this line continues to proceed to lower temperatures, and p continues to rise to infinitely great; or that after having reached certain minimum of temperature, it proceeds to higher temperatures, again before p has become infinitely great. If the latter should be the case there is in the p,T-line of the coexistence a point in which $\frac{dp}{dT}=\frac{\eta_l - \eta_s}{v_l - v_s}=\infty$, or $v_l = v_s$. And for the points of the line of coexistence which lie higher, v_l has then become $> v_s$, and the above described case occurs again. Nor is there a difference of significance when v always remains greater than v_l. At the temperature of the disappearance of the solid state, which is then lower than the temperature of the triple point, this disappearance takes place at a pressure equal to infinity. Since, however, v_s is always greater than v_l, we can hardly continue to speak of "expelled". The volumes v_s and v_l have now however both become equal to v_0, and on rise of the temperature it is again only the liquid which can exist.

Assuming here again that liquids under a very high pressure, and so in a very small volume, almost equal to v_0, assume the viscous state, we might point out the following difference. If $v_s < v_l$ these substances have the solid state in volumes which are little greater than v_0 at temperatures somewhat below that of the disappearance of the solid state; they have the viscous state on increase of volume, and on further increase of the volume they have the liquid state. If on the other hand $v_s > v_l$, the succession of the 2 solid states is reversed.

At the highest temperature for the existence of the solid state the difference between solid and viscous has probably disappeared under infinite pressure.

Physics. — *"On a system of curves occurring in* EINSTEIN'S *theory of gravitation"*. By CH. H. VAN OS. (Communicated by Prof. H. A. LORENTZ).

In Prof. P. EHRENFEST'S communication on EINSTEIN'S gravitation theory (Vol. XV, p. 1187) a system of ∞^2 curves occurs which is determined by the condition that a hyperboloid:

$$H \equiv A(x^2 + y^2 - z^2) + Bx + Cy + Dz + E = 0, \quad . \quad . \quad (1)$$

a so-called "light hyperboloid" can always be brought through two of these curves. This system will be examined more closely here.

The curves are intersections of the hyperboloids H. As they must also have a conic $K\infty$

$$x^2 + y^2 - z^2 = 0, w = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

at infinity in common, so that their intersections must degenerate, the considered curves are either straight lines or conics.

A. All the curves are straight lines. Then they must either all pass through one point, so that the H's degenerate into planes, or (at most with the exception of one) as generatrices of the H's, form angles of 45° with the z-axis, hence be so-called "light lines".

B. Not all the curves are straight lines. Through one conic pass ∞^1 H's, because these H's have also $K\infty$, so a biquadratic spacial curve in common. Hence they form a sheaf:

$$H_1 + \lambda H_2 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let us take $\frac{B}{A} = \xi, \frac{C}{A} = \eta, \frac{D}{A} = \zeta, \frac{E}{A} = \tau$ as coord. of a point in an R_4, the image point of the corresponding H. The image point of an H of the sheaf (3) then becomes:

$$\frac{\xi_1 + \lambda\xi_2}{1 + \lambda}, \quad \frac{\eta_1 + \lambda\eta_2}{1 + \lambda}, \quad \frac{\zeta_1 + \lambda\zeta_2}{1 + \lambda}, \quad \frac{\tau_1 + \lambda\tau_2}{1 + \lambda}.$$

The image points of the H's of this sheaf therefore form a straight line, the image line of this sheaf.

Two arbitrary sheaves of the system have always an H in common, viz that passing through the base curves of these sheaves. Hence the homologous straight lines intersect each other in the image point of this H. The image lines of the considered sheaves form a system of ∞^2 straight lines in R_4 such that two of these straight lines always intersect. Now two cases may be distinguished:

a. All the straight lines pass through one point. Then all the sheaves have the H whose image point this is, in common, and accordingly their base curves all lie on one hyperboloid. This case is trivial.

b. Not all the straight lines pass through one point. If a plane is brought through two of them, every other will have the points in which it intersects the two first, in common with this plane, and therefore lie entirely in this plane. Then all the image lines and image points lie in one plane. If $(\xi_1, \eta_1, \zeta_1, \tau_1)$, $(\xi_2, \eta_2, \zeta_2, \tau_2)$, $(\xi_3, \eta_3, \zeta_3, \tau_3)$ are three of these image points, the coordinates of the others are:

$$\xi = \frac{\lambda_1\xi_1 + \lambda_2\xi_2 + \lambda_3\xi_3}{\lambda_1 + \lambda_2 + \lambda_3}, \quad \eta = \frac{\lambda_1\eta_1 + \lambda_2\eta_2 + \lambda_3\eta_3}{\lambda_1 + \lambda_2 + \lambda_3} \text{ etc.;}$$

if this is substituted in (1), *we find for the general equation of the H's of the system:*

$$\lambda_1 H_1 + \lambda_2 H_2 + \lambda_3 H_3 = 0.$$

So these form a net. If we put $z = it$, this is changed into a net of spheres. So all the H's, just as all the spheres of a net, pass through two points Ω_1 and Ω_2. Hence the curves which are their intersections also pass all through these points.

Through the joining line Γ of Ω_1 and Ω_2 pass ∞^1 planes. As there are ∞^2 curves, ∞^1 curves lie in each of them.

All of these pass through Ω_1 and Ω_2, and through the points of intersection of their plane with $K\infty$, and so they form a sheaf. Now the general shape of the system is determined.

The following cases may now be distinguished.

I. Γ lies at infinity.

a. Ω_1 and Ω_2 do not lie both on $K\infty$. Then the plane at infinity has a conic and another point in common with each of the H's, and so it constitutes part of it. So the H's degenerate into planes, the curves into straight lines.

b. Ω_1 and Ω_2 both lie on $K\infty$. The curves in each plane have two pairs of coinciding points at infinity in common, and so they are concentric, similar, and similarly placed. The centres in the successive planes are the centres of parallel sections of one of the H's and lie therefore on one straight line.

II. Γ does not lie at infinity. Now the following cases are possible:

1. The angle of Γ with the z-axis is $> 45^\circ$
2. ,, ,, ,, Γ ,, ,, z-axis ,, $= 45^\circ$
3. ,, ,, ,, Γ ,, ,, z-axis ,, $< 45^\circ$

a. Ω_1 and Ω_2 are real. *b.* They coincide.
c. They are conjugate imaginary. *d.* One of them lies at infinity.
e. Both lie at infinity.

In the cases 1*d*, 1*e*, 3*d*, and 3*e* the plane at infinity has besides $K\infty$ another point in common with each of the H's, so that the H's degenerate into planes.

In the cases 2*a*, 2*b*, and 2*c* Γ intersects $K\infty$, and has therefore 3 points, viz. this point of intersection, Ω_1 and Ω_2 in common with each of the H's; so Γ is a common generatrix of it. As the H's have moreover $K\infty$ in common, their further sections, i.e. the considered curves, are straight lines.

At 2*d* the curves in every plane have a twopoint contact, at 2*e* a three-point contact at infinity.

In the cases 1*b* and 1*c* the tangents of all the curves form angles $> 45^\circ$ with the z-axis. If they are considered as world lines, the corresponding velocities are $>$ the light-velocity, which is physically excluded.

REMARK.

It is easy to extend the problem raised by Prof. EHRENFEST to an n-dimensional laboratory, or to an $(n+1)$-dimensional "world".

It must again be possible to bring through two of the world-lines a ruled surface, on which two systems of light-lines lie in such a way that every straight line of the 1st system intersects every line of the 2nd system. If through two straight lines of the first system an R_3 is brought, every straight line of the 2nd system has 2 points in common with this R_3, and is therefore entirely contained in it; the whole surface lies then in an R_3, and is a hyperboloid H.

The curves do not all lie in the same R_n; therefore the hyperboloids do not do so either. Now two H's lying in different R_3's have at most a conic in common in the plane of intersection of these R_3's; so the curves are straight lines or conics.

A. The curves are in general straight lines.

B. " " " " " conics. We will only consider this case.

An H can always be brought through two curves, hence an R_3 can be brought through their planes; these planes therefore always intersect each other along a straight line.

a. All the planes do not pass through one straight line. If an R_3 is brought through two of them, every other plane has two straight lines in common with this R_3, and so it lies entirely in it. Then all the curves would lie in one and the same R_3, which is contradictory to what was put.

b. So all the planes pass through one straight line Γ. Now the curves will all cut Γ in the same two points.

For, if an H *is* brought through 2 of the curves, their points of intersection with Γ are also the points of intersection of this H with Γ. If there were more than two, Γ would be a generatrix of H, but then a plane through Γ could further only have a straight line in common with H, and so the two curves would be straight lines.

In this way it is proved that the world lines are either straight lines or hyperbolae, which all pass through 2 same points of a straight line Γ. In each of the ∞^{n-1} planes through Γ lie again ∞^1 curves, forming a sheaf. The number of fields of gravitation arising in this way in an n-dimensional laboratory therefore amounts to ∞^{2n+2}.