3. The temperature coefficient was small: 1.06—1.11 for 10°.
4. The velocity of the pinacone formation is greatly dependent on the alcohol; for instance, the methyl alcohol and the allyl alcohol were oxidised much more slowly than other primary and secondary alcohols.
5. The velocity of the pinacone formation is greatly dependent on the ketone, the benzophenone is attacked rapidly, most of the ketones as yet examined less rapidly, many not at all.
6. The ratio of these velocities in different alcohols is constant.
7. The active light of the ketone reduction is sure to be situated in the spectrum between 400 and 430 $\mu\mu$ and very probably in, or adjacent to, the rays 404.7 and 407.8 of the mercury quartz lamp.
8. The ratio of the velocities of the pinacone formation in sunlight and in mercury light is the same.
9. When two ketones are present simultaneously one of them absorbs a part of the rays required by the other ketone; this also appears when the light passes through a solution of the one ketone and falls on that of the other.

Particularly in the case of the powerfully absorbing ketones the hindrances are stronger than was to be expected.

Delft, October 1914.

Physics. — "*Simplified deduction of the formula from the theory of combinations which* PLANCK *uses as the basis of his radiation-theory.*" By Prof. P. EHRENFEST and Prof. H. KAMERLINGH ONNES.

(Communicated in the meeting of Oct. 31, 1914).

We refer to the expression

$$C_P^N = \frac{(N-1+P)!}{P!(N-1)!} \quad \ldots \ldots \ldots \quad (A)$$

which gives the number of ways in which N monochromatic resonators $R_1, R_2, \ldots R_N$ may be distributed over the various degrees of energy, determined by the series of multiples $0, \varepsilon, 2\varepsilon \ldots$ of the unit energy ε, when the resonators together must each time contain the given multiple $P\varepsilon$. Two methods of distribution will be called identical, and only then, when the first resonator in the one distribution is at the same grade of energy as the same resonator in the second and similarly the second, third, ... and the Nth resonator are each at the same energy-grades in the two distributions.

Taking a special example, we shall introduce a symbol for the distribution. Let $N = 4$, and $P = 7$. One of the possible distributions is the following: resonator R_1 has reached the energy-grade 4ε (R_1 contains the energy 4ε), R_2 the grade 2ε, R_3 the grade 0ε (contains no energy), R_4 the grade ε. Our symbol will, read from left to right, indicate the energy of R_1, R_2, R_3, R_4 in the distribution chosen, and particularly express, that the total energy is 7ε. For this case the symbol will be:

$$\Vert\, |\overset{\varepsilon}{-}|\overset{\varepsilon}{-}|\overset{\varepsilon}{-}|\overset{\varepsilon}{-}|\, O \,|\overset{\varepsilon}{-}|\overset{\varepsilon}{-}|\, O O \,|\overset{\varepsilon}{-}|\, \Vert$$

or also more simply:

$$\Vert\; \varepsilon\,\varepsilon\,\varepsilon\,\varepsilon\; O\; \varepsilon\,\varepsilon\; O O\; \varepsilon \;\Vert$$

With general values of N and P the symbol will contain P times the sign ε and $(N-1)$ times the sign **O**[1]. The question now is, how many *different* symbols for the distribution may be formed in the manner indicated above from the given number of ε and **O**? The answer is

$$\frac{(N-1+P)!}{P!(N-1)!} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Proof: first considering the $(N-1+P)$ elements $\varepsilon \ldots \varepsilon$, $O \ldots O$ as so many distinguishable entities, they may be arranged in

$$(N-1+P)! \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

different manners between the ends $\Vert\;\Vert$. Next note, that each time

$$(N \;\; 1)!\,P! \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

of the combinations thus obtained give the *same* symbol for the distribution (and give the same energy-grade to each resonator), viz. all those combinations which are formed from each other by the permutation of the P elements ε[2]) or the $(N-1)$ elements **O**. The number of the *different* symbols for the distribution and that of the

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1) We were led to the introduction of the $(N-1)$ partitions between the N resonators, in trying to find an explanation of the form $(N-1)!$ in the denominator of (A) (compare note 1 on page 872). Planck proves, that the number of distributions must be equal to the number of all "combinations with repetitions of N elements of class P" and for the proof, that this number is given by the expression (A), he refers to the train of reasoning followed in treatises on combinations for this particular case. In these treatises the expression (A) is arrived at by the aid of the device of "transition from n to $n+1$", and this method taken as a whole does not give an insight into the origin of the final expression.

2) See appendix.

distributions themselves required is thus obtained by dividing (2) by (3) q. e. d. [1]).

APPENDIX.

The contrast between PLANCK'*s hypothesis of the energy-grades and* EINSTEIN'*s hypothesis of energy-quanta.*

The permutation of the elements ε is a purely formal device, just as the permutation of the elements **0** is. More than once the analogous, equally formal device used by PLANCK, viz. distribution of P energy-elements over N resonators, has by a misunderstanding been given a physical interpretation, which is absolutely in conflict with PLANCK's radiation-formula and would lead to WIEN's radiation formula.

As a matter of fact PLANCK's energy-elements were in that case almost entirely identified with EINSTEIN's light-quanta and accordingly it was said, that the difference between PLANCK and EINSTEIN consists herein that the latter assumes the existence of mutually independent energy-quanta also in empty space, the former only in the interior of matter, in the resonators. The confusion which underlies this view has been more than once pointed out [2]). EINSTEIN really considers P similar quanta, existing *independently of each other*. He discusses for instance the case, that they distribute themselves irreversibly from a space of N_1 cm³ over a larger space of N_2 cm³ and he finds using BOLTZMAN's entropy-formula: $S = k \log W$, that this produces a gain of entropy [3]):

$$S - S_0 = k \log \left(\frac{N_2}{N_1}\right)^P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

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[1]) It may be added, that the problem of the distribution of N resonators over the energy-grades corresponds to the following: On a rod, whose length is a multiple $P\varepsilon$ of a given length ε, notches have been cut at distances ε, 2ε, etc. from one of the ends. At each of the notches, and only there, the rod may be broken, the separate pieces may subsequently be joined together in arbitrary numbers and in arbitrary order, the rods thus obtained not being distinguishable from each other otherwise than by a possible difference in length. The question is, in how many different manners (comp. Appendix) the rod may be divided and the pieces distributed over a given number of boxes, to be distinguished from each other as the 1st 2nd, ... Nth, when no box may contain more than one rod. If the boxes, which may be thought of as rectangular, are placed side by side in one line, they form together as it were an oblong drawer with (N—1) partitions, formed of two walls each, (comp. the above symbol in its first form, from which the second form was derived by abstracting from the fact, that each multiple of ε forms one whole each time), and these double partitions may be imagined to be mutually exchanged, the boxes themselves remaining where they are. The possibility of this exchange is indicated by the form of the symbol chosen.

As a further example corresponding to the symbol we may take a thread on which between P beads of the same kind, (N—1) beads of a different kind are strung, which divide the beads of the first kind in a 1st, 2nd ... Nth group.

[2]) P. EHRENFEST, Ann. d. Phys. **36**, 91, 1911, G. KRUTKOW, Physik. Zschr. **15**, 133, 363, 1914.

[3]) A. EINSTEIN, Ann. d. Phys. **17**, 132, 1905.