vertical directions. The new figure in this paper has been adjusted in such a position, that it will correspond to the text of p. 797, if only the words vertical and horizontal (line 9 and 10 from beneath) are interchanged on reading.

Groningen, June 1914.

Laboratories for Physics and for Inorganic Chemistry of the University.

Physics. — "FRESNEL'S *coefficient for light of different colours.*" (First part). By Prof. P. ZEEMAN.

One of the empirical foundations of the electrodynamics of moving bodies in the domain of optics is FIZEAU's celebrated experiment on the carrying along of the light waves by the motion of water. Let w be the velocity of water relative to an observer, then for him the velocity of light propagated in the water would be

$$C_a = \frac{c}{\mu} \pm w$$

if the dynamical laws for the addition of velocities were perfectly general.

In this equation μ designs the index of refraction of water, c the velocity of light in vacuo, and we must take the upper or the lower sign, according as the light goes with or against the stream. FIZEAU demonstrated that not the entire velocity w but only a fraction of it comes into action. This particular fraction appeared to be approximately equal to $1 - \frac{1}{\mu^2}$, FRESNEL's coefficient. Hence we must write in place of the above given formula:

$$C_a = \frac{c}{\mu} \pm \varepsilon w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where

$$\varepsilon = 1 - \frac{1}{\mu^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For water ε is equal to seven-sixteenths.

The extremely important role which the formulae (1) and (2) have had in the theory of aberration, in the development of LORENTZ's electronic theory needs not to be exposed here, and it is hardly necessary to state that equation (1) is now regarded as a simple confirmation of EINSTEIN's theorem concerning the addition of velocities.

I may be permitted however to point out the smallness of the second term of formula (1). The velocity which we are able to obtain in a column of water transmitting light is of the order of magnitude of 5 metres per second. We have thus to find a difference of velocity of 5 metres in $\frac{3\,10^8}{4/3}$ m., i. e. of one part in fifty millions.

This was done by FIZEAU [1]) in one of the most ingenious experiments of the whole domain of physics. FIZEAU divided a beam of light issuing from a line of light in the focus of an object-glass into two parallel beams. After traversing two parallel tubes these beams pass through a second lens, in the focus of which a silvered mirror is placed. After reflection the rays are returned to the object glass, interchanging their paths. Each ray thus passes through the two tubes. A system of interference fringes is formed in the focus of the first lens. If water is flowing in opposite directions in the two tubes, one of the interfering beams is always travelling with the current and the other against it. When the water is put in motion a shift of the central white band is observed: by reversing the direction of the current the shift is doubled.

The ingenuity of the arrangement lies in the possibility of securing that the two beams traverse identical ways in opposite directions. Every change due for example to a variation of density or of temperature of the moving medium equally influences the two beams and is therefore automatically compensated.

One can be sure that a shift of the system of interference fringes, observed when reversing the direction of the current must be due to a change of the velocity of propagation of the light.

The tubes used by FIZEAU had a length of about 1,5 metres and an internal diameter of 5,3 m.m., whereas the velocity of the water was estimated at 7 metres. With *white light* the shift of the central band of the system of interference fringes observed by reversing the direction of flow was found from 19 rather concordant observations equal to 0,46 of the distance of two fringes; the value calculated with FRESNEL's coefficient is 0,404.

The result is favourable to the theory of FRESNEL. The amount of the shift is less than would correspond to the full velocity of the water and also agrees numerically with a coefficient $1-\frac{1}{\mu^2}$, if the uncertainty of the observations is taken into account.

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[1]) H. FIZEAU. Sur les hypothèses relatives à l'éther lumineux et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur. Ann. de Chim. et de Phys. (3) **57** 385. 1859.

FIZEAU's experiments, though made by a method which is theoretically as simple as it is perfect, left some doubts as to their accuracy, partly by reason of the remarkable conclusions as to relative motion of ether and matter to which they gave rise, and these doubts could only be removed by new experiments.

35 years after FIZEAU's first communication [1]) to the Académie des Sciences, MICHELSON and MORLEY [2]) repeated the experiment. They intended to remove some difficulties inherent to FIZEAU's method of observation and also, if possible, to measure accurately the fraction to be applied to the velocity of the water. MICHELSON uses the principle of his interferometer and produces interference fringes of considerable width without reducing at the same time the intensity of the light. The arrangement is further the same as that used by FIZEAU but performed with the considerable means, which American scientists have at their disposal for important scientific questions. The internal diameter of the tubes in the experiment of MICHELSON and MORLEY was 28 m.m. and in a first series the *total* [3]) length of the tubes was 3 metres, in a second series a little more than 6 metres.

From three series of experiments with *white light* MICHELSON found results which if reduced to what they would be if the tube were 2×5 metres long and the velocity 1 metre per second, would be as follows:

"Series	Δ = double displacement
1	0,1858
2	0,1838
3	0,1800"

"The final weighted value of Δ for all the observations is $\Delta = 0{,}1840$. From this by substitution in the formula, we get $\varepsilon = 0{,}434$ with a possible error of $\pm\, 0{,}02$".

For light of the wavelength of the D-lines we calculate $1 - \frac{1}{\mu^2} = 0{,}437$. This agreement between theory and observation is extremely satisfactory.

A new formula for ε was given by LORENTZ [4]) in 1895 viz.:

[1]) Comptes rendus 33, 349, 1851.

[2]) A. A. MICHELSON and E. W. MORLEY, Influence of motion of the medium on the velocity of light. Am. Journ of Science (3) 31, 377, 1886.

[3]) *Viz.* the sum of the lengths of the ways in the moving medium, traversed by each of the interfering beams, or approximately twice the length of one of the tubes.

[4]) H. A. LORENTZ Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern, p. 101, 1895. See also Theory of Electrons p. 290.

$$\varepsilon = 1 - \frac{1}{\mu^2} - \frac{1}{\mu}\lambda\frac{d\mu}{d\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the wavelength of the sodium lines this becomes:

0.451.

We see, therefore, that the value deduced by formula (3) deviates more from the result of the observations than the value given by the simple formula (2).

"Sollte es gelingen, was zwar schwierig, aber nicht unmöglich scheint, experimentell zwischen den Gleichungen (3) und (2) zu entscheiden, und sollte sich dabei die erstere bewähren, so hätte man gleichsam die DOPPLER'sche Veränderung der Schwingungsdauer für eine künstlich erzeugte Geschwindigkeit beobachtet. Es ist ja nur unter Berücksichtigung dieser Veränderung, dass wir die Gleichung (3) abgeleitet haben". [1])

It seemed of some importance to repeat *with light of different colours* FIZEAU's experiment, now that the correspondence between theory and observation had become less brilliant, and in view of the fundamental importance of the experiment for the optics of moving bodies.

From the point of view of the theory of relativity the formula (3) is easily proved, as has been pointed out by LAUE [2]), neglecting terms of the order $\frac{w^2}{c^2}$. Recently, however, again some doubt as to the exactness of LORENTZ's term has been expressed. I may refer here to a remark by MAX B. WEINSTEIN [3]) in a recent publication and to a paper by G. JAUMANN [4]). The last mentioned physicist gives an expression for the coefficient ε, which for water does not differ much, but in other cases deviates very considerably from FRESNEL's coefficient.

The interference fringes were produced by the method of MICHELSON. The method of observation introduced will be described later on. The incident ray *s l a* meets a slightly silvered plate at *a*. Here it divides into a reflected and a transmitted part. The reflected ray follows the path *a b c d e a f*, the transmitted one the path *a e d c b a f*. These rays meeting in the focal plane of *f* have

[1]) LORENTZ. Versuch u. s. w., 102.

[2]) M. LAUE. Die Mitführung des Lichtes durch bewegte Körper nach dem Relativitätsprinzip. Ann. d. Phys. **23**, 989. 1907.

[3]) MAX B. WEINSTEIN. Die Physik der bewegten Materie und die Relativitätstheorie. Leipzig. 1913, see note on p. 227 of his publication.

[4]) G. JAUMANN. Elektromagnetische Theorie. Sitzungsber. d. Kaiserl. Ak. der Wiss. Wien. mathem. naturw. Kl. **117**, 379. 1908, especially p. 459.

pursued identical, not only equivalent, paths, at least this is the case for that part of the system of interference fringes which in white light forms the centre of the central band.

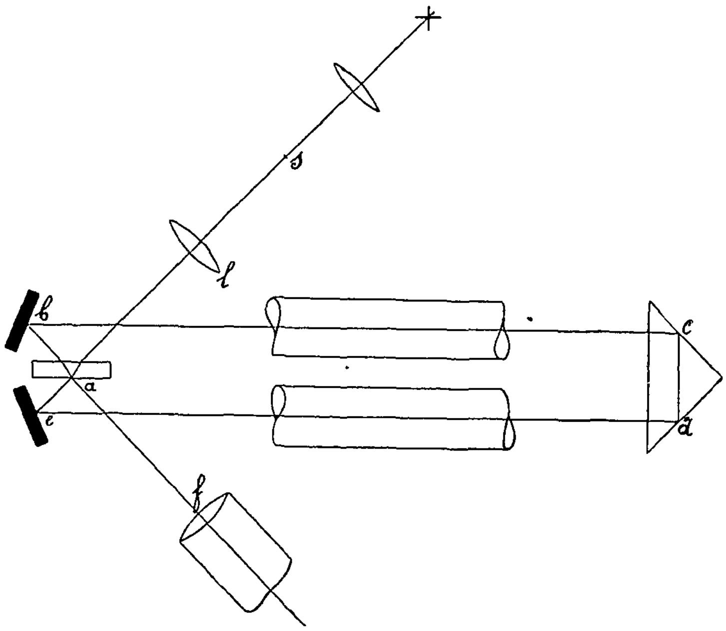

Fig. 1.

In order to verify the formula (3) it is necessary that the light be monochromatic. Further it seems of immense advantage to have a water current which remains constant during a considerable time. For observations with violet light this even becomes strictly necessary, because visual observations are impossible with the violet mercury line (4358) used. MICHELSON obtained a flow of water by filling a tank, connected with the apparatus; by means of large valves the current was made to flow in either direction through the tubes. "The flow lasted about three minutes, which gave time for a number of observations with the flow in alternating directions". In view of my experiments the municipal authorities of Amsterdam permitted the connection of a pipe of 7.5 cm. internal diameter to the main water conduit. There was no difficulty now photographing the violet system of interference fringes, though the time of exposition with one direction of flow was between 5 and 7 minutes. The pressure of the water proved to be very constant during a series of observations; the maximum velocity in the axis of the tubes, of 40 m.m. internal diameter and of a total length of 6 metres, was about 5,5 metres.

Before recording some details of my experiments, I may be permitted to communicate the general result that for water *there exists a dispersion of* FRESNEL's *coefficient* and *that formula* (3) *and therefore the third term of* LORENTZ *is essentially correct.*

I wish to record here my thanks to Mr. W. DE GROOT phil. nat. cand. and assistant in the physical laboratory for his assistance during my experiments with the final apparatus.

The difficulties encountered in these experiments were only surmounted after two reconstructions of the apparatus. Great annoyance gave the inconstancy of the interference fringes, when the pressure of the water or the direction of flow were changed. Then not only the width of the interference bands, but the inclination of the fringes were undergoing uncontrollable variations. All these defects were perfectly eliminated by the use of wide tubes and by arranging the end plates in the manner indicated in Fig. 3.

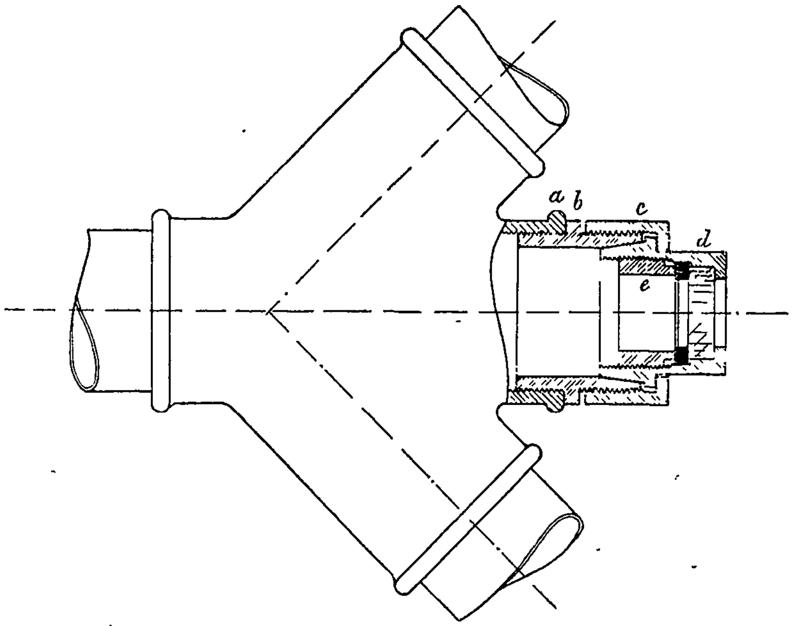

Fig. 3.

I am indebted to Mr. J. VAN DER ZWAAL, instrumentmaker in the laboratory for his carefully carrying out my instructions and designs in the mechanical construction of the apparatus.

In fig. 2A a side aspect, and in Fig. 2B a horizontal projection of the arrangement on a scale of about $^1/_{15}{}^{th}$ is given (see Plate).

The interferometer is at the right side, at the left the rectangular prism is placed.

The mounting of this prism is only sketched and was in reality more stable than might be inferred from the drawing.

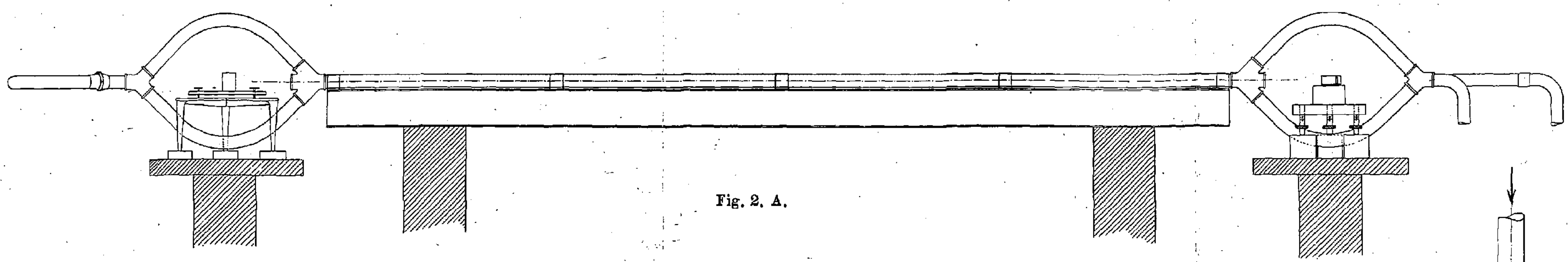

Fig. 2. A.

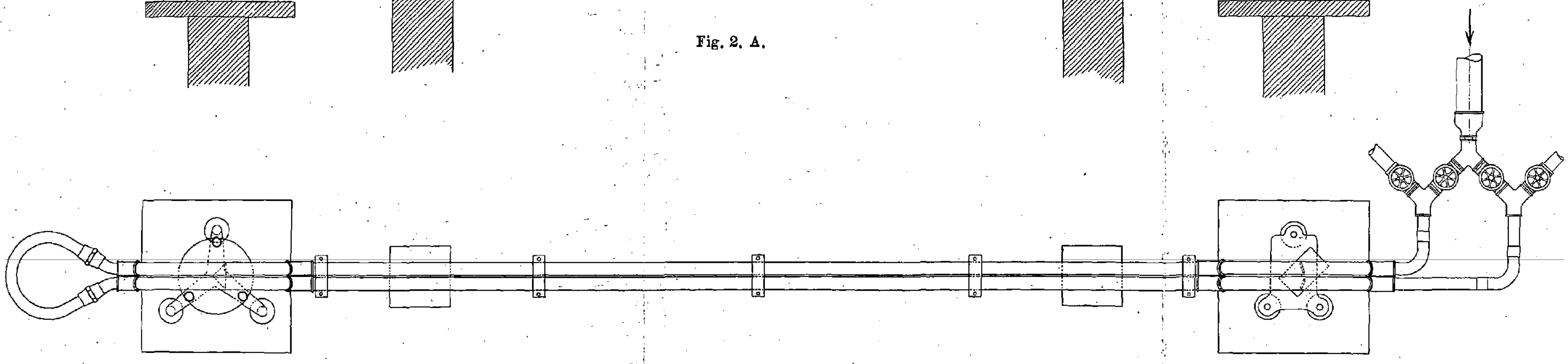

Fig. 2. B.

Prism and interferometer were mounted on the piers cemented to the large brick pier of the laboratory The tubes are entirely disconnected from the interferometer and mounted on a large iron I girder; this girder is placed upon piers of freestone cemented to large plates of freestone fixed to the wooden laboratory floors. In this manner the adjustment of the interferometer cannot be disturbed by vibrations proceeding from the tubes. At the right of the horizontal projection the four large valves may be seen, by turning which the current was made to flow in either direction through the tube systems.

The mountings containing the glass plates by which the tubes are closed are not given in the Plate. One of these mountings containing the plane parallel plates of glass is drawn to scale in Fig. 3 at one half of the natural size The four plates of glass are by HILGER, they are circular of 24 m.m. diameter and 10 m.m thick; in a second series of observations plates 7 m.m. thick have been used. The accuracy of parallelism of the plates is excellent; they are indeed cut from echelon plates. The general plan adopted for the construction of the plate mountings is this: one can only be sure that no change will occur in the position of the plates during the course of an experiment, if this position is *entirely definite.* In order to attain this the glass plate rests upon the inner, accurately grinded, surface of the brass piece d. This piece d fits accurately into the conical inner part of a piece b, itself rigidly screwed to the tube a. Parts d and b are connected by means of the counter nut c. The glassplate is held against d by the nut e. There is no objection to the presence at the *inside* between e and d of rings of hard india-rubber and of brass. *(To be continued)*

Physics. — *"A new relation between the critical quantities, and on the unity of all substances in their thermic behaviour."* (Conclusion). By Dr. J. J. VAN LAAR. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of April 24, 1914).

By way of supplement we shall add the calculation of three more isotherms *below* the critical temperature, for which (loc. cit.) data are known from the unsaturated vapour region. If the β-values above T_k were somewhat too high on the whole, now we shall find values which are *much too low,* lower even than β_0, and therefore impossible. These deviating values can only be explained, when with low temperatures and large volumes *association in the vapour* is assumed.