, 4, and 5 follows a curve $1^{(2)}$, $3^{(2)}$, $4^{(2)}$ and $5^{(2)}$ in the concentration-diagram.

As the same is also true for the other equilibria (3), (4) and (5), four curves start, therefore, from each of the points 1, 2, 3, 4 and 5 in the concentration-diagram. Hence it is apparent, therefore, that at some distance from the invariant point in the $P,T$-diagram, several changes in form of the quadrangles of the concentration-diagram may occur, by which the partition of the regions in the $P,T$-diagram is changed. We call this the deformation of the regions.

In order further to elucidate those considerations, we take a simple example viz. a ternary system in the invariant point of which the phases:

watervapour $= G$, solution $= L$ and the salts $Z_1$, $Z_2$ and $Z_3$ occur.

We assume that those phases are situated with respect to one another as in fig. 1. Now we have the monovariant equilibria:

$$(Z_1) = Z_2 + Z_3 + L + G; \quad (Z_2) = Z_1 + Z_3 + L + G; \quad (Z_3) = Z_1 + Z_2 + L + G$$
$$(L) = Z_1 + Z_2 + Z_3 + G; \quad (G) = Z_1 + Z_2 + Z_3 + L.$$

In fig. 1 only three of these equilibria are drawn; curve $La$ represents $(Z_3)$ consequently the saturationcurve of $Z_1 + Z_2$ under its own vapour-pressure; curve $Lb$ represents $(Z_1)$ and curve $Lc$ represents $(Z_2)$. Consequently curve $Lb$ is the saturationline of $Z_2 + Z_3$

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and curve $Lc$ the saturationline of $Z_1 + Z_3$ under its own vapour-pressure. As long as those curves do not come too close to the triangle $Z_1\ Z_2\ Z_3$, the vapour-pressure of those equilibria increases with the temperature; we shall assume this also here.

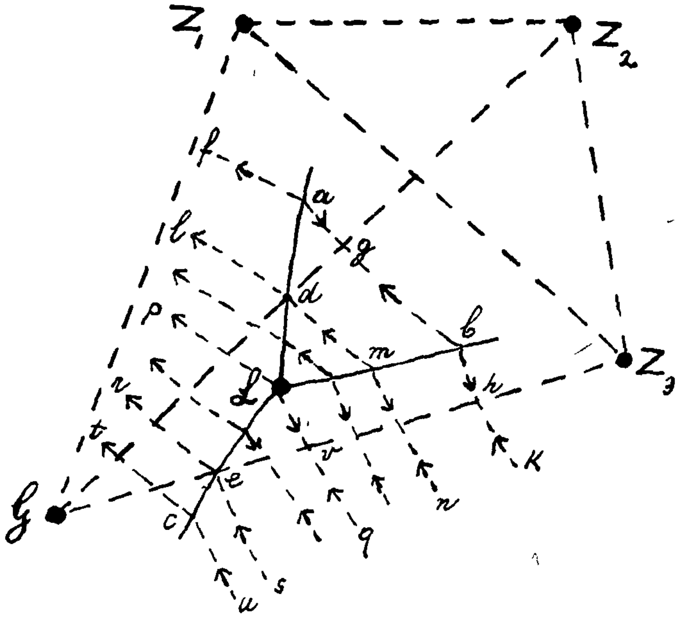

Fig. 1.

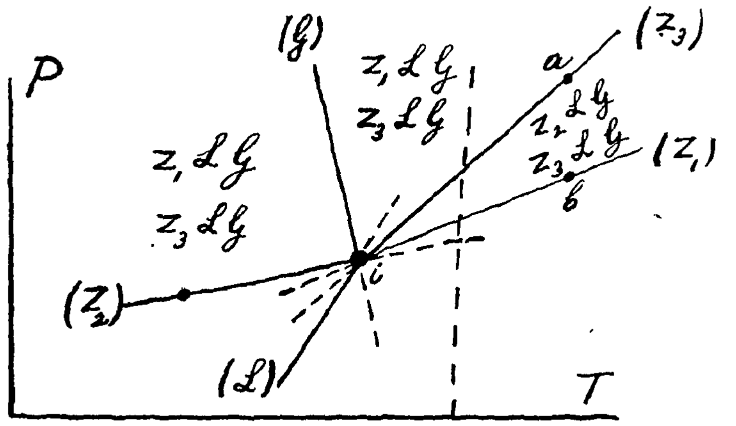


Fig. 2.

The $P,T$-diagram is drawn in fig. 2; we may deduce it easily in different ways. Of course it satisfies also the rule of the diagonal order of succession. The phases form viz. in fig. 1 a monoconcave quintangle [as in fig. 3 *(II)*] with the sides $GZ_1$, $Z_1Z_2$, $Z_2Z_3$, $Z_3L$ and $LG$. When we trace this quintangle in diagonal direction, then we find the same order of succession as the curves in fig. 2.

It appears from a comparison of figs. 3 $(II)$ and 4 $(II)$ [we have to bear in mind that the figs. 4 (II) and 6 $(II)$ have to be interchanged] that $(Z_1)$ and $(Z_3)$ and also $(Z_2)$ and $(G)$ must form a bundle.

Now we draw in the $P,T$-diagram only the regions which contain liquid and vapour. The region $Z_1$ $L$ $G$ is situated between its limiting-curves $(Z_2)$ and $(Z_3)$; the region $Z_2$ $L$ $G$ between $(Z_1)$ and $(Z_3)$; the region $Z_3$ $L$ $G$ between $(Z_1)$ and $(Z_2)$.

Now we draw a vertical line in the $P,T$-diagram; this is dotted in fig. 2. As far as this line is situated in the region $Z_1$ $L$ $G$, it represents the equilibrium $Z_1$ $L$ $G$ at constant temperature, consequently all the solutions which are saturated with solid $Z_1$ at that temperature under their own vapourpressure. In the concentration-diagram (fig. 1) these solutions are represented by a curve, "the saturationcurve of $Z_1$ under its own vapourpressure". [For a fuller examination of these curves confer the communications I—XVIII over "ternary equilibria"].

In fig. 1 curve $fa$ represents the solutions, which are saturated at the temperature $T_a$, curve $dl$ the solutions which are saturated at $T_d$, curve $Lp$ the solutions saturated at $T_L$ with solid $Z_1$ under their own vapourpressure etc. All dotted curves, which proceed in fig. 1 starting from $cL$ and $La$ towards the left, are, therefore, saturationcurves of $Z_1$ under their own vapourpressure. All dotted curves between $La$ and $Lb$ are saturationcurves of $Z_2$, under their own vapourpressure for instance the curves $agb$ and $dm$. All dotted curves, going to the right starting from $cL$ and $Lb$ are saturationcurves of $Z_3$ under its own vapour-pressure. In fig. 1 the bivariant region $Z_1LG$ is situated, therefore, at the left of the curves $cL$ and $La$, the bivariant region $Z_2LG$, therefore, between the curves $La$ and $Lb$; the bivariant region $Z_3LG$ at the right of the curves $cL$ and $Lb$.

The regions $Z_1LG$ and $Z_3LG$ exist, therefore, in stable condition, as well above as below the temperature $T_L$ of the invariant point; the region $Z_2LG$, however, only above this temperature. This is also in accordance with the $P,T$-diagram; herein a line parallel to the $P$-axis, intersects at temperatures below $T_i$ only the regions $Z_1LG$ and $Z_3LG$; at temperatures above $T_i$ besides those also the region $Z_2LG$.

Previously we have deduced that a saturationcurve under its own vapour-pressure shows a point of maximum- and a point of minimum-pressure; these points are situated on the conjugation-line solid-vapour. On the parts of these curves, drawn in fig. 1 only points of maximum-pressure occur. [Compare this figure with fig. 1 in Communication XV on "Equilibria in ternary systems"]. The saturation-curves under their own vapour-pressure of $Z_3$ have, therefore, their point of maximum-pressure on the line $GZ_3$. Consequently on curve $bK$ the pressure has to increase in the direction of the little arrows and it must be a maximum in $h$. The same applies to the other curves of the region $Z_3LG$. On curve $cu$, however, no maximum of pressure occurs; this is metastable here. As it must, however, be situated on the line $GZ_3$, it follows that the pressure has to increase from $u$ towards $c$.

In the region $Z_2LG$ the curves must have their point of maximum-pressure on the line $GZ_2$, in the region $Z_1LG$ on the line $GZ_1$; hence it follows that the pressure increases along the curves in the direction of the arrows.

Let us consider now the region $Z_2LG$. At a change of $T$ and $P$ in fig. 1 the phases $Z_2$ and $G$ remain unchanged in place, the solution $L$ however traces the region between the curves $La$ and $Lb$. Triangle $Z_2LG$ may, therefore, have its angle-point $L$ sometimes on the one side, sometimes on the other side of the line $GZ_2$ and casually on this line.

In the $P,T$-diagram (fig. 2) this same region is situated between the curves $ia$ and $ib$; in fig. 3 this region is drawn once more with its limiting curves $(Z_1) = ib$ and $(Z_3) = ia$. We take in this figure a point $m$ on $ib$ and on $ia$ a point $d$ corresponding with the points

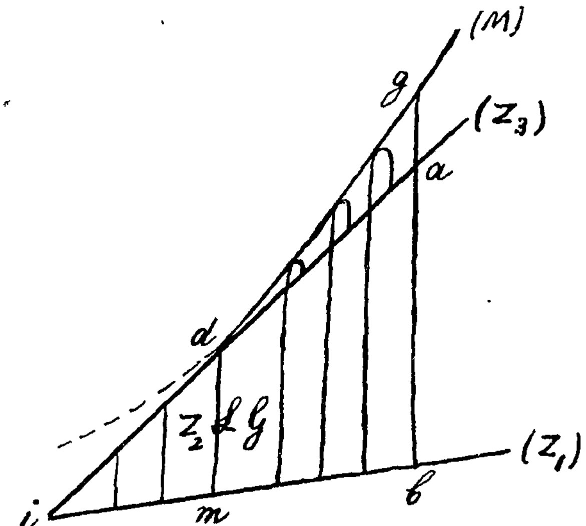


Fig. 3.

$m$ and $d$ of fig. 1. As $T_d = T_m$, in fig. 3 the line $dm$ must be parallel to the $P$-axis. The same applies to the line $ab$, when $a$ and

$b$ are the same points in fig. 3 as in fig. 1. In general viz. each saturation-curve under its own vapour-pressure from fig. 1 is represented in the $P,T$-diagram (figs 2 and 3) by a line parallel to the $P$-axis.

Now we take a temperature $T_x$ between $T_L$ and $T_d = T_m$. The saturation-curve under its own vapour-pressure of $Z_2$ is situated at this temperature $T_x$ in fig. 1 between point $L$ and curve $dm$. When we follow this curve, starting from a point on $Lb$, then the pressure increases, as is apparent from fig. 1. This is in accordance with fig. 3, in which curve $id$ is situated above curve $im$. Hence it follows that each point of the region $Ldm$ of fig. 1, must be situated in fig. 3 within the region $idm$. Region $Ldm$ of fig. 1 is, therefore, represented in fig. 3 by region $idm$.

Let us now take a temperature $T_y$, higher than $T_d$, for instance $T_y = T_a = T_g = T_b$ (fig. 1). On curve $agb$ the pressure increases as well if we start from $a$ as from $b$, it reaches its maximum in $g$. In fig. 3 the point $g$ must be situated, therefore, not only above point $b$, but also above point $a$. The region $Z_2LG$ covers in fig. 3, therefore, not only the line $ab$, but also the line $ag$; consequently it extends over the point $a$. It appears from fig. 1 that a similar extension occurs for each temperature $T_y$ higher than $T_d$.

Starting from curve $mb$ (fig. 3) the region $Z_2LG$ finishes, therefore, not at once in curve $da$; it extends viz. first over this curve $da$. up to a curve $dg$, then it turns to curve $da$, in order to finish in this curve. We call $dg$ the turning-line of the region $Z_2LG$. We may imagine, therefore, the region $Z_2LG$ between the parts of the curves $mb$ and $dg$, as consisting of two leaves, the one of which starts from $mb$ and the other from $da$, they pass into one another in the turning-line $dg$. Between the curves $da$ and $dg$ those leaves cover one another. In order to represent this reversion of the region in fig. 3 some lines have been drawn which unite a point of $da$ with a point of $mb$ and which touch the turning-line.

The region $Z_2LG$ starts, therefore, from $id$ towards the right, from $da$ however towards the left, after having reached the turning-line, it goes, however, again towards the right.

The turning-line $dg$ from fig. 3 corresponds of course with the line $dg$ from fig. 1. In the communications on equilibria in ternary systems several of these lines have been discussed in detail under the general name of $M$-curve. I only mention here, that it touches curve $ia$ in $d$ and continues further, but then in metastable condition.

When we consider the equilibrium $Z_2LG$ in its whole extension, viz. without taking into consideration which parts are stable or metastable, then each leaf of this region extends itself over the curves $(Z_1)$ and $(Z_3)$. The turning-line of this region has a form, as curve $xyzu$ in fig. 5; we imagine the curves $ia$ and $ib$ entirely within this turning-line and the point of contact $d$ from fig. 3 between $x$ and $y$ in fig. 5.

In fig. 3 we now have an example of that which we have called above the deformation of a region; we see that it is connected here with the occurrence of point $d$, in fig. 1 the point of intersection of curve $La$ with the line $GZ_2$.

In the invariant point, and also as long as the liquid of the equilibrium $(Z_3) = Z_1 + Z_2 + L + G$ is represented by a point of curve $Ld$, the reaction in this equilibrium $(Z_3)$ is:

$$Z_1 + L \rightleftarrows Z_2 + G$$

When, however, the liquid is represented by the point $d$, then the reaction is:

$$L \rightleftarrows Z_2 + G$$

and when the liquid is represented by a point of $da$:

$$L \rightleftarrows Z_1 + Z_2 + G.$$

When the equilibrium $(Z_3)$ therefore follows, the curve $La$, then the phase-reaction gets another form in the point $d$. As it appears from fig. 3 in the $P,T$-diagram the deformation of the region begins in the point $d$.

Previously we have deduced: each region, which covers a curve $(F_p)$, contains the phase $F_p$. In fig. 3 the region $Z_2LG$ covers, however, the curve $(Z_3)$ [viz. the part $da$] and yet this region does not contain the phase $Z_3$. When we bear in mind the first condition, viz. that we are allowed to consider regions only, which are situated not too far from the invariant point, then this region $Z_2LG$ does not cover the curve $(Z_3)$.

We may imagine the point $d$ indeed in the vicinity of $i$, but not coinciding with it. For, in this case in fig. 1 the point $L$ would coincide with point $d$; three of the five phases of the invariant equilibrium, viz. $G$, $L$ and $Z_2$ should then be situated on a straight line, so that the invariant equilibrium should show a particularity which we have excluded up to now. For, in the three types of concentration-diagrams which are represented in the figs. 1 (II), 3 (II) and 5 (II), no three points are situated on a straight line. When this is the case, then we have a transition-type, to which we shall refer later.

We shall also show that also the second condition, mentioned above, has a meaning in some cases.

For this we consider the bivariant region $Z_3LG$. In the $P,T$-

diagram (fig. 2) this region is situated between the curves *ic* and *ib* and it extends over the curves ($G$) and ($Z_3$). In fig. 4 this region is drawn once more with its limit-curves; the other curves have been omitted.

Now we take a temperature $T_x$ higher than $T_e$ (fig. 1). When we take $T_x = T_b$ then the saturation-curve under its own vapour-pressure of $Z_3$ is represented in the concentration-diagram (fig. 1) by curve *bhk* and in the *P,T*-diagram (fig. 4) by the line *hbk*, parallel to the *P*-axis. It is apparent from fig. 1 that the pressure in the point *h* is higher than in *b*, in fig. 4 *h* must, therefore, be situated higher than *b*. When we take $T_x = T_L = T_i$, consequently the temperature of the invariant point, then the saturationcurve under its own vapourpressure of $Z_3$ is represented in fig. 1 by curve *Lvq*, in fig. 4 by the straight line *viq*. As it is apparent from fig. 1

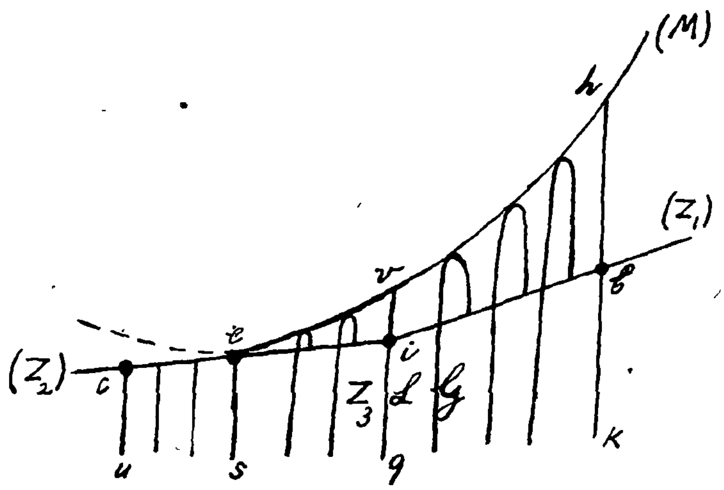


Fig. 4.

that the pressure is higher in *v* than in point *L*, in fig. 4 the point *v* must, therefore, be situated higher than *i*. As this is valid for each temperature $T_x$, higher than $T_e$, the region $Z_3LG$ must have, therefore, a turning-line which is represented in fig. 4 by curve *evh*.

Now we take a temperature $T_y$, lower than $T_e$ e.g. $T_y = T_c$. In fig. 1 the saturation curve under its own vapourpressure is represented by *cu*; hence it is apparent that the pressure decreases starting from *c*, so that in fig. 4 the point *u* must be situated below *c*.

The bivariant region $Z_3LG$ has, therefore, quite another form above $T_e$ than below $T_e$. Below $T_e$ it falls viz. starting from its limit-curve [consequently from the part $ce$ on curve $(Z_3)$]; above $T_e$ it rises starting from its limit-curves first up to its turning-line $evh$ and afterwards it falls. This is represented again in fig. 4 by some lines, which touch the turning-line. Below $T_e$ the region consists, therefore in stable condition of one single leaf only, above $T_e$, however, it consists of two leaves. The one falls starting from the turning-line and it finishes in the curves $ei$ and $ib$; the other falls also starting from the turning-line, but it extends moreover below the curves $ei$ and $ib$.

When we consider the region $Z_3LG$ in its whole extension then we may again represent the turning-line by curve $xyzu$ from fig. 5; we imagine the curves $ib$ and $ic$ in fig. 4 within this turning-line and the point of contact anywhere on branch $xy$ of the turning-line.

Here we have a deformation of a region, more important than in fig. 3. The region covers here, viz. its limit-curves $(Z_1)$ and $(Z_2)$ already in the vicinity of the invariant point, which is not the case in fig. 3. Also we see that this region does not occupy in fig. 4 the whole space between the curves $(Z_1)$ and $(Z_2)$, but a part only. Consequently this is different to that which we should mean to be allowed to deduce from fig. 2. Also several other properties seem to be no more valid now. When we take e. g. the rule: each region which covers a curve $(F_p)$ contains the phase $F_p$; the region $Z_3LG$ covers here viz. the curves $(Z_1)$ and $(Z_2)$ and yet it contains neither the phase $Z_1$ nor $Z_2$. Also the property: a regionangle is always smaller than 180° seems to be no more true now; the region $Z_3LG$ extends itself viz. in fig. 4 over the invariant point $i$, so that the region-angle is 360°.

All those contradictions disappear, however, when we take into consideration the conditions 1 and 2.

When we consider viz. in accordance with the first condition, only pressures and temperatures, which differ a little only from those of the invariant point or in other words, when we take from the curves $(Z_1)$ and $(Z_2)$ only parts in the vicinity of the point $i$, then the region $Z_3LG$ occupies indeed the space between the curves $(Z_1)$ and $(Z_2)$.

The other contradictions disappear when we take into consideration the second condition; this is apparent from the following.

When we take away from fig. 4 the leaf $cevhkqsu$, so that the leaf $evhbi$ remains only then all contradictions have disappeared. The region-angle is then smaller than 180° and the region $Z_3LG$ covers no more its limit-curves $(Z_1)$ and $(Z_2)$.

The liquids of the remaining region $evhbi$ in fig. 4 are represented in fig. 1 by points of $evhbL$; hence it is apparent that all properties are true again now, as long as the liquid of the equilibrium $Z_3LG$ is represented by a point of $evhbL$.

Consequently the liquid is allowed to change its composition only starting from $L$ (fig. 1) up to the line $eZ_3$; correspondingly on this line the equilibrium shows something particular; here the triangle $Z_3LG$ passes viz. into a straight line.

In our previous considerations we have assumed everywhere that each point of a region $(F_1F_2)$ represents one single bivariant equilibrium $(F_1F_2)$ only. This is also the case when we take in fig. 3 a point of the region $Z_2LG$ between the curves $ia$ and $ib$; no more, however, when this point is situated between $da$ and $dg$. Then it represents two equilibria $Z_2LG$, which differ from one another by the composition of the liquid $L$. The liquid of the one equilibrium is situated in fig. 1 at the one side, that of the other equilibrium at the other side of the line $dZ_2$.

In fig. 4 each point of the region $Z_3LG$, which is situated within $evhbi$ represents two equilibria $Z_3LG$; the liquid of the one equilibrium is situated in fig. 1 at the one side, that of the other equilibrium at the other side of the line $eZ_3$. Also, however near to the point $i$ we take this point within $evhbi$, yet it always represents two different equilibria. The point $i$ itself represents still two different equilibria; in the one equilibrium the liquid has the composition, indicated by point $L$ in fig. 1; in the other equilibrium the liquid is situated somewhere on $vq$.

Hence it is apparent that this property is true again when we take into consideration in fig. 3 the first condition and in fig. 4 the second.

After this discussion of some examples, we shall now consider the general case. For this we take the field

$$(F_1F_2) = F_3 + F_4 + \dots F_{n+2}$$

first in its whole extension, consequently without taking into consideration which parts are stable or metastable. When all phases have an unvariable composition, then nothing particular can take place on change of $P$ and $T$; this is the case, however, when one or more phases with variable composition occur. We take from the equilibrium $(F_1F_2)$ a complex $X$ and we change the pressure at constant $T$ or the temperature under constant $P$. Now the phases of this complex change their composition; we may imagine that at a certain moment between them a phase-reaction becomes possible. This is the case e.g. when in a binary system two points coincide which are first situated separated from one another; when in a ternary system three points, which first form a triangle, take their place on a straight line, when in a quaternary system four points, which first form a tetrahedron, fall in a plane; in general when between the phases of the equilibrium a reaction:

$$a_3F_3 + a_4F_4 + . \; . + a_{n+2}F_{n+2} = 0 \quad . \; . \; . \; . \; . \; (3)$$

can occur.

[We might imagine the phases of $(F_1F_2)$ also in such a way that they satisfy (3) in the invariant point not always, but casually. In both cases the phases have then already something particular in the invariant point. The corresponding $P,T$-diagram forms then a transition-type, to which we shall refer later].

When between the phases of the complex $X$ reaction (3) can occur, then at constant $T$ the pressure —, and under constant $P$ the temperature is for this complex a maximum or minimum.

When the temperature is a maximum (minimum) under constant $P$, then the complex $X$ no more exists above (below) this temperature; below (above) this temperature then however at each $T$ two equilibria $X'$ and $X''$ may occur, in which the variable phases have different compositions. When the pressure is a maximum (minimum) at constant $T$, then the complex $X$ no more exists under higher (lower) pressures; under lower (higher) pressures however two different equilibria $X'$ and $X''$ occur again.

Hence it is apparent that the bivariant field $(F_1F_2)$ is limited by a curve $(M)$ which is defined, because in the equilibrium:

$$(M) = (F_1F_2) = F_3 + F_4 + \ldots F_{n+2}$$

reaction (3) occurs.

Each point of this region $(F_1F_2)$ represents, therefore, two different equilibria $(F_1F_2)'$ and $(F_1F_2)''$ which pass into one another at the limit of this field. Curve $(M)$ is, therefore, the turning-line of this field. Consequently the field consists of two leaves, which cover one another and which we shall call leaf $(F_1F_2)'$ and leaf $(F_1F_2)''$.

In fig. 3 $dg$ is the turning-line of the field $Z_2LG$; each equilibrium $Z_2LG$ has on this turning-line at constant $T$ a point of maximum-pressure and under constant $P$ a point of minimum-temperature. The same applies in fig. 4 to the equilibrium $Z_3LG$.

In our previous considerations "Equilibria in ternary systems I—XVIII" we have fully examined different ternary turning-lines under the name of $M$-curves. They may have different forms, we find one of those in fig. 5 which represents a general form of the turning-lines $dg$ (fig. 3) and $eh$ (fig. 4).

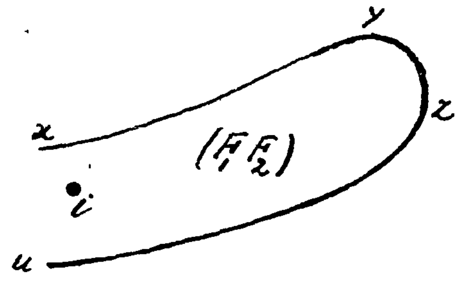


Fig 5.

As the field $(F_1F_2)$ consists of two leaves, each point of this field represents two equilibria $(F_1F_2)'$ and $(F_1F_2)''$; both equilibria consist under the same $P$ and $T$, but they differ from one another by the compositions of the variable phases.

We imagine in fig. 5 the curve $(F_1)$ to be drawn. As the equilibrium $(F_1)$ contains all phases of the equilibrium $(F_1F_2)$, curve $(F_1)$ cannot come outside the field $(F_1F_2)$, therefore, also not outside the turning-line. When curve $(F_1)$ meets therefore the turning-line somewhere in a point $d$, then $d$ is not a point of intersection, but a point of contact of those curves. In this point of contact curve $(F_1)$ passes from the one leaf into the other.

When we imagine in fig. 5 a curve within the turning-line, then we see that this curve must have points of maximum- or minimum pressure and temperature.

For the deduction of the $P,T$-diagramtypes and of the properties of their fields we have used the following properties [deduced in communication I]:

each point of a field $(F_1\, F_2)$ represents one single equilibrium $(F_1\, F_2)$ only;

the stable part of a field $(F_1\, F_2)$ extends itself between the stable parts of its limit-curves $(F_1)$ and $(F_2)$ without covering them;

a field-angle is smaller than 180°.

Now the question arises in how far those properties are still valid now. For this we imagine in the field $(F_1\, F_2)$ a point $i$ on the leaf $(F_1F_2)'$. The curves $(F_1)$ and $(F_2)$ are situated, starting from this point, first in the leaf $(F_1\, F_2)'$; in their point of contact with the turning-line they pass into the other leaf.

When we deduce the properties mentioned above, just as in Comm. I, then it appears that they are valid, when we leave out of consideration the leaf $(F_1F_2)''$.

When the invariant point $i$ is situated in the leaf $(F_1F_2)'$, then we shall say that the equilibria of this leaf are situated within, and those of the leaf $(F_1F_2)''$ outside the turning-line. We do not say that with respect to the $P$ and $T$ of those equilibria, but with respect to the compositions of their phases. In order to convert viz. an equilibrium $(F_1F_2)'$ continually into an equilibrium $(F_1F_2)''$, the first one must pass starting from a point of the leaf $(F_1F_2)'$ through the turning-line into the leaf $(F_1F_2)''$.

Then we may say: the properties are valid, as long as the equilibria $(F_1)$, $(F_2)$ and $(F_1 F_2)$ are situated within the turning-line of the field $(F_1 F_2)$

As in a $P,T$-diagram several fields are situated around the invariant point, we have to take into consideration the turning-line of each field; then we may say: the properties are valid as long as we consider those parts of the curves and the fields, which are situated within the corresponding turning-lines.

We have to bear in mind that "within the turning-line" means here "belonging to the same leaf on which the invariant point is situated".

The meaning of "not too far" and "not too much" in the conditions 1 and 2 is consequently indicated here somewhat more exactly.

Above we have already stated that we may imagine that the phases of the equilibrium $(F_1 F_2)$ satisfy reaction (3) casually in the invariant point; then the point $i$ is situated in fig. 5 accidentally on the turning-line. Then the two curves $(F_1)$ and $(F_2)$ come in contact with one another and with the turning-line in this point $i$. The corresponding $P,T$-diagram forms then, as we have already mentioned, a transitiontype, to which we shall refer later.

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(*To be continued*).

**Physics.** — "*On the Symmetry of the Röntgen-patterns of Triclinic and some Rhombic Crystals, and some Remarks on the Diffraction-Images of Quartz*". By Prof. Dr. H. HAGA and Prof. Dr. F. M. JAEGER.

(Communicated in the meeting of March 25, 1916).

§ 1. In the following paper we wish to communicate in the first place the results of the experiments, which as a sequel to our previous studies, were made with crystals of the *triclinic system*. The crystals of each of the two symmetry-classes of this system: those of the *triclinic-pedial* and those of the *triclinic-pinacoïdal* class,—of which crystals the first mentioned are wholly *unsymmetrical*, while the second possess only *centrical* symmetry,—will of course necessarily behave in *the same way*, as far as the diffraction-phenomenon of Röntgen-rays is concerned. But because the centre of symmetry cannot manifest itself in the structure of the Röntgen-images in any way, all obtained Röntgenogrammes will