The elements summed up above for the Kalooë fauna do not occur in the British Indian devonian, which yielded only a few trilobites with which I cannot identify my species. Melocrinus, which according to my preliminary examination appears to occur at Kalooë, is according to the palaeontological handbooks of ZITTEL, NICHOLSON and LYDEKKER a silurian-devonian genus and the species Melocrinus typus is typical for devonian strata, but the exact determination of species being impossible on the spot no certainty as to the age of the fossils has as yet been obtained. I have sent the fossils to Prof. Dr. JONKER of Delft, who probably later will publish full details.

Also in *South Yunnan* (China) devonian is known, the fauna of which has been described by v. LOCZY in "Reize des Grafen Szechenyi", Bd. I, p. 682. In North Yunnan VON RICHTHOFEN collected devonian fossils, more fully described by KAYSER (v. RICHTHOFEN, China, Bd. IV, p. 75). From the Himalayas only an insignificant devonian fauna has become known. I leave it to palaeontologists to make more elaborate comparisons based on more extensive lists of fossils.

It is desirable to study the stratigraphy of the strata near Kalooë well and thus also to determine the position of the white dolomites. I have found those chiefly as boulders in the Aloer Karangpoetih, although they occur in situ in some places of the river, and therefore I am of opinion that the Goenoeng Karangpoetih partly rests on them. A limestone with grey-black flints, as occurs on the northern foretop of this mountain, I did not find mentioned in any of the descriptions of limestones of other parts of Sumatra. Mr. J. B. SCRIVENOR wrote to me that he did not know this type of rock in Malacca either.

The author hopes in a subsequent paper if circumstances permit to be able to deal more fully with this trilobite lime of Kalooë, the discovery of which he only wished to announce in this first communication. *Pangkalan Brandan*, North Sumatra, Aug. 1915.

Prof. MOLENGRAAFF made some remarks in addition to the preceding article.

Mathematics. — "*On the nature of the limiting surface for multiple space transformations.*" By Dr. K. W. RUTGERS. (Communicated by Prof. JAN DE VRIES).

(Communicated in the meeting of February 26, 1916).

1. If $r+1$ surfaces of order p are given by means of the equations $f_1=0,\ldots, f_{r+1}=0$, an r-fold infinite linear system S_r of surfaces, determined by these $r+1$ surfaces, is represented by $\lambda_1 f_1+\lambda_2 f_2+\ldots+\lambda_{r+1} f_{r+1}=0$.

On any surface Φ of S_r an $(r-1)$-fold infinite system of twisted curves is produced by the intersection of the remaining ones, of which system of twisted curves we will suppose that any of them is of the genus *one*.

If we take $r > 2$ a net of elliptical twisted curves arises at least on each surface Φ, so that any surface of S_r is rational, i.e. may be represented point for point in a plane [1]); in such a representation the system of twisted curves will have to correspond to the likewise $(r-1)$-fold infinite system of plane elliptical curves $(r > 2)$. Suchlike linear systems of plane elliptical curves may always be reduced by means of birational transformation to a system of cubics with μ single base-points $(0 \leq \mu \leq 7)$ or to a system of quartics with two two-fold base-points. [2])

If n is the number of free intersections of two curves (at the same time the number of points in which three surfaces of S_r meet moreover apart from the base-curves and base-points), the dimension of the system of curves in each of the cases proves to be equal to n; the dimension of the system of surfaces S_r is therefore $r = n+1$.

2. Between two spaces Σ and Σ' a $(1, n)$-fold transformation is achieved, if we establish a collinear correspondence between the planes of a space Σ' and the surfaces of a triply infinite linear system S_3, of which the elements intersect, apart from the base-curves and base-points, moreover in n points.

If S_r is chosen out of an r-fold infinite system, as has been above supposed, any surface Φ is to be represented in a plane in such a way, that the ∞^2 intersections with the other surfaces correspond to a net of cubics or of quartics. We exclude the latter case and assume moreover that all the base-points of the c_3-system are different for the representation of Φ.

3. For the space transformations are now of importance: *the surface of* Jacobi, Φ_J, in the space Σ and the *limiting surface* Φ'_J in the space Σ' corresponding with it point for point.

Taking into consideration the above mentioned suppositions we shall prove that *the limiting surface* Φ'_J *must always be a cone.*

Its order may be determined in the following way.

The curve of intersection of a surface Φ of S_3 with Φ_J corre-

[1]) M. Noether, "*Ueber Flächen, welche Schaaren rationaler Curven besitzen*", Math. Ann., 3, p. 161, 1871.

[2]) G. B. Guccia, "*Generalizzazione di un teorema di Noether*", Rendiconti del Circolo Matematico di Palermo, tomo I, fasc. 3, 1886.

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sponds in the representation on a plane to the curve of JACOBI of the net of curves c_3; the latter is of order 6 with μ nodes in the μ base-points of the net (the genus is consequently $10-\mu$) and is moreover intersected by an arbitrary c_3 in $18-2\mu$ or (as $\mu = 9-n$) in $2n$ points. The intersection of two surfaces of S_3 has therefore $2n$ free intersections with Φ_J, a straight line in Σ' touches the limiting surface Φ'_J in as many points, so that the limiting surface is of order $2n$.

The genus of the curve of JACOBI of the c_3-net corresponds to that of the intersection of a surface of S_3 with Φ_J, and this curve corresponds again point for point to the plane intersection of Φ'_J. The latter too has therefore the genus

$$10-\mu = n+1.$$

The rank of Φ'_J too is easy to determine. A surface of S_3 with a node D corresponds in Σ' to a tangent plane at Φ'_J in the corresponding point D'. The curve formed by the points of contact of the tangent planes drawn at Φ'_J out of a point P', corresponds to the loci of the nodes of the net of surfaces of S_3, determined by the points $P_1, \ldots, P_n$ corresponding to P', i. e. to the curve of JACOBI ϱ_J of this net of surfaces.

ϱ_J is intersected by a surface Φ of S_3 in as many points as there are curves with a node in a pencil of the c_3-net, consequently in 12, i.e. *the rank of Φ_J is equal to* 12.

4. From what has been mentioned above it ensues that the plane intersection of Φ'_J is of order $2n$, of genus $n+1$ and of class 12, by which the remaining numbers of PLUCKER are known; it appears inter alia that this intersection must possess $30-n$ bitangents.

A bitangent of Φ_J happens to be the representation of the intersection of two surfaces of S_3, which possesses two nodes, in other words is degenerate.

In order to determine the *class* and the *order* of the congruence of bitangents of Φ'_J we investigate how often the curve of intersection of a surface Φ with the remaining surfaces of S_3 is degenerate, and how often such a degeneration will pass through one or more of n coupled points $P_1, \ldots, P_n$ (which determine a net out of S_3).

5. From the representation of Φ in a plane it appears that the degenerate curves of intersection are represented in different ways, viz. by:

a. a c_3, possessing a node in *one* of the μ base-points of the c_3-system. Number $\mu = 9-n$.

b. a c_3, which has degenerated into a straight line passing through two of the base-points and a conic passing through the $\mu-2$ remaining ones.

$$\text{Number } \frac{\mu(\mu-1)}{2.1} = \frac{(9-n)(8-n)}{2.1}.$$

c. a c_3, which has degenerated into a straight line passing through one of the base-points and a conic passing through the $\mu-1$ remaining ones.

Number $n-2$ for each base-point. Total amount $(9-n)(n-2)$.

d. a c_3, which has degenerated into a straight line passing through none of the base-points and a conic passing through μ base-points.

$$\text{Number } \frac{(n-2)(n-3)}{2.1}.$$

The total amount therefore is $30-n$, corresponding to $30-n$ bitangents of the plane intersection of Φ'_T, and on account of the supposed difference of the base-points those of the *a* case give rise to $(9-n)(8-n)$ of the 1^{st} class, those of *b* to $\frac{1}{2}(9-n)(8-n)$ of the 1^{st} class, those of *c* to $(9-n)$ of the $(n-2)^{nd}$ class, those of *d* to *one* of the class $\frac{(n-2)(n-3)}{2.1}$.

6. For each of these the order is to be determined now. In the *a* case, the part of the degenerate intersection, represented in the plane by a base-point, has *one* free intersection with any surface of S_3. Such a curve can only occur as part of a degenerate curve of intersection of two surfaces of a net determined by the n points $P_1, \ldots, P_n$, if it passes through one of these points and consequently has no free intersection any more with a surface of the net of surfaces.

This, however, leads to an impossibility as in that case a surface is always to be obtained passing through two curves of the same kind, which does not occur.

In the *b* case the same is the case for the parts represented by the straight line.

For *c*, the curve, represented by the straight line, has two free intersections with any surface, and is only to be taken as part of a degenerate intersection, if it passes through two of the points $P_1, \ldots, P_n$.

That this is also impossible ensues from the fact that on a surface no two equal curves (represented as a straight line passing through two points) will intersect in a point.

For d we find the same impossibility: now, the part represented by a straight line must pass through 3 of the points $P_1, \ldots, P_n$, while here on the same surface Φ no intersection in *two* points can occur.

It is therefore evident that of none of the degeneracies a part can pass through *one* of the base-points $P_1, \ldots, P_n$. Through an arbitrary point P_1' in Σ' no bitangent of Φ'_J passes. All the bitangents are therefore lying in $(30-n)$ separate bitangent planes.

The surface itself must be a *cone*.

Chemistry. — "*The photo-oxidation of alcohol with the co-operation of ketones. Contribution to the knowledge of the photochemical phenomena.*" II. By Prof. J. BÖESEKEN and Dr. W. D. COHEN.

(Communicated in the meeting of March 25, 1916).

At the meeting of 31 Oct. 1914 some particulars were communicated by us as to this reaction; since then we have made a closer study of the same and a few important results thereof are found in the dissertation of one of us[1]).

The reaction scheme:

2 ketone + alcohol = pinakon + aldehyde-(ketone),

deduced from comparative velocity measurements with increasing quantities of alcohol could be confirmed in various ways.

First of all it was demonstrated that in this reaction the aliphatic alcohol was converted quantitatively into aldehyde (ketone, respectively) and not into the correlated aliphatic pinacone.

The acetone obtained in the reaction:

$$2\ C_6H_5COC_6H_5 + CH_3CHOHCH_3 = [(C_6H_5)_2C(OH)]_2 + (CH_3)_2CO$$

was thus weighed in the form of the *p*-nitrophenylhydrazone and the *cyclo*hexanone formed in the reaction:

$$2\ C_6H_5COC_6H_5 + C_6H_{11}OH\ (\text{cyclohexanol}) = [(C_6H_5)_2COH]_2 + C_6H_{10}O,$$

was determined by the "oxime method".

That, in the photo-oxidation of aromatic alcohols, the said reaction scheme was also correct was proved as follows:

If the reaction had taken place according to the equation:

$$C_6H_5COC_6H_5 + C_6H_5CHOHC_6H_5 = [(C_6H_5)_2\ C(OH)]_2$$

which also would have been in harmony with the unimolecular course of the reaction, then when taking the two following mixtures:

[1]) Dissertation W. D. COHEN, Delft 1915.