The equation (6) has in the course of time been used for the determination of μ, of r_1 and of ε. It is, however, doubtful whether the accuracy, needed to derive a real correction to our present knowledge of any of these constants, could be attained even by a series of observations such as is proposed by E. W. BROWN in his address to the British Association in Australia. It certainly should determine the parallax within a fraction of $\pm$ 0″.01 to be of real value. To make this possible the selenocentric coordinates, especially the radius-vector of the Crater Mösting A, or any other feature of the lunar surface which is used for the determination, must be accurately known. The determinations of the height of Mösting A over the mean radius are:

HAYN[1])	$+ 2''.2 \pm 0''.6$	effect on π' . . .	0″.037
STRATTON[2])	$+ 3\ .0 \pm 0\ .7$	„ „ „ . . .	0 .049.

The difference between the two determinations makes a difference in the parallax larger than the uncertainty due to any of the constants r_1, g_1, μ or ε.

Our conclusion is thus that the value (8) of the lunar parallax is more accurate than any that can at present be derived by direct observations.

Geodesy. — *"On Isostasy, the Moments of Inertia, and the Compression of the Earth"*. By Prof. W. DE SITTER.

1. The hypothesis of isostasy is strictly speaking a compound of two hypotheses, viz.:

A. Up to a certain distance from the centre the constitution of the earth is in agreement with the theory of CLAIRAUT, i. e. the equipotential surfaces are surfaces of equal density, and the density never increases[3]) from the centre outwards. [Apart from this condition it may vary in any manner, even discontinuously.] The last

[1]) Selenographische Koordinaten. III. (1907). Abh. der K. Sächs. Ges. der Wiss. Band XXX. page 74.

[2]) Memoirs of the R. A. S. Vol. LIX, Part IV, page 276.

[3]) Strictly speaking it is not necessary that always $\frac{d\Delta}{db} \leq 0$. It is sufficient if, for all values of b, $\int_0^b \beta^3 \frac{d\Delta}{d\beta}\, d\beta \leq 0$, and $\int_0^b \beta^5\, \varepsilon\, \frac{d\Delta}{d\beta}\, d\beta \leq 0$.

equipotential surface which satisfies these postulates is called the *isostatic surface*, and will be denoted by S_0.

B. In the crust outside S_0 the distribution of mass is such that over sufficiently large areas of S_0 there is the same mass as there would be with a certain normal distribution. How exactly this normal distribution is supposed to be, is generally not explicitly stated. In any case with the normal distribution the whole mass of the crust would be inclosed between S_0 and a certain *normal surface* S.

The actual surface of the earth is neither an equipotential surface, nor a surface of equal density. The actual surfaces of the oceans may be supposed to be parts of one and the same equipotential surface, which is called the *geoid*. The figure of this geoid is derived from geodetic measures made on the continents or from determinations of the intensity of gravity made on the continents and on the sea. It has been found that the geoid differs very little from an ellipsoid of revolution. This "ellipsoid of reference" may be taken to be identical with the normal surface, or more precisely the several ellipsoids of reference found from each separate investigation are considered to be approximations to the normal surface. The latter is thus determined as the ellipsoid best fitting the several partial ellipsoids of reference.

2. On the basis of the theory of isostasy we must consider the isostatic surface S_0 as primarily given, though of course its figure is unknown, and must be determined from that of S. Now the relation between S_0 and S is not very explicitly stated by the different authors on the subject.

The most natural assumption evidently is that S would be a equipotential surface and a surface of equal density. The normal surface satisfying these conditions, which are those of the theory of CLAIRAUT, will be called the *ideal surface* of the earth, and will be denoted by S_1.

When HELMERT originally introduced the method of condensation, he supposed the radius-vector of the surface of condensation to be proportional to that of the normal surface: $r_0 = r\,(1-\alpha)$. In the reductions according to the theory of isostasy the isostatic surface S_0 corresponds to HELMERT's surface of condensation. The normal surface would then be given by $r = r_0\,(1-\alpha)^{-1}$. This surface may be called the *proportional surface*, and will be denoted by S_2.

Some authors also state as a definition that the depth of the isostatic surface below the normal surface is constant. We should thus have $r = r_0 + Z$. The surface so defined may be called the *equidistant surface*, and will be denoted by S_3.

Let

$b =$ the aequatorial radius } of any surface,
$\varepsilon =$ the compression }

Further

$$\eta = \frac{b}{\varepsilon}\frac{d\varepsilon}{db},$$

then we have approximately

$$\varepsilon_1 - \varepsilon_0 = \frac{\eta\varepsilon}{b}(b_1 - b_0).$$

For the earth we have $\eta_1 = 0.561$. Taking $\frac{b_1 - b_0}{b} = 0.0179$, and $\varepsilon = 0.00338$, we find

$$\varepsilon_1 - \varepsilon_0 = + 0.000034$$

The difference of the numerators is

$$\varepsilon_1^{-1} - \varepsilon_0^{-1} = - 3.0 \ ^{1)}.$$

---

1) A better approximation is obtained by also taking into account the variation of ν. Let

$\Delta =$ the density at } any equipotential surface,
$D =$ the mean density within }

and

$$\zeta = - \frac{b}{D}\frac{dD}{db},$$

then the theory of CLAIRAUT gives, neglecting the second order in ε

$$\zeta = 3\left(1 - \frac{\Delta}{D}\right)$$

$$b\frac{d\eta}{db} = 2\zeta(1+\eta) - 5\eta - \eta^2.$$

If the crust were constituted in accordance with the theory of CLAIRAUT, it would consist of a solid crust entirely covered by an ocean of a depth of about 2.4 km. The bottom of this ocean would be an equipotential surface, say S_b. For S_1 we have now

$$\Delta_1 = 1.03 \quad D_1 = 5.52$$

from which we find

$$\zeta_1 = 2.44.$$

Then, with $\eta_1 = 0.561$, we find

$$b_1\left(\frac{d\eta}{db}\right)_1 = 4.50.$$

Therefore, since $b_1 - b_b = 0.00038\, b_1$, we have

$$\eta_b = \eta_1 - (b_1 - b_b)\left(\frac{d\eta}{db}\right)_1 = 0.559.$$

For the surface S_b we then have

For the proportional surface we have, of course,

$$\varepsilon_2 = \varepsilon_0.$$

The equidistant surface is not an exact ellipsoid, but it differs only in quantities of the second order in ε from the ellipsoid whose compression is,

$$\varepsilon_3 = \frac{\varepsilon_0}{1 + \frac{1}{3}\varepsilon_0 + k} = 0.979\ \varepsilon_0.$$

where $k = \frac{Z}{b}$. Therefore

$$\varepsilon_3 - \varepsilon_0 = -\ 0.000070$$
$$\varepsilon_3^{-1} - \varepsilon_0^{-1} = +\ 6.1.$$

The depth of the isostatic surface below the normal surface is in the three cases

$$r_1 - r_0 = kb\,[1 + \varepsilon\,(1+\eta)\,(\tfrac{1}{3} - sin^2\,\varphi)],$$
$$r_2 - r_0 = kb\,[1 + \varepsilon\,(\tfrac{1}{3} - sin^2\,\varphi)],$$
$$r_3 - r_0 = kb.$$

or, expressed in kilometers

$$r_1 - r_0 = 114 + 0.59\,(\tfrac{1}{3} - sin^2\,\varphi),$$
$$r_2 - r_0 = 114 + 0.38\,(\tfrac{1}{3} - sin^2\,\varphi)$$
$$r_3 - r_0 = 114.$$

The difference between the three definitions of the relation of the isostatic and the normal surfaces is thus considerable, especially in its effect on the compression. If the undisturbed surface of the different oceans are parts of one and the same equipotential surface, which is the geoid, and if at the same time the geoid does not differ more than a few tens of meters [1]) from an ellipsoid of revolution,

---

$$\Delta_b = 2.73 \qquad \zeta_b = 1.52 \qquad b_b\left(\frac{d\eta}{db}\right)_b = 1.63.$$

Further if we put $\bar{b} = \frac{1}{2}(b_1 + b_0)$, we have $b_b - b_0 = 0.0177\ \bar{b}$, and consequently

$$\eta_0 = \eta_b - 0.0177 \times 1.63 = 0.530.$$

Taking now

$$\bar{\eta} = \tfrac{1}{2}(\eta_1 + \eta_0) = 0.546, \qquad \bar{\varepsilon} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_0), \qquad b_1 - b_0 = 0.0181\ \bar{b},$$

we find

$$\varepsilon_1 - \varepsilon_0 = 0.0181\ \bar{\eta}\,.\,\bar{\varepsilon} = 0.0099\ \bar{\varepsilon}.$$

Taking $\bar{\varepsilon} = 0.00336$, we have

$$\varepsilon_1 - \varepsilon_0 = 0.000033.$$
$$\varepsilon_1^{-1} - \varepsilon_0^{-1} = -\ 2.9.$$

1) HELMERT, Geoid und Erdellipsoid, Zeitschr. der Ges. für Erdkunde, 1913, p. 17—34.

we cannot but take this latter as the normal surface. In that case the normal surface is very nearly an equipotential surface. The deviations of the geoid from the ellipsoid, or, which is the same thing, of the normal surface from the equipotential surface, are caused by the irregularities in the crust. They would be very much larger — in fact of the order of 1000 meters [1]) — if there were no isostatic compensation. If this point of view is adopted, then the normal surface can differ only very little from the "ideal" surface S_1 as defined above. This will be assumed in what follows and no further reference will be made to the surfaces S_2 and S_3. They were only discussed here to point out the necessity of precision in the definition of the relation between the isostatic and the normal surfaces.

3. Let $A < B < C$ be the moments of inertia of a body about the axes of x, y, z. If the body rotates about the axis of z with the velocity ω, then the outer surface, if it is an equipotential surface, is very nearly [2]) an ellipsoid whose principal axes are

$$b, \quad b(1-\nu), \quad b(1-\tfrac{1}{2}\nu)(1-\varepsilon).$$

If $C-A$ and $C-B$ are of the first order of smallness, and $B-A$ of the second order, and if

$$J = \tfrac{3}{2}\,\frac{2\,C-A-B}{2\,Mb^2}, \qquad K = \tfrac{3}{2}\,\frac{B-A}{Mb^2},$$

then to the second order inclusive we have

$$\varepsilon = J + \tfrac{1}{2}\,\varrho_1 + \varepsilon^2 - \tfrac{1}{2}\,\varepsilon\,\varrho_1 - \tfrac{3}{8}\,B_4 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\nu = K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The radius of the equator in longitude λ is $b\,[1-\nu\,\sin^2(\lambda-\lambda_0)]$, if λ_0 be the longitude of the axis of x. The compression of the meridian in longitude λ is thus $\varepsilon_\prime = \varepsilon + \tfrac{1}{2}\,\nu\cos 2\,(\lambda-\lambda_0)$. Consequently ε is the average compression of the meridians.

The value of ϱ_1 in (1), viz.

$$\varrho_1 = \frac{\omega^2 r_1}{g'_1} = 0.0034496,$$

can be assumed to be exactly known. Further

$$B_4 = 0.0000029.$$

The equation (1) can thus be written

$$\varepsilon = J + 0.0017287 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1')$$

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1) Helmert, Höhere Geodäsie, II, p. 356.

2) The deviation from the ellipsoid is $-\varkappa b \sin^2 2\,c$, where

$$\varkappa = \tfrac{5}{8}\,\varepsilon\varrho - \tfrac{7}{8}\,\varepsilon^2 + \tfrac{35}{32}\,B_4 = 0.0000051,$$

or $b\varkappa = 3\,26$ meters Darwin, Scientific Papers, Vol. III, p. 102.

and the uncertainty in the numerical part is no more than a few units in the last decimal place given.

We also need the ratio

$$H = \frac{2\,C - A - B}{2\,C}.$$

For the ideal surface we have $A_1 = B_1$, and consequently

$$J_1 = \tfrac{1}{2} \frac{C_1 - A_1}{M b_1{}^2}, \qquad H_1 = \frac{C_1 - A_1}{C_1},$$

The true moments of inertia A and B may however be unequal.

The ratio H can be determined with great accuracy from the constant of precession. The best modern determinations of this constant are (for 1850):

NEWCOMB (with corrections by HOUGH and HALM) [1]	$p_1 = 50''.2486$
BOSS [2]	50 .2511
DYSON and THACKERAY [3]	50 .2503

We can thus take

$$p_1 = 50''.2500 \pm 0''.0010.$$

The lunisolar precession then becomes

$$p = 50''.373.$$

If now we take for the mass of the moon

$$\mu^{-1} = 81.50 \pm 0.07,$$

we find

$$H = 0.0032775 \pm 0.0000022.$$

The uncertainty is almost entirely due to μ and not to p.

So far no assumptions have been made regarding the constitution of the earth. The theory of CLAIRAUT now leads to a determination of the ratio of J and H. We are thus able from H to compute J, and then ε from (1'). RADAU's transformation of CLAIRAUT's differential equation gives, to the first order of ε [4]),

$$g = \frac{J}{H} = \tfrac{1}{2} \frac{C}{Mb^2} = 1 - \tfrac{2}{5} \frac{\sqrt{1+\eta}}{F_0} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where, also to the first order, $\eta = 3 - 5 \dfrac{J}{\varepsilon}$, and F_0 is a certain

1) Monthly Notices, Vol. LXX; p. 587. See also: The Observatory, July 1913, p. 299.

2) Astronomical Journal, Vol. XXVI. p. 148.

3) Monthly Notices, Vol. LXV, p. 443.

4) This and other formulas of the theory of CLAIRAUT will be collected in the following paper.

mean value of a function F of η which differs very little from unity for values of η between 0 and η_1.

If the formula (3) is extended to the second order, it becomes very complicated. The range of F_0 becomes wider, and therefore also of g and ε. The formula has been elaborated by DARWIN [1]) and VÉRONNET [2]). The formulae given by these two authors are very different. DARWIN starts from a definite assumption regarding the constitution of the earth, and thus finds a definite value of ε. VÉRONNET introduces no assumptions, and consequently only gives limits for ε. Introducing the above value of H we find:

DARWIN . . . . $\varepsilon^{-1} = 296.03$.

VÉRONNET . . . $295.84 < \varepsilon^{-1} < 296.68$.

The lower limit of ε^{-1} corresponds to the case of homogeneity, the upper limit to concentration of the whole mass in the centre. There can be no doubt, but that the actual distribution is nearer the first limit. The agreement of the results of DARWIN and VÉRONNET is thus complete, and we can adopt the value derived from DARWIN's formula. The m. e. of ε^{-1} due to the uncertainty of H is ± 0.16. From the agreement of the results of DARWIN and VÉRONNET we may conclude that any probable hypothesis regarding the constitution of the earth differing from that of DARWIN would not cause in ε^{-1} a difference exceeding say ± 0.10. We thus estimate the total uncertainty of ε^{-1} at ± 0.19.

4. However, the value of H used above is the ratio of the *true* moments of inertia. The equation (3) on the other hand is only applicable to the *ideal* surface. We must thus try to derive the values of J_1 and H_1 for the ideal surface from the true values J and H, and at the same time determine the difference $\varepsilon - \varepsilon_1$ of the compressions of the normal and the ideal surfaces. This will be done on the basis of the hypothesis of isostasy.

The normal surface is the ellipsoid best fitting the geoid. The potential on the geoid depends on the true moments of inertia. The compressions ν and ε of the normal surface are therefore derived by the equations (1) or (1') and (2) by using the true values of J and K. The equation (1) or (1') also applies to the ideal surface. Consequently

[1]) The theory of the figure of the earth to the second order of small quantities. Scientific Papers, Vol. III, p. 78 - 118.

[2]) Rotation de l ellipsoide hétérogène et figure exacte de la Terre. Journal des Math. 1912, 4me fascicule.

86*

$$\varepsilon - \varepsilon_1 = J - J_1.$$

The change in H due to the change in C in the denominator is very small (of the order of $^1/_{300}$) compared with the effect of the change in the numerator. Consequently

$$J - J_1 = g\,(H - H_1).$$

and

$$\varepsilon - \varepsilon_1 = g\,(H - H_1) = 0.502\,(H - H_1). \quad . \quad . \quad . \quad . \quad (4)$$

The part contributed towards the moments of inertia by an element of mass m at latitude φ, longitude λ, and distance from the centre r is

$$dC = mr^2 \cos^2 \varphi\,,$$
$$dA = mr^2\,[1 - \cos^2 \varphi \cos^2 (\lambda - \lambda_0)]\,,$$
$$dB = mr^2\,[1 - \cos^2 \varphi \sin^2 (\lambda - \lambda_0)]\,,$$

from which

$$d\,[C - \tfrac{1}{2}(A + B)] = mr^2\,(1 - 3 \sin^2 \varphi)$$
$$d\,[B - A] = mr^2 \cos^2 \varphi \cos 2\,(\lambda - \lambda_0).$$

If now over a surface element ω of the ideal surface the height of the continent is h_1 and the mean density Δ, then the mass is $m = \omega \Delta h_1$. If Z_1 is the depth of the isostatic surface below the ideal surface, the defect of density needed to compensate this mass, if equally distributed over the whole depth, is $\delta = \Delta \frac{h_1}{Z_1}$. The change in Σmr^2 produced by the continent and its isostatic compensation then is, if r_1 be the radius vector of the ideal surface:

$$d\,(\Sigma mr^2) = \int_{r_1}^{r_1 + h_1} \Delta \omega\, x^2 dx - \int_{r_1 - Z_1}^{r_1} \delta \omega x^2 dx = \Delta \omega h_1\,(Z + h_1)(r_1 - \tfrac{1}{3} Z_1 + \tfrac{1}{3} h_1), \quad . \quad (5)$$

Similarly for an oceanic element, let d_1 be the depth of the bottom of the ocean below the ideal surface and Δ' the difference of density between the water and the mean density of the crust. The compensating excess of density below the sea then becomes $\delta' = \frac{d_1}{Z_1 - d_1}\,\Delta'$, and the change in Σmr^2 is

$$d'\,(\Sigma mr^2) = \Delta' \omega d_1\,[(- Z_1 + 2 d_1)\, r_1 + \tfrac{1}{3} Z_1^2 + \tfrac{1}{3} Z_1 d_1]. \quad . \quad . \quad (6)$$

It has been found sufficiently exact for our purpose instead of (5) and (6) to use the approximate formulas

$$d\,(\Sigma mr^2) = q \,.\, h_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$
$$d'(\Sigma mr^2) = -\,0.57\, q \,.\, d_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

The height h_1' above the ideal surface is the sum of the height h above the normal surface and the height h' of the normal above the ideal surface. This latter is

$$h' = (\varepsilon - \varepsilon_1)\, b_1 \left(\tfrac{1}{3} - \sin^2 \varphi\right).$$

Taking $Z_1 = 0.0179\ r_1$, and $\Delta_1 = 2.70$, and integrating over the whole surface we find for this part of $H - H_1$, using also (4):

$$\delta' H = 0.023\, (\varepsilon - \varepsilon_1) = 0.012\, (H - H_1) \quad . \quad . \quad . \quad . \quad (7)$$

The principal part of $H - H_1$ is due to the deviation of the actual surface from the normal surface. This has been computed by (5′) and (6′), replacing h_1 and d_1 by h and d respectively. The value of the constant q depends on Z and on the units used. I have adopted $\Delta = 2.70$, $\Delta' = 1.70$[1]), $Z = 114$ km.

The surface of the earth was divided into compartments of about 100 square degrees. For each compartment the value of

$$Q = q\omega\, (\alpha_1 h - 0.57\, \alpha_2 d)$$

was computed, where α_1 and α_2 are the fractions of the compartment covered by land and by sea respectively (so that $\alpha_1 + \alpha_2 = 1$). Further

$$P = Q\, (1 - 3 \sin^2 \varphi)$$
$$R = Q \cos^2 \varphi \cos 2\lambda$$
$$S = Q \cos^2 \varphi \sin 2\lambda.$$

The units had been so chosen that

$$\delta \frac{2C - A - B}{2C} = 10^{-7}\, \Sigma P$$

$$\delta \frac{B - A}{C} = 10^{-7} \{\Sigma R \,.\, \cos 2\lambda_0 + \Sigma S \,.\, \sin 2\lambda_0\},$$

The longitude λ_0 is determined by

$$\Sigma S \cos 2\lambda_0 - \Sigma R \sin 2\lambda_0 = 0.$$

I found the following results. (See table p. 1304).

We find thus

$$\delta \frac{2C - A - B}{2C} = -\ 0.00000512$$

$$\delta \frac{B - A}{C} = +\ 0.00000205,$$

and the axis of minimum moment of inertia (A) is situated in the longitude

$$\lambda_0 = 86.^\circ 5 \text{ West of Greenwich.}$$

This computation, of course, is rather rough. It would perhaps be worth while to repeat it with greater care. The small influence of the continents, especially of Asia, is somewhat surprising. This

[1]) The normal density of the crust in the upper few kilometers *below* the normal surface was thus taken to be 2.73, and the density of the land projecting above that surface 2.70.

Parts of the world.	ΣP	ΣR	ΣS
1. North Polar Area	+ 2 44	— 0.02	+ 0.03
2. Europe	— 0.83	+ 0.39	— 0.47
3. Asia	— 1.51	— 5.72	— 0.19
4. North-America	— 3.64	— 1.36	— 1.28
5. Northern Atlantic Ocean	— 5.00	— 0.23	— 11.36
6. South-America	+ 3 21	— 2.16	+ 2.56
7 Southern Atlantic Ocean	— 0.45	— 11.65	— 6.36
8. Africa	+ 3.55	+ 2 22	— 3 29
9. Indian Ocean	— 2.58	+ 15.11	+ 7 09
10 Indian Archipelago and Australia	— 2.14	+ 1.12	— 1.57
11. Pacific Ocean	— 29.97	— 17.96	+ 17.97
12. South Polar Area	— 14.27	— 0.03	+ 0.02

is due to the remarkable fact that the great mountainous regions of the earth (Himalaya, the Alps, Rocky Montains, the higher part of South Africa) are situated on or near the neutral latitude of which the *sine* is $\sqrt{1/_3}$ [$\varphi = 35°.3$].

The value of δH found here is not yet exact, for if the crust were built according to the theory of CLAIRAUT it would consist of a solid crust covered by an ocean of a mean depth of about 2.4 km In the above computation this ocean has been taken of the density 2.73 instead of 1.03. To remedy this we must apply a correction, which by the theory of CLAIRAUT is

$$\delta_1 (C-A) = \tfrac{8}{15} \pi \int_{b_1-2.4}^{b_1} \Delta' \frac{d}{d\beta} (\beta^5 \varepsilon)\, d\beta = \tfrac{8}{15} \pi \,.\, 2.4\, (5+\eta)\, b^4 \varepsilon.$$

This gives

$$\delta_1 H = + \; 0.00000213.^{1)}$$

The bottom and the surface of this ocean would be ellipsoids of revolution, the neglect has therefore no effect on the value of $B-A$.

There now remains

$$\delta H = - \; 0.00000299.$$

1) There is an error of computation in this number. It should be +0.00000260. The final value then becomes $\varepsilon^{-1} = 295.95$. The difference from the value in the text in negligible. (Added in the English translation.)

Adding this to $\delta' H$ as given by (7) we have altogether

$$H - H_1 = -0.00000299 + 0.012\,(H - H_1)$$

or

$$H - H_1 = -0.0000031,$$

Then we find by (4)

$$\varepsilon - \varepsilon_1 = -0.0000016$$
$$\varepsilon^{-1} - \varepsilon_1^{-1} = +0.14.$$

From

$$H = 0.0032775$$

we find thus

$$H_1 = 0.0032806.$$

DARWIN's equation then gives

$$\varepsilon_1^{-1} = 295.82,$$

and from the equation of VÉRONNET we find

$$295.62 < \varepsilon_1^{-1} < 296.46.$$

It has already been mentioned that DARWIN's value may be assumed to be very near the truth. Adopting this and adding the value of $\varepsilon^{-1} - \varepsilon_1^{-1}$, which has been found above, we have[1])

$$\varepsilon^{-1} = 295.96.$$

It is very difficult to estimate the uncertainty of the correction $H - H_1$, since it depends not only on the correctness of the data used, but also, and probably for the greater part, on the exactness of the hypothesis that the compensating defect or excess of density is distributed equally over the whole depth Z. The whole correction to ε^{-1} however only amounts to 0.07, and its uncertainty is almost certainly overestimated if we take it equal to the whole amount, ± 0.07. Combining this with the m e. ± 0.19 due to the uncertainty of H, and of DARWIN's hypothesis, the total uncertainty of ε^{-1} is found to be ± 0.20.

The greater part of this is due to the uncertainty of H, and this is wholly due to that of the adopted value of the moon's mass. Consequently, in order to improve our knowledge of ε we must determine μ, which is found from the lunar inequality of the sun's longitude and the solar parallax. A correction of $+0.05$ to the adopted value of μ^{-1} would give -0.10 in ε^{-1}.

For the ideal surface $B_1 = A_1$, or $K_1 = 0$. Therefore for the normal surface

$$\nu = K = \tfrac{3}{2} \frac{C}{Mb^2} \cdot \frac{B - A}{C} = 0.00000103.$$

The longest radius of the equator, in the longitude 86°.5 is thus

1) See note on p. 1304.

6.4 meters longer than the shortest radius. The compression of the meridian ε, varies between $\varepsilon + \frac{1}{2}\nu$ and $\varepsilon - \frac{1}{2}\nu$. For central Europe, $\lambda = -30^\circ$, we find:

$$(\varepsilon_E)^{-1} = 295.98$$

and for North-America, $\lambda = 100^\circ$

$$(\varepsilon_A)^{-1} = 295.92.$$

5. The methods mostly used for the determination of the compression of the earth are:

I. From geodetic measures,
II. From the intensity of gravity,
III. From the moon's parallax,
IV. From the lunar theory.

By the first method the geodetic measures made in the United States of America give

$$\varepsilon^{-1} = 297.0 \pm 1.2 \quad \ldots \ldots \quad (I)$$

This agrees within the limits of the mean error with the value 296.0 found above.

From a great number of determinations of the intensity of gravity HELMERT derived

$$\varepsilon^{-1} = 298.3 \pm 1.1 \quad \ldots \ldots \quad (II)$$

This result agrees with the final result from the American determinations, viz.:

$$\varepsilon^{-1} = 298.4 \pm 1.5 \quad \ldots \ldots \quad (II')$$

In judging the value of these results it must be remembered that both the direction (method I) and the intensity (method II) of gravity, before they are used for the determination of the figure of the geoid, or of an ellipsoid of reference, need certain corrections, which have been applied by different investigators more or less in agreement with the hypothesis of isostasy. All investigators however use approximate formulas, and it is not clear which of the definitions, treated in art. 2 above, has been adopted. The American investigators take a constant depth below the *actual* surface of the earth (under the sea even below the *bottom*). HELMERT uses the reduction as in free air [1]), thus assuming that the isostatic compensation is complete.

Now it is of course impossible from the observations to decide between the three cases of art. 2, and also the corrections computed under the three assumptions will be very nearly equal. But small

[1]) The American observations reduced by the free air method give instead of (II') $\varepsilon^{-1} = 292.1 \pm 1.7$. See BOWIE, Effect of topography and isostatic compensation upon the intensity of Gravity, second paper, p. 26.

differences in the radius of curvature, or in the values of g, have a large influence on the compression, and it seems not impossible that the resulting value of ε has been influenced by inaccuracies in the reductions. Discussing the large difference between the compressions found by BESSEL ($\varepsilon^{-1} = 299.15$) and CLARKE (293.47) partly from the same observations, HELMERT [1]) asserts that this difference can be fully explained by a difference of a few meters in the adopted height of the geoid over the normal surface. If this is so, we can expect that considerably larger differences of the isostatic reduction will lead to similar effects [2]).

For these reasons it appears to me that the agreement of the three values (I), (II) and (II') can only be accidental. It is not at all certain a priori whether they refer to the same normal surface, and their uncertainty undoubtedly is considerably larger than would be inferred from the mean errors. [3])

From the lunar parallax we found in the preceding paper

$$\varepsilon^{-1} = 293.4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(III)}$$

We also showed that the value 296.0 cannot be said to be excluded by the observations.

The lunar theory gives J, from which ε is found by the equation (1'). The principal term, which is commonly used for the deter-

1) Geoid und Erdellipsoid, l.c. p. 18.

2) The values of ε derived from the American determinations by different methods of reduction (and different combinations of stations) are widely divergent. Thus e.g. from the observations in the United States and in Alaska by the isostatic method 300.4 ± 0.7 and by the free air method 291.2 ± 0.7. See BOWIE, l c p. 26. The former of these should properly be quoted instead of (II') as the final result from the American determinations.

3) HELMERT's formula of 1901, from which (II) is derived, reduced to the Potsdam system, is

$$g = 9.78030\,[1 + 0.005302 \sin^2 \varphi - 0.000007 \sin^2 2\,\varphi] \quad . \quad . \quad (\alpha)$$

With the compression $\varepsilon^{-1} = 296.0$, and a constant correction of $+0.00011$ this becomes

$$g = 9.78041\,[\,1 + 0.0052764 \sin^2 \varphi - 0.0000074 \sin^2 2\,\varphi] \quad . \quad (\beta)$$

The residuals of these two formulas for different zones of latitude are as follows, expressed in units of 0.00001:

Zone	5°	15°	25°	35°	45°	55°	65°	75°
(α)	+7	0	−20	+6	+6	+11	−7	−3
(β)	−4	−9	−4	+3	+8	+17	+3	+10

The m. e. of each of these residuals is ± 11. The residuals β naturally are somewhat systematic, but they are not larger than (α), and can very well be due to errors of observation or inaccuracies in the reductions. A new discussion on the basis of the theory of isostasy, and including the valuable material, which has become available since 1900, is very desirable. [Note added in the English translation].

mination of J, is a periodic term in the latitude, whose period is one month and whose coefficient is, by BROWN's theory:[1])

$$B = -[3.7046]\,J - 0''.017.$$

From the observations BROWN finds[2])

$$B = -8''.19 \pm 0''.06 - [0''.40 \pm 0''.20]\,.\,T,$$

where T is the time expressed in centuries and counted from 1850.0. If we take the mean epoch of the observations, i.e. about 1875, we find[3]) $J = 0.001633$, and consequently

$$\varepsilon^{-1} = 297.3 \pm 1.3. \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(IV)}$$

It appears to me that this determination is not very reliable, chiefly on account of the large and uncertain coefficient of T in the observed value. BROWN proposes to use it not to determine ε, but the inclination of the ecliptic and its secular variation. It seems very doubtful whether a correction to these elements thus determined would be a real improvement to our knowledge of them derived from other sources.

A great weight is attributed by BROWN to the determination of J from the motion of the perigee and the node. He finds

$$\varepsilon^{-1} = 293.5 \pm 0.5 \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(IV')}$$

In deriving the m.e. no account has been taken of the uncertainty of the theoretically determined part of these motions due to other causes. Among these other causes, however, is the figure of the moon, which is very imperfectly known. It will be shown in the following paper that it is very well possible to adopt such values for the quantities defining this figure, that the motions of the perigee and the node are in agreement with the value $\varepsilon^{-1} = 296.0$. Smaller values of ε however lead to very improbable conclusions regarding the constitution of the moon.

All our discussions thus lead to the conclusion that none of the other determinations is equal in accuracy to, or can throw a doubt on the determination from the constant of precession. We must therefore adopt as final value of the compression the result of this determination, viz:

$$\frac{1}{\varepsilon} = \mathbf{295.96 \pm 0.20}.$$

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1) Part V, Chapter XIII. (Memoirs of the R. A. S, Vol. LIX, Part I). On p. 80 the inequality is given as $-8''.355\, sin\,(w_1 + \psi)$. This should be $-8''.553$.

2) Monthly Notices, Vol. LXXIV, p. 564. BROWN gives probable errors, which I have changed to mean errors.

3) The theoretical value for 1875, corresponding to $\varepsilon^{-1} = 297.0$ is $-8''.312$, the observed value is $-8''.28$. The difference is therefore $O - C = +0''.03$ and not $-0''.03$ as stated by BROWN, l.c. p. 565.