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Astronomy. — *"On the figure of the planet Jupiter."* By Prof. W. DE SITTER.

The potential function of a body possessing axial symmetry is [1])

$$V = m\left[\frac{1}{r} + \sum_{1}^{\infty} \frac{B_i}{r^{i+1}} P_i(\sin\delta)\right],$$

where

$$B_i = \frac{1}{m}\int \varrho^i P_i\left(\frac{\zeta}{\varrho}\right) dm.$$

In these formulas the axis of symmetry is chosen as axis of ζ. The coordinates of the element of mass dm are ξ, η, ζ; $\varrho^2 = \xi^2 + \eta^2 + \zeta^2$, and the integrals must be extended over the whole body. Further δ is the planetocentric latitude, and P_i are the zonal harmonics of order i. If the origin of coördinates is taken in the centre of gravity, we have

$$B_1 = 0.$$

If the plane of ξ, η is a plane of symmetry, then also

$$B_3 = 0, \quad B_5 = 0, \ldots \text{ etc.}$$

I adopt from the theory of the four large satellites

$$\frac{B_2}{b^2} = -0.01462.$$

The motion of the perijove of satellite V then gives

$$\frac{B_4}{b^4} = +0.00058.$$

By analogy we can conclude

$$\frac{B_6}{b^6} = -0.00002.$$

The effect of the term in B_6 can thus never amount to more than a few units in the fifth decimal place, even at the surface of the planet. Since the values of B_2 and B_4 are uncertain to a larger amount than this, we neglect B_6 altogether.

If now the body rotates about the axis of ζ with the velocity ω, we must, at the surface, have

$$V_0 = fm\left[\frac{1}{r} + \frac{B_2}{r^3} P_2(\sin\delta) + \frac{B_4}{r^5} P_4(\sin\delta)\right] + \tfrac{1}{2}\,\omega^2 r^2 \cos^2\delta = const.$$

If we put

$$\varrho = \frac{\omega^2 b^3}{fm},$$

where b is the equatorial semidiameter, and

[1]) TISSERAND, Méc. cél. II p. 319—322.

$$\frac{fm}{V_0} = a,$$

then we find

$$\frac{r}{a} = 1 + \frac{B_2}{r^2} P_2(\sin \delta) + \frac{B_4}{r^4} P_4(\sin \delta) + \tfrac{1}{2}\varrho \frac{r^3}{b^3} \cos^2 \delta \; . \quad . \quad . \quad (1)$$

The value of ω is different for different latitudes. At the equator the period of rotation is nearly

$$T_0 = 9^h 50^m.5.$$

In higher latitudes in the northern hemisphere it is about

$$T_1 = 9^h 55^m.6$$

while in the southern hemisphere the average is

$$T_1' = 9^h 55^m.2.$$

In the northern hemisphere T_1 appears to increase somewhat from the equator to the pole, while in the southern hemisphere there seems to be a slight decrease [1]). These results are, however, still rather uncertain and it seems better to adopt a mean value.

For $T_0 = 9^h 50^m.5$ we have $\varrho_0 = 0.09047$
and for $T_1 = 9\ 55\ .5$ „ „ $\varrho_1 = 0.08971$

If now we write the equation (1) first for a point on the equator $[r{=}b]$, and then for the pole $[r{=}b(1{-}\varepsilon_1)]$, taking both times $\varrho = \varrho_1$, we find the following condition determining ε_1

$$\frac{\varepsilon_1}{1-\varepsilon_1} - \tfrac{1}{2}\varrho_1 = -\tfrac{1}{2}\frac{B_2}{b^2}[1+2\lambda^3] + \tfrac{3}{8}\frac{B_4}{b^4}[1-\tfrac{8}{3}\lambda^5],$$

where

$$\lambda = \frac{1}{1-\varepsilon_1}.$$

I thus find

$$\varepsilon_1 = 0.06494 = {}^1/_{15\cdot 40}$$

From the eclipses of the satellites observed at Harvard College I derived [2]):

From satellite	I	$\varepsilon =$ 0.0604 ±	.0030
„ „	II	0764 ±	15
„ „	III	.0544 ±	30
„ „	IV	.0649 ±	10

It is well known that also the values of ε derived by different observers from micrometrical measures of the diameters are very discordant. They range from about 0.055 to 0.075. The value derived

1) STANLEY WILLIAMS, Observatory 1913, page 465.
2) Monthly Notices LXXI page 96.

here from the equation (1) is probably more exact than any of these.

This value of ε_1 has been used for the computation of the values of the radiusvector given in the second, third and fourth columns of the following table. The third and fourth columns were computed by the equation (1), using for ϱ the values ϱ_0 and ϱ_1 respectively. The table gives $\frac{r}{b} - 1$.

δ	Ellipsoid	Equipotential surface		Difference		Diff. in km.
		ϱ_0	ϱ_1	ϱ_0	ϱ_1	
0°	0 00000	+ 0.00042		+ 0 00042		+ 30
5	— .00055	— .00014	— 0.00056	+ 40	— 0.00001	+ 28
10	— .00216	— .00181	— .00222	+ 35	— 6	0
15	— .00478	— .00452	— .00490	+ 26	— 12	— 8
20	— .00830		— .00850		— 20	— 14
30	— .01750		— .01786		— 36	— 26
40	— .02843		— .02890		— 47	— 34
50	— .03968		— .04014		— 46	— 33
60	— .04990		— .05026		— 36	— 26
70	— .05799		— .05819		— 20	— 14
80	— .06317		— .06322		— 6	— 4
90	— 06494		— .06494		0	0

The deviation from the ellipsoid thus consists of a protuberance along the equator, produced by the increase of the velocity of rotation, and a depression in mean latitudes [1]). The transition probably takes place rather suddenly somewhere near the latitude 7°.

We have up to now taken no account of the variability of ω in

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[1]) If quantities of the order of ε^3 are neglected, the deviation from the ellipsoid is easily shown to be (for constant ω) of the form

$$-\varkappa \sin^2 2\delta ,$$

where

$$\varkappa = \frac{5}{8}\varepsilon\varrho - \frac{7}{8}\varepsilon^2 + \frac{35}{32}\frac{B_4}{b^4} = 0.00058.$$

The actual depression is only about $^1/_5$ of this.

For the earth the value of $\varkappa$ is of the order of 0.0000005 = 3 meters.

the higher latitudes. A difference of $0^m.4$ in T corresponds to a difference of 0.00006 in $\frac{1}{2}\ \varrho$. Therefore, if we had used for each latitude its own value of ω or ϱ, only the last decimal of $r/_b$ would have been affected. In that case, however, we must also dismiss the assumption $B_3 = 0$. Of the true value of B_3 we know nothing, but we can assert with considerable certainty that it will be of the same order of magnitude as the difference between the northern and southern rotations, i. e. that it will, like the other causes of uncertainty discussed above, not exceed the fifth decimal place.

The deviations from the ellipsoid are, of course, far beyond the reach of direct micrometrical measures. In fact they are always below 0".01. The effect on the times of the phenomena of the satellites is, at latitude 60°, $0^s.034$ for satellite I and $0^s.070$ for satellite IV, which also is beyond the accuracy of the observations. Thus for all practical purposes we can treat the surface of Jupiter as a true ellipsoid.

Chemistry. — "*The Allotropy of Cadmium V*". By Prof. ERNST COHEN and W. D. HELDERMAN.

The heat of Transformation in the reaction $Cd(\alpha) \rightleftarrows Cd(\gamma)$.

1. As we pointed out some time ago in our sixth communication on the thermodynamics of standard cells,[1]) in calculating the chemical energy of the WESTON cell we have to take into account that cadmium is able to exist in different allotropic modifications. While this problem will be treated later in full, it may be pointed out here that it is very important to know the quantity of heat which is involved in the reaction

$$\mathrm{Cd}(\alpha) \rightleftarrows \mathrm{Cd}(\gamma).$$

The investigations to be described here have reference to this problem.

2. Up to the present such a heat of transformation of a metal has only been determined in one single case. Some months ago BRÖNSTED [2]) carried out some measurements on the heat of the transformation

grey tin $\rightarrow$ white tin.

1) Chem. Weekblad **11**, 740 (1914).
2) Zeitschr. f. physik. Chemie **88**, 479 (1914),