lead circuit closed. Although the matter requires further elucidation, it would seem to follow from this measurement, that a few of the layers of the coil are short-circuited. After opening the circuit a residual moment remained in the coil as before which was destroyed on raising the temperature slightly above that of the helium-bath.

Physics. — *"Further experiments with liquid helium. K. Appearance of beginning paramagnetic saturation."* By Prof. H. KAMERLINGH ONNES. Communication N°. 140*d* from the Physical Laboratory at Leyden.

(Communicated in the meeting of May 30, 1914).

The question, whether paramagnetic substances would show a saturation-effect at high field-strengths, has always been considered a very important one. Although it could hardly be assumed, that the susceptibility would remain independent of the field at higher strengths than were attainable, still so far at the highest fields available it appeared to be the case. LANGEVIN's theory brought the explanation, why so far all attempts to find paramagnetic saturation-effects had remained unsuccessful. According to this theory the magnetisation appears to be determined by the expression $a=\frac{\sigma_m H}{RT}$, where σ_m is the magnetic moment of the molecules per gramme-molecule, R the gas-constant, T the absolute temperature and H the field. As long as a remains below 0,75, the changes of the susceptibility with the field escape the ordinary method of observation and at the ordinary temperature even a substance as strongly paramagnetic as oxygen gives for a with a field of 100000 not more than about 0,05. As I pointed out at the 2[nd] International Congress of Refrigeration at Vienna (1910) this theory shows that lowering the temperature is the means by which the observation of paramagnetic saturation might be attained and that helium-temperatures are the most suitable for the purpose. In fact as the absolute temperatures to which one may descend by means of helium are 70 and even 150 times lower than the normal temperature, the result will be equivalent to raising the magnetic field at which the observation is made 70 or 150 fold.

I have lately at last been able [1]) to fulfil my desire to attack by

[1]) Viz. by the acquisition a short time ago of an electromagnet (built according to WEISS's principle and utilising his friendly advice) the interferrum of which leaves sufficient room at fields of 20000 for experiments with liquid helium.

this method the problem of paramagnetic saturation which is also fundamental to WEISS's theory of ferromagnetism. In the first place it was necessary to have a substance which might be expected to obey CURIE's law, which also follows from LANGEVIN's theory, down to helium-temperatures; in the second place the substance must have a high value of σ_m. Both properties I hoped to find combined in crystallized gadolinium-sulphate, a quantity of which Professor URBAIN some time ago had very kindly put at my disposal.

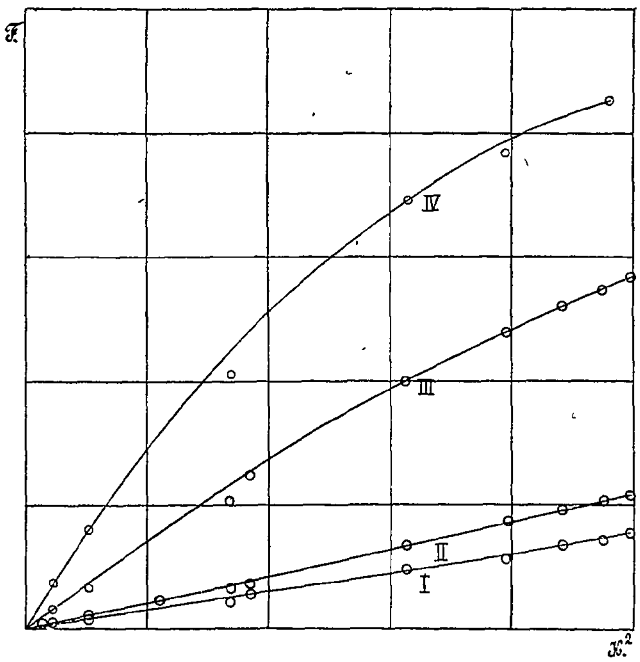
Earlier investigations in conjunction with PERRIER and OOSTERHUIS had shown, that gadolinium-sulphate follows CURIE's law down to the freezing point of hydrogen and does not show any sign of saturation, which as LANGEVIN's theory shows, if it existed at that temperature, would be ferromagnetic in its nature, as paramagnetic saturation at the value of a which could be reached would not yet be clearly observable. The number of magnetons calculated according to WEISS is large (38). That gadolinium-sulphate would still obey CURIE's law at helium-temperatures I felt justified to infer from the fact, that it is a "diluted" paramagnetic substance. The gadolinium-atoms, separated as they are e.g. by the water of crystallisation, are at great distances from each other, and this Dr. OOSTERHUIS and I in Comm. N°. 139e found a favourable circumstance to CURIE's law being valid down to very low temperatures.

The experiments have given a confirmation of LANGEVIN's theory which is at least qualitatively even now complete. Before an opinion can be formed as to the quantitative agreement various corrections will first have to be investigated. The most important of these which must not be neglected, especially when the validity of CURIE's law is to be tested, is the demagnetising action of the paramagnetisation itself, as the latter attains exceptionally high values. As an instance I may mention that with 0,345 gram of gadolinium-sulphate at 2° K. in a field of 15 kilogauss there was observed an attraction amounting to over 100 grammes. Another circumstance that one should keep in mind is that the object of observation consists of small crystals packed on each other.

The measurements consisted in determining the attraction in a non-homogeneous field, the gadolinium-sulphate in the cryostat being cooled first in liquid hydrogen under normal pressure, next in hydrogen under reduced pressure, next in helium boiling at ordinary pressure and finally in helium under 4 mms, the apparatus and the fields being the same each time.

The measurements at the boiling point of hydrogen (20.°3 K.) had the object to obtain the force at a given point for a given strength of field for controlling the values derived from ballistic calibrations. I hope to return to the details of the measurements and the arrangement of the apparatus afterwards, when an accurate quantitative comparison of the results with LANGEVIN's theory will have been made. It is as yet impossible to decide, in how far deviations are present which might be attributed to the existence of a small zero-point energy which would manifest itself in the manner in which the saturation changes, as well as in a deviation from CURIE's law at weak fields. It seems, however, that these deviations are not sufficiently large to disturb the general aspect.

On this occasion I wish to confine myself to communicating the general aspect of the results as laid down in the adjoining graphic representation on which the experimental numbers may also be read with sufficient accuracy. The curves represent the observed attractive force as a function of the square of the field on the axis between the poles. This field was read as a function of the current from a calibration curve.



Curve I refers to 20.°3 K. curve II to 14,°7 K, III to 4.°25 K. and IV to 1.°9 K. Each division along the horizontal axis corresponds to about 90 kilogauss, along the vertical axis to 25 grammes. The ratio of the force to the square of the field on the axis of the poles per unit of susceptibility was about $^1/_8$, neglecting small changes in the topography of the field.

If the susceptibility does not depend on the field and its topography remains the same, the curves are straight lines. The small deviations from the straight line at 20.°3 K are probably chiefly due to errors in the topography of the field, seeing that according to earlier more accurate determinations we had to await within the limits of the experiments a susceptibility independent of the field and therefore in this graph a straight line. By means of the deviations from the straight line at 20.°3 K the curves for the other temperatures have been provisionally corrected. It will be seen that for a given field these curves are the more strongly curved the lower the temperature to which one descends, in accordance with LANGEVIN's theory. Within the limits of accuracy to be expected in connection with the neglect of the various corrections referred to above the tangents of the angles of elevation of the tangents to the curves at the origin appear to be inversely proportional to the temperature as required by CURIE's law and the deviations of the curves from the tangents as expressed by the ratio between the ordinates of both for a given abscissa are strikingly similar to the deviations of LANGEVIN's curve for the magnetisation as a function of the field expressed in the same manner. The nature of paramagnetic magnetisation is very clearly revealed in these measurements at helium-temperatures.

Mathematics. — *"On some integral equations."* By W. KAPTEYN.

1. In a memoir "Recherches sur les fonctions cylindriques" (Mém. Soc. Roy. Sc. Liége 3ième Série t. VI 1905) we gave the solution of the integral equation

$$f(x) = \int_0^x \varphi(\beta)\, I_0(x-\beta)\, d\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in this form

$$\varphi(x) = \frac{df}{dx} + \int_0^x f(\beta) \frac{I_1(x-\beta)}{x-\beta}\, dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where the functions I_k represent BESSEL's functions of order k.

This solution rests upon the relation

$$\int_0^x I_k(\beta) \frac{I_n(x-\beta)}{x-\beta}\, d\beta = \frac{I_{n+k}(x)}{n} \begin{pmatrix} n = 1\ 2.3\ .. \\ k = 0\ 1.2\ .. \end{pmatrix} \quad . \quad . \quad . \quad (3)$$

from which the following theorem may be deduced.