

Citation:

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Or will the entropy of the gas, which is monatomic from a thermodynamic point of view at very low temperatures, perhaps not be represented by (16), but have a value $kN \log 2$ smaller? This seems improbable, at least at first sight. A perfectly satisfactory answer to this question is probably to be expected only from a general theory of quanta.

However, something can be said about the division by $N!$ also without having recourse to the solid phase. Suppose we have the general theory of quanta. We come to the conclusion that for the determination of the thermodynamic probability we have not to reckon with infinitely small regions, but with such of a definite finite extent. This, however, holds only without reservation for systems the molecules of which are all different. Of a gas for which this is the case, we could not say that the entropy was proportional to the mass; it would much sooner contain a term $k \log N!$. Now, however, the entropy of a mixture of different gases is $k \log \frac{N!}{n_1! n_2! \dots}$, greater than when the gases are equal, which can be thermodynamically derived for large values n_i etc., while it seems natural to consider it also as valid for small n 's, (n_i etc. are of course the numbers of molecules for the different kinds, N is $= \sum n_i$). If now all n 's are $= 1$, in other words, if the gas consists of nothing but different molecules, the entropy will be $k \log (N!)$ greater than for a gas consisting of nothing but equal molecules. For the latter we shall then have to subtract $k \log (N!)$ from the original entropy expression. Such considerations have originally led me to the division by $N!$ and to the formula (16).

Physics. — “*On interference phenomena to be expected when Röntgen rays pass through a di-atomic gas.*” By Prof. P. EHRENFEST.
(Communicated by Prof. H. A. LORENTZ.)

(Communicated in the meeting of February 27, 1915).

As is known W. FRIEDRICH¹⁾ has ascertained that a beam of Röntgen rays passing through yellow wax and other amorphous *solid* substances gives interference rings on a photographic plate placed behind it. *Liquid* paraffin also gives a ring, which, however, does not represent a maximum of darkness, but an inflection point of

¹⁾ W. FRIEDRICH, Eine neue Interferenzerscheinung bei Röntgenstrahlen: Phys. Zsch. 14, (1913), p. 317.

the decrease of darkness. FRIEDRICH shortly discusses two possible explanations of the observed phenomenon.

a The solid amorphous bodies may be imagined as built up of small crystals. The interference spots of the different individual bodies uniformly turned in all directions unite on the photographic plate to rings.

b. For the solid amorphous substances and particularly of the liquids is "die Anordnung der Teilchen eine vollkommen regellose". In the case of the passage of Röntgen rays through such an amorphous medium we should meet with a phenomenon analogous to that of the passage of rays of light through a glass plate strewn with lycopodium powder.¹⁾

FRIEDRICH cites the circumstance that also for liquid paraffin an interference ring occurs, in favour of the second explanation, and he therefore expresses the supposition that here we should have to do "mit Beugung am Molekül resp. Atom".

E. HUPKA²⁾ tries, if I have understood him correctly, to give another explanation, in which the mean distance of the molecules is decisive as "grating constant".

A trustworthy introduction of statistical considerations on which the calculation of the dark rings must rest, does not seem easy to me even for liquids on account of the compact arrangement of the molecules and particularly on account of the unknown complexes (association) of adjacent molecules.

I may be therefore allowed to point out briefly that the problem is considerably simpler in case of transition of Röntgen rays *through a di-atomic gas*. Whether the *experimental* difficulties can be surmounted, I cannot judge; in case this should be so, some new data might be obtained in this way on the situation of the atoms in the gas molecule.

§ 1. Let homogeneous plane Röntgen rays fall on an isolated di-atomic gas molecule. Both atoms emit secondary waves which interfere in the whole space. We consider the interference in an arbitrary point *P* of a plane *E* (photographic plate), which lies normal to the direction of incidence of the Röntgen ray at the distance *D* behind the molecule. *D* may be considered as infinitely great with

¹⁾ DRUDE, Optik I, Afd. Kap. IV; M LAUE, Beugungserscheinungen an vielen unregelmässig verteilten Teilchen Sitzber. d. preuss. Akad. 1915

²⁾ E HUPKA, Die Interferenz der Röntgenstrahlen Samml. Vieweg, Heft 18 1914), p. 62.

respect to the central distance a of the two atoms of the molecule, which are supposed to be equal for the sake of simplicity.

If consecutively the molecule axis is given all possible directions by the molecule being turned round one of the two atoms A_1 , the phase difference with which the secondary waves of A_1 and A_2 interfere in the point P changes, hence the intensity of the radiation there.

We calculate the *mean* intensity in the point P and inquire: *in what way does this mean intensity vary with the situation of P on the plate E ?*

For reasons of symmetry the mean intensity is of course the same for all those points P for which the direction molecule \rightarrow point P forms the same angle φ with the direction of incidence of the Röntgen rays. With increase of φ the mean density however changes oscillatorily namely as:

$$1 + \frac{\sin 2\pi\varrho}{2\pi\varrho} \dots \dots \dots (1)^1$$

if

$$\varrho = \frac{2a}{\lambda} \sin \frac{\varphi}{2} \dots \dots \dots (2)$$

[λ is the wavelength of the Röntgen rays; a the distance of the two atoms (supposed as points²⁾) from each other].

The consecutive maxima and minima of (1) are in the following ratio to each other³⁾:

$$2 : 0,78 : 1,13 : 0,91 : 1,07$$

and lie at:

$$2\pi\varrho = 0; 4,49; 7,72; 10,90; 14,07.$$

$\frac{a}{\lambda}$	φ_0	φ_1	φ_2	φ_3
$\frac{1}{2}$	0°	90°	—	—
1	0	41	71°	114°
2	0	21	34	50
3	0	14	22	32

¹⁾ See appendix.

²⁾ It is convenient to confine ourselves for the present to this schematisation, till experiment shall give an indication for possibly necessary refinement of the scheme.

³⁾ Comp the tables for $\frac{\sin X}{X}$ in JAHNKE u. EMDE. Functionentafeln.

The preceding table gives the corresponding values of φ for different values of $\frac{a}{\lambda}$.

§ 2. Instead of a single di-atomic gas molecule now a whole gas mass is irradiated; the dimensions of the irradiated quantity, however, will be comparatively small compared with the distance between the gas and the photographic plate E (e.g. 1 mm. to 5 cm.).

We state ¹⁾:

The dark rings on the photographic plate will then — except for a slight diminution in sharpness — continue to be represented by equation (1).

The decrease of sharpness corresponds with the slight changes in situation and size, which the rings (1) undergo when the centre of the molecule discussed in § 1 is made to pass consecutively through all the points of the small irradiated region.

§ 3. In the experimental realisation of these rings we are confronted with difficulties which are indeed very great, but yet possibly not insurmountable.

1. The probably very slight intensity of the whole secondary radiation. In any case we shall choose vapours the atoms of which will be as heavy as possible ²⁾.

2. The incident radiation must be as homogeneous as possible or anyway possess such a distribution of intensity in the spectrum that at least the first ring does not fade away entirely. In order to be still able to calculate α in the latter case from the distribution of light and dark, the spectrum distribution must be known from interference figures for crystals.

3. If the first ring is to fall on favourable values of φ , $a : \lambda$ must certainly be greater than one (see the table in § 1).

4. Possibly the dark rings that originate from the amorphous glass vessel in which the vapour is contained; might be troublesome ³⁾. Then we should have to replace the amorphous glass e.g. by mica.

A P P E N D I X.

The secondary waves which two atoms A, B send to a definite

¹⁾ See appendix.

²⁾ Or perhaps solutions; but here the phenomena are theoretically more complicated.

³⁾ They have not occurred in FRIEDRICH'S experiments with yellow wax.

point P of the photographic plate, give at that place conjointly the disturbance of equilibrium

$$M \sin p(t - \tau_A) + M \sin p(t - \tau_B) \dots \dots \dots (a)$$

The time average of the square of (a), — taken over a period, is:

$$\frac{M^2}{2} [1 + 2 \cos p(\tau_A - \tau_B) + 1] = M^2 [1 + \cos p(\tau_A - \tau_B)] \dots (b)$$

If the two atoms A, B belong to *different* molecules of the gas, the quantity $\cos p(\tau_A - \tau_B)$ assumes equally often equally large positive and negative values during the time of exposure in consequence of the independent movement of the molecules, so that here the time average of $\cos p(\tau_A - \tau_B)$ becomes zero.

It is different with two atoms belonging to *the same* molecule. We split up here the taking of the time average into two phases: *I.* All possible orientations of the axis of the di-atomic molecule, one atom being fixed. *II.* Repetition of this average value determination for all possible situations of that atom inside the (irradiated) gas space.

Mean value determination I.

Let CA be an incident Röntgen ray, AD the secondary ray that travels from atom A to the point P of the photographic plate. What is the locus of all the situations of the atom B , for which the difference of path

$$CDA - EBF = \Delta$$

has one and the same value? ¹⁾ Answer: Describe round A a sphere with radius equal to the fixed distance of the atoms $AB = a$. Intersect this sphere with the plane $B'YB$, which is normal to the plane of drawing $C'AD$, and parallel to the straight line AU ²⁾. The circle BB' along which this plane intersects the sphere is the required locus; for all its points:

$$EBF = E'B'F'$$

Hence all of them give one and the same difference of path with respect to CAD , which can also be represented by

$$\Delta = XYZ - CAD$$

finally also by

$$\Delta = VAW = 2(AY) \sin \frac{\rho}{2} \dots \dots \dots (\gamma)$$

¹⁾ AB is so small compared with AP and BP that AP and BP may be considered as parallel, hence Δ as the difference of path with which the secondary waves from A and B arrive in P .

²⁾ A mirror parallel to the plane $B'YB$ would just reflect the rays EB', YX, EB in the direction $B'F', YZ, BF$.

$$\frac{2ap}{c} \sin \frac{\varphi}{2} = 2\pi Q$$

is put for shortness.

Thus we get the dark rings:

$$M^2 \left(1 + \frac{\sin 2\pi Q}{2\pi Q} \right)$$

which were mentioned in § 1¹⁾.

Determination of the mean value II.

The distances from Rontgen tube to molecule, and from molecule to photographic plate being enormously large in comparison with the distance $AB=a$, we could act up to now as if we had to deal with an interference problem of FRAUENHOFER. If for the determination of the mean value II the molecule is made to occupy all situations in the irradiated space, these molecule displacements are practically still infinitesimal compared with the distance from Rontgen tube, but not compared with the distance from molecule to plate. In connection with this in the slight displacements of the molecule *parallel* to the photographic plate the dark rings move over an equally large distance. In displacement *normal* to the plate a slight enlargement or diminution of the rings occurs. We see that this brings about a slight fading of the rings.

Geology. — “*On the granitic area of Rokan (Middle-Sumatra) and on contact-phenomena in the surrounding schists.* By Dr. H. A. BROUWER. (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated in the meeting of June 27, 1914).

Between Rokan and Loeboek Bandhara the Rokan Kiri cuts a granitic mass of about $4\frac{1}{2}$ km. in width, which on its northwestern and southeastern sides is adjoined by tertiary sandstones and conglomerates, which however, along a portion of the south west limit, are separated from the granite by a narrow strip of schists.

During the exploration of this area some facts were collected concerning the various facies of the granites and the contactphenomena in the surrounding schists. The schists dip towards the granitic mass

¹⁾ The factor M^2 varies of course also with φ ; perhaps in the same way as $\cos^2 \varphi$. In § 1 we have however disregarded this variation for the present to get a first survey.