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After 2 hours the curvatures amounted to:

— mg	C	+ mg
2.5	1.5	1.5

The influence of the longitudinal component is therefore once more evident.

The term longitudinal component of gravity is of course only a phrase. No way of explaining it physiologically has so far been found.

The phototropic curvatures of the coleoptiles of Avena when illuminated at different angles, showed a very marked deviation from the expected sine relation. As ARISZ (l. c. 1914) has justly argued, the paraboloid shape of the apex must be a very important factor in this connection.

In geotropic reactions another factor must also be taken into account, namely, the polarisation of separate cells.

It is generally assumed that a difference exists in the sensitiveness to pressure of the protoplasm lining the inner and the outer walls of the cells. The idea that there may be a similar difference of sensitiveness between the apical and basal part of each cell, may therefore not be summarily rejected. In this way the longitudinal component can also be explained. In the rotating apices of climbing plants where I could establish its influence on growth as well as on the nature of geotropic curvature, this is probably the right conception. The paraboloid vegetation point of a stem which bents at its end like a hook, may here take up any sort of position and hardly deserves consideration in connection with gravitational stimuli.

Utrecht, March 1915. *Botanical Laboratory.*

Astronomy. — "*On the mean radius of the earth, the intensity of gravity, and the moon's parallax.* By Prof. W. DE SITTER.

1. NEWCOMB has more than once[1]) pointed out that the mean radius of the earth is more appropriate for use as a standard of reference, than the equatorial radius, which is always used in astronomical practice. The mean radius in fact, which — if we neglect quantities of the second order in the compression — is also the mean radius of curvature, is more nearly the quantity actually

[1]) Researches on the motion of the moon, second paper, page 41
Tables of the sun, page 12, footnote.

determined by geodetic measures, which are practically all made in mean latitudes.

The several definitions of the mean radius[1]) are identical to the first order of the compression ε. I adopt as mean radius the radius at the geographical latitude whose sine is $\sqrt{1/3}$, and which is given by the formula

$$r_1 = b\,[1 - \tfrac{1}{3}\varepsilon + \tfrac{5}{9}\varepsilon^2 + \ldots\ldots] \quad \ldots\ldots \quad (1)$$

HELMERT has recently[2]) collected the following determinations of b, from which I derive the value of r_1 by means of the corresponding value of ε.

1. From four European arcs, all reduced with BESSEL's $\varepsilon^{-1} = 299.15$.

$$b = 6378150 \qquad r_1 = 6371077$$

2. From arcs in India and South-Africa, reduced with $\varepsilon^{-1} = 298.3$.

$$b = 6378332 \qquad r_1 = 6371237$$

3. From the geodetic measures in the United States, reduced with $\varepsilon^{-1} = 296.96$.

$$b = 6378388 \qquad r_1 = 6371268$$

It will be seen that the agreement of the several values of r_1 is much better than of b.

Combining these values of r_1 with the weights assigned by HELMERT to the corresponding values of b, we find

$$r_1 = \mathbf{6371237 \pm 49} \quad \ldots\ldots\ldots \quad (2)$$

The mean error has been derived from the residuals. If the values of b are combined in the same way we find from the residuals the mean error $\pm$ 66.

2. A similar reasoning applies to the acceleration of gravity. HELMERT[3]) finds

$$g = 9.78030\,\{1 + 0.005302\,\sin^2\varphi - 0.000007\,\sin^2 2\varphi\}.$$

1) HELMERT, Höhere Geodäsie, I, pages 64—68.

2) Geoid und Erdellipsoid. Zeitschr. der Ges. für Erdkunde, 1913, page 17.

3) Encyclopädie der Math. Wiss.; Band VI. 1 B, Heft 2, page 95. The alternative formula given there, viz :

$$g = 9.78028\,\{1 + 0.005300\,\sin^2\varphi - 0.000002\,\sin^2 2\,\varphi\}$$

must be dismissed, since for theoretical reasons the coefficient of $\sin^2 2\varphi$ must be included between the limits —0 0000055 and —0.0000088. The theoretical expression of the coefficient is $+\frac{11}{1}\varepsilon^2 - \frac{5}{2}\varepsilon\rho - \frac{105}{32}B_4$, where B_4 is necessarily positive, and smaller than $\frac{1}{7}J$. Taking $\varepsilon = 0.00338$, $\rho = 0.00345$, $J = 0.00165$, we find the stated limits. The value of the coefficient in the formula of the text corresponds to DARWIN's value of B_4 viz: 0,0000029.

For $\sin^2 \varphi_1 = 1/_3$, this gives

$$g_1 = 9\ 79752. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

HAYFORD and BOWIE [1]) have

$$g = 9.78038 \{1 + 0.005304 \sin^2 \varphi - 0.000007 \sin^2 2\varphi\},$$

from which

$$g_1 = 9.79762.$$

The fundamental determination at Potsdam by KÜHNEN and FURTWÄNGLER, viz: $g_P = 9.81274$, combined with the value of ε, which will be derived in the following paper, viz: $\varepsilon^{-1} = 296.0$, gives

$$g_1 = 9.79755 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3')$$

I adopt [2]) this last value (3').

We then find the attraction of the earth by the formula

$$g'_1 = \frac{fM}{r_1^{\,2}} = g_1 \{1 + \tfrac{2}{3} \varrho_1 + \tfrac{10}{3} \varepsilon^2 - \tfrac{122}{63} \varepsilon\varrho_1 - \tfrac{115}{36} B_4\}, . \qquad (4)$$

where

$$\varrho_1 = \frac{\omega^2 r_1^{\,3}}{fM} = \frac{\omega^2 r_1}{g'_1} = 0.0034496\ ^{3}),$$

$$\varepsilon = 0.00338, \qquad\qquad B_4 = 0.0000029,$$

which gives

$$g'_1 = \mathbf{9.82014} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

3. Now let $\pi' = \dfrac{\sin \pi}{\sin 1''}$ be the constant of the lunar parallax. By BROWN's theory we have

$$\pi' = [0.0003940] \frac{b}{a},$$

where the number in square brackets is a logarithm, and by KEPLER's third law

$$a^3 n^2 = fM(1+\mu)\ ^{4}).$$

We find thus

1) Effect of Topography and Isostatic compensation upon the intensity of gravity (second paper) U. S. Coast and Geod. Survey, special publ. No. 12, page 25.

2) In the original Dutch communication the value (3) was adopted. The difference is negligible.

3) The quantity which is commonly used is

$$\varrho_0 = \frac{\omega^2 b}{g_0} = \varrho_1 + \tfrac{3}{2} \varrho_1^{\,2} = 0.0031676.$$

4) Strictly speaking this value of M is not exactly the same as that used in (4), since the latter is exclusive of the atmosphere. The mass of the atmosphere is 0.000000865 M. The effect on π' is 0''.001.

$$\pi'^3 = \frac{[0.0011820]}{\sin 1''} \cdot \frac{n^2}{1+\mu} \cdot \frac{b^3}{r_1^3} \cdot \frac{r_1}{g_1'} \cdot \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

Using now the value (1) of b, or

$$\frac{b}{r_1} = (1 + \tfrac{1}{3}\varepsilon)(1 - \tfrac{1}{9}\varepsilon^2),$$

and the values (2) and (5) of r_1 and g_1', and taking

$$\mu^{-1} = 81.50 \pm 0.07$$

we find

$$\pi' = 3418''.695\,(1 + \tfrac{1}{3}\varepsilon). \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

The uncertainty of the numerical factor may be estimated as follows:

due to r_1	. . .	$\pm$ 0''.008
,, ,, g_1	. . .	$\pm$ 0 .006
,, ,, μ	. . .	$\pm$ 0 .010.

In the following paper we will derive the value

$$\varepsilon^{-1} = 296.0 \pm 0.2.$$

This gives

$$\boldsymbol{\pi' = 3422''.544 \pm 0''.015} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

The mean error includes the effects of r_1, g_1 and μ as given above, to which has been added:

due to ε . . $\pm$ 0''.0025.

From the recent determination of the lunar parallax by the observatories at Greenwich and the Cape [1]) — assuming the corrections given to be applicable to HANSEN's parallax 3422''.07 — we find the following comparison:

ε^{-1}	*Cape-Greenwich*	*Formula* (7)
293	3422''.60	3422''.58
294	.54	.57
295	48	.55
296	.42	54[5]
297	.36	.53.

This would give:

$$\varepsilon^{-1} = 293.4, \qquad \pi' = 3422''.58.$$

With $\varepsilon^{-1} = 296.0$ would correspond the observed value $\pi' = 3422''.42$, which is 0''.12 smaller than (8). It does not appear impossible to ascribe this quantity to errors of observation, especially to a constant error of pointing on the Crater Mosting A by the observers at Greenwich and the Cape.

[1]) Monthly Notices, Vol. LXXI, page 526.

The equation (6) has in the course of time been used for the determination of μ, of r_1 and of ε. It is, however, doubtful whether the accuracy, needed to derive a real correction to our present knowledge of any of these constants, could be attained even by a series of observations such as is proposed by E. W. Brown in his address to the British Association in Australia. It certainly should determine the parallax within a fraction of $\pm$ 0''.01 to be of real value. To make this possible the selenocentric coordinates, especially the radius-vector of the Crater Mösting A, or any other feature of the lunar surface which is used for the determination, must be accurately known. The determinations of the height of Mosting A over the mean radius are:

Hayn [1])	$+ 2''.2 \pm 0''.6$	effect on π' . . .	0''.037
Stratton [2])	$+ 3\ .0 \pm 0\ .7$	,, ,, ,, . . .	0 .049.

The difference between the two determinations makes a difference in the parallax larger than the uncertainty due to any of the constants r_1, g_1, μ or ε.

Our conclusion is thus that the value (8) of the lunar parallax is more accurate than any that can at present be derived by direct observations.

Geodesy. — *"On Isostasy, the Moments of Inertia, and the Compression of the Earth"*. By Prof. W. de Sitter.

1. The hypothesis of isostasy is strictly speaking a compound of two hypotheses, viz.:

A. Up to a certain distance from the centre the constitution of the earth is in agreement with the theory of Clairaut, i. e. the equipotential surfaces are surfaces of equal density, and the density never increases [3]) from the centre outwards. [Apart from this condition it may vary in any manner, even discontinuously.] The last

[1]) Selenographische Koordinaten. III. (1907). Abh. der K. Sächs. Ges. der Wiss. Band XXX. page 74.

[2]) Memoirs of the R. A. S. Vol. LIX, Part IV, page 276.

[3]) Strictly speaking it is not necessary that always $\frac{d\Delta}{db} \leq 0$. It is sufficient if, for all values of b, $\int_0^b \beta^3 \frac{d\Delta}{d\beta}\, d\beta \leq 0$, and $\int_0^b \beta^5\, \varepsilon \frac{d\Delta}{d\beta}\, d\beta \leq 0$.