Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

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This may be also shown in the following manner. The direction of the solution path of F is determined by 5 (IV), that of the boundary curve by 16 (IV) and that of the four-phase curve by 8.

If now between the phases occurs a reaction of group c we get:

$$\frac{y-\beta}{x-\alpha} = \frac{y_1-y}{x_1-\alpha} = \frac{y_1-\beta}{x_1-\alpha}.$$

. From this relation follows:

$$\frac{M}{N} = \frac{x - \alpha}{x_1 - x}$$

so that 5 (IV) passes into 16 (IV).

With the aid of the above relation we readily find from (2) and (3) the formula 16 (VI), with which the above mentioned property is indicated.

(To be continued).

Physics. — "Further experiments with liquid helium. H. On the galvanic resistance of pure metals etc. VII. The potential difference necessary for an electrical current through mercury below 4°.19 K." By Prof. H. KAMERLINGH ONNES. (Continued.)

§ 11. Local nature of the loss of heat by a mercury thread enclosed in a glass capillary carrying a current, when the temperature sinks below 4°.19 K. While the supposition that the thread should accidentally consist of some other substance than mercury for a small part of its length, is in contradiction to the regularity of the potential phenomena, yet on the other hand the supposition that the mercury thread has a microresidual resistance similar to the ordinary resistance in OHM's law (therefore independent of the strength of current, see (4), gives rise to no less difficulties ¹). Such a microresistance proper to the mercury will be evenly distributed over the whole thread. If we calculate from the potential differences observed during the warming up at low temperatures and the strength of current to which they belong, the resistance of the thread under the conditions of the experiment, then we find that the thread, when the threshold value of the strength of current is only very slightly exceeded, must for a part of its length be partly heated distinctly above the vanishing point. Let us take for example the experiments

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¹) Besides those mentioned in § 9, the difficulties here treated also present themselves if we try to explain the potential phenomena by an even distribution of additive mixtureresistance.

of Dec. 1911 in table I. We find from the threshold value of the current at 4°.19 K., that the resistance of the thread at this temperature may be put at $< 3.10^{-6} \Omega$. In the experiment at 3°.65 K. we find that when the strength of current rises to 1 amp. the resistance, $11.5.10^{-6} \Omega$, was already distincily greater than when the whole thread was at 4°.19 K., while the ends must still be at 3°.65 K. The portion that comes above the vanishing point by this heating, as it assumes ordinary resistance, need only be very small to pro-

duce the potential differences observed; in the case in point only-0.1 mm. If we assume that the giving off of heat to the bath may be calculated by the same data as were found for it above the vanishing point in § 7, then we find that, if the whole surface of the thread were at the mean of the temperature of the bath and of the vanishing point, the loss of heat per second should be about -20000 microjoules, while in reality only 14.0 microjoules, or about 1400 times less, are given off.

We conclude from this that the rise of temperature in the thread, which is in a bath of a temperature below the vanishing point is only local. If there were anywhere else a rise of temperature (although of a smaller amount) the thread must have ceased to give off heat to the glass to a perceptible degree, except at certain points. The heat could therefore only flow to the extremities or the remaining points of conduction. This might be the consequence, for instance, of the mercury having come away from the glass everywhere except at the places indicated. But this is contradicted by the fact that in freezing the mercury adheres to the glass, and that immediately above the vanishing point the contact has not yet ceased. The supposition that everywhere where the temperature remains above the vanishing point (and perhaps close to it) the mercury thread gives off heat, and that it does not where the temperature is lower, is confirmed by the way in which the resistance disappears below the vanishing point (see Table II and fig. 7). If we determine, from the proportion of the resistance remaining to that just by the vanishing point, the length of the portion of the thread which is at the temperature of the vanishing point, then the JOULE-heat that it must give off at the existing strength of current corresponds more or less to that which is to be expected at the assumed difference of temperature of bath and vanishing point if the heat is given off to the glass over the whole length of that portion; more or less, for there remain unexplained and apparently systematic differences, with which perhaps the difference of the curves for different strengths of current in fig. 7 is connected.

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In supposing, however, that the development of heat which brings a part of the thread to the temperature of the vanishing point is of a local nature, we give up the supposition that the microresidual resistance is evenly distributed over the thread. Assuming the whole of the path of the current to be of pure mercury, there could possibly only be an *apparent microresidual resistance*, in consequence, for instance, of the mercury not being homogeneous, or not free from mechanical tension. These disturbances would then be the cause of threads showing a resistance throughout, while the pure homogeneous tension-free mercury would have an imperceptible microresidual resistance.

If we remember that with lead the increase of resistance by pressure ¹) becomes less at low temperatures, and has almost disappeared at hydrogen temperatures, then it is not probable that tensions, although they could cause PELTIER-effects, and although their regularity corresponds to that of the phenomena, should really play a part in the disturbances.

It would be more natural to suppose a lack of homogeneity in the thread, which might be the consequence of difference of the state of crystallization. When we turn down a block of very pure KAHLBAUM-lead on the lathe, we can sometimes see a moiré effect on the surface, which indicates different alternating states of crystallization, each of which extends over more than a centimetre. In this way a thread of solid mercury might consist of a series of differently crystallized portions, the dividing surfaces of which would be at the same time usually cross sections of the thread.

At a dividing surface of this kind, a local heating such as we have treated above, might take place, at the expense of current energy. For instance a transitional resistance might give an apparent microresidual resistance to such a dividing surface. But the relation between the threshold value of current density and the temperature of the bath, points (see. § 8) rather to a PELTIER-effect at this transitional place. We should then have to imagine that when the current density reaches the threshold value, the temperature at the dividing surface between two states of crystallisation, even if not high enough to occasion a thermoelectric force equal to the potential difference observed, yet reaches the vanishing point, and that, therefore, by further increase of the current density ordinary resistance must appear at this dividing surface. The length of the thread which. takes an ordinary resistance would then increase with the excess of

1) H. KAMERLINGH ONNES and BENGT BECKMAN. Comm. No. 182b, Nov. 1912.

the development of heat above that which produces locally the vanishing point temperature; it would be further determined by the circumstances under which the excess of the heat developed would be given off. When we compare the potential difference observed in the different cases, there are one or two things that seem to confirm this supposition 1).

Taking all this together we are brought back to the idea that the potential phenomena must be ascribed to "bad places", although in a different sense to that in § 9. But the regularity of the phenomena remains a weighty objection to this hypothesis ²). For although, with the explanation of the local development of heat by a difference in the states of crystallization, the difficulty disappears which in the explanation by foreign resistances arose out of the circumstance that the whole section must be blocked up, still the appearance of a dividing surface between two states of crystallization is governed by chance. In any case, to come to an explanation on this principle, we should have to assume, that there are various PELTIER-places of the kind meant in each mercury thread of any length and that they are not too unevenly distributed.

But in this manner we should add a new indefinite hypothesis to the one which has to be tested and it is only by a complete quantitative working out of a perfectly definite theory that the question with which we are dealing can be answered: for the answer involves some far-reaching inferences. If we might assume that the potential phenomena in mercury-threads at a current density exceeding the threshold value are entirely due to disturbances then, on account of the systematic connection of the potential phenomena, there would be every reason to assume that we get a truer idea of the actual degree of conductivity of the superconductive mercury, the lower the temperature at which we determine the threshold value of current density of a thread ³). And as at the lowest temperatures the disturbances still have an influence, although a smaller one, the actual conductivity would therefore have to be placed higher, perhaps a good deal higher, than the value found in § 7, which was already $0.5.10^{10}$ times that at the ordinary temperature,

¹) Too indefinite to be published.

²) The existence of a real microresidual resistance is also made probable by that the ratios between the resistances for the mercury in the capillary tube and the frozen mercury thread at $4^{\circ}.25$ K. seems to run parallel to the threshold values, so that the difference of the threshold values might be ascribed to differences of the local deviations of the cross sections from the mean.

³) In this train of thought there is no reason for not supposing that the conductivity assumes its large value immediately below the vanishing point.

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in other words the conductivity of the super-conducting mercury might *practically* be considered *infinite*.

§ 12. Failure of the relations of WIEDEMANN and FRANZ and of LORENZ with super-conductors. a. If the conclusion concerning the non giving off of heat to the glass by a mercury thread below 4° .19 K. which we discussed in § 11, were applicable, we should arrive at a different view concerning the potential phenomena, from that arrived at above. If the mercury has an appreciable real micro-residual-resistance, so that heat is developed throughout the thread, and if we need not take any account of apparent microresidual-resistances, the distribution of temperature in the part of the wire that is below the vanishing point, is governed by the ordinary formula for the rise of temperature of a wire conveying a current without external conduction of heat.

Let us keep as near as possible to the well known ordinary case in order to show the nature of the phenomena that are to be expected in the case in point, and for the sake of simplicity, as it is principally a question of order of magnitude, let us assume that below the vanishing point the ratio of the electric conductivity k to that of heat λ , is given by the same formula as holds approximately above the vanishing point, with the difference that the constant has a different value 10⁷ times smaller, so that while above the vanishing point;

 $\frac{\lambda}{k} = aT \quad \text{with } a = 0,023.10^{-6} \text{ (watt, ohm, degree}^{-1}\text{)},$

below the vanishing point

 $\frac{\lambda}{k} = a'T \quad \text{with } a' = a.10^{-7}.$

We arrive at the low value which we ascribe to α' amongst other things in consequence of the fact that λ remains of the same order of magnitude below the vanishing point as above it, as appears when on the supposition that all the heat in the experiments is developed in the middle of the thread and only flows away at the extremities, we deduce an upper limit for the heat conductivity ¹). ⁻¹) This conclusion is confirmed by preliminary determinations of the heat conductivity of mercury above and below the vanishing point made by me and Mr. G. HOLST. We conclude from these that this constant does not undergo any considerable change at the vanishing point, and the same is true for the specific heat, which we have also investigated, however important this point may be for the electric conduction.

[Our preliminary yet very uncertain values are: for the conductivity between $4^{\circ}.5-6^{\circ}.5$ K., k = 0.25 cal. cm. sec., between $3^{\circ}.8-4^{\circ}.2$ K., k = 0.46 cal. cm. sec., for the specific heat between $4^{\circ}.2-6^{\circ}.5$ K., $C_{\mu} = 0.0014$ and between $3 - 4^{\circ}$ K., $C_{\mu} = 0.00053$ (Added in translation)].

With the assumption indicated the maximum temperature T_{max} of a thread, the extremities of which are at the temperature T_b , witha potential difference of E volts at the extremities is determined by

$$T^2_{max} - T^2_b = \frac{1}{4a'} E^2_b.$$

From this formula can be seen at once that the well known property of good conductors, that comparatively small potential differences, when external heat conduction is excluded, produce considerable heating, which may even lead to melting, becomes enormously more prominent in the superconducting condition.

In fact we find that at the smallest, potential difference E of 0.5 microvolts, which is only a little above that which at 2°.45 K. is first observed, such comparatively great heating can take place, that even at the lowest values of T_b , T_{max} rises to 4°.20 K. At higher bath temperatures of course smaller potential differences are sufficient to reach the vanishing point, or at the same potential difference a' can be placed lower, at 4°.18 K. for instance a' = a. 10^{-5} .

With the rough estimation of a' given, and assuming that the mercury thread where its temperature has fallen below the vanishing point gives off no heat to the glass '), we can, therefore, without the assumption of heating caused by local disturbances, predict phenomena such as threshold value of the current density and the differences of potential, that appear at greater current densities.

At current strengths below the threshold value, the thread will all along be in the condition of superconduction, without external heat conduction, at current densities above the threshold value this only exists for portions below the vanishing point temperature; for the portion of the thread that is above the vanishing point, the regime of ordinary conduction with loss of heat at the surface comes in its place ²). In this way there can, however, be no question of the deduction of the law of dependence of the thres-

²) The divergence of the lines for 0,4 and 0,004 amp. in fig. 7 may also indicate the transition from the one regime to the other.

¹) This calls our attention to the question of the distribution of temperature along a thread through which a current passes without external conduction of heat for different laws of dependence of λ, k and T. Laws might be imagined, which would cause the rise of temperature to run through the values from '0 to $T^{max} - T_b$ practically within a very small length of the thread, in which case the heating by a microresidual resistance could not be distinguished from a heating caused by a local disturbance For the present, however, we adhere to the simpler supposition that the thread gives off no heat to the glass.

hold value on the temperature, because it is determined by the temperature function, which we have arbitrarily assumed as constant a' while we have seen that in the train of reasoning followed it might have very different values at different temperatures, from $a' = 10^{-5}a$ to $a' = 10^{-7}a$. And it is very questionable if, when the necessary data are known for working out the sketch taking note 1 into consideration, the potential phenomena would correspond quantitatively to those observed. For the supposition with regard to the absence of external conduction of heat, upon which the theory in this § is based, might be untrue. (Cf. § 16 d of VIII).

It would be of great importance ') to cool by immediate contact the thread over its whole surface with liquid helium; if the potential phenomena are to be ascribed to a real micro-residual resistance of the mercury, then the threshold value of the current density could probably be raised considerably higher than was now possible. This is too difficult with mercury. Thus for further experiments the use of tin and lead (see § 1) was indicated, these metals being more easily manipulated than solid mercury, and with them the conditions of the external conduction of heat being more easily regulated ²). We shall treat of these investigations in future papers.

 β . We may here add a few remarks concerning the supercon- 'ducting condition.

The experiments described above leave no doubt that for mercury below $4^{\circ}.19$ K. there is no question of an approximate validity even as regards the order of magnitude of the relations established by WIEDEMANN and FRANZ and by LORENZ. The failure of this relation between λ , k and T indicates a difference between the super-conducting and the ordinary conducting state which may be regarded as a *characteristic difference* of both.

Both according to § 11 and to § 12α , we come to a conductivity of mercury which is say 10^{10} times as great, or even more, than that at the ordinary temperature. If we assume that the number of free electrons per unit of volume at the transition from the ordinary to the super-conducting condition undergoes no important change, and then calculate according to the ordinary electron theory from the conductivity the free path of the electrons, we arrive at values which are comparable to the *lengths* of the mercury threads used in

¹⁾ Less, when the particular circumstances mentioned in note 1 should exist.

²) The purity of both can probably not be made so high as that of mercury so that disturbance₃ from a trace of additive admixture resistance in the superconductive state do not seem impossible.

the experiments, in fact are considerably larger '). With such large free paths there would be every reason to believe that the peculiarities of the movements of the electrons pointed out in § 4, which are not consistent with OHM's law, would begin to play a part (which perhaps might resemble a Peltier-effect such as seems to reveal itself in the potential phenomena). It is, however, questionable whether the whole hypothesis developed in § 4 in connection with Comm. N° 119, concerning the movement of free electrons through the metal and which is also mentioned in § 10, must not be replaced by an essentially different one for the super-conducting condition, according to which the movement of the electrons is carried on by the current for considerable distances, but each separate electron which takes part in the progress, only moves one molecular distance.

To illustrate this idea we may take as an example the well known case of the propagation of a blow by a row of billiard balls which just touch each other. In a super-conductor the flow of electricity might consist in this, that an electron jumping across onto an atom of the super-conductor from one side causes an electron on the other side of the atom²) to jump onto the next one, etc. till finally at the further end of the superconducting wire as many electrons would be carried away in the direction of the current, as were thrown in at the beginning²).

¹) Taking the free path at ordinary temperature at 10-7 cm., it becomes 10^{2} cm. at $2^{0}.45$ K., yet taking no account of the decrease of the number of free electrons. We do not consider collisions of the electrons mutually, as these would cause microresidual-resistance phenomena.

²) To express it more accurately, in the same layer of atoms taken across the path of the current, more passes over in a given time than is sent out (or thrown back) through the same layer in the same time to the side from which the electrons taken up come. We here give only the simplest possible sketch, to characterise the super-conducting condition.

³) The taking up of an electron on one side of an atom and the giving off on the other side of one to another atom, would then be accompanied by a moving up of the electrons (through or) over the surface of the atom, by which each electron moves along a part (if the number of electrons on the surface of an atom is large, then a small part) of the diameter of the atom. The connection of the electrons of two different atoms with each other and with these atoms probably does not differ very much from the connection between the electrons of one atom with each other and with the atom, so that the passing of an electron from the one atom to the other in the super conducting state would be similar to the movement of the clectrons in a single atom. The conductivity of the super-conductor would thus be that of the atoms united into one continuous whole (see § 4).

If the numerous electrons in the atom, which belong to the framework of it, in the described process only pass on the blow from the one electron that jumps on to the atom, onto the one that is given off without themselves taking part in the

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The migration speed is thereby propagated through the superconductor without the performance of work ¹). If the super-conducting metal is converted into an ordinary conducting metal by heating above the vanishing point, (if the point is not much exceeded it will still be strongly conducting) then, according to this hypothesis the OHM resistance is due to the action of the vibrators (between the atoms) which bring the atoms to a distance from each other such that the electrons cannot jump from one atom to another without doing work, but in traversing the space made by the vibrators between the atoms give off some of the energy taken up by them. ²) The representation given of the conduction in the super-conductors seems thus to be most easilŷ combined with the conduction theory developed by LENARD.

In my rough sketch (Comm. N° . 119) of the application of the quanta-theory to the electron-theory of conductors, in order to judge

movement and if the moving electrons are the valency electrons, then our hypothesis, although arrived at by a different road, may be regarded as an application to the super-conducting state of the hypothesis of STARK concerning the movement of the framework of the valency electrons along the shearing surfaces of the metal crystals. It thus shows the usefulness of the fundamental idea of STARK. As in the above hypothesis this idea is supplemented by the notion of the free moving electrons of the original electron theory viz. the jumping across of the electrons, the connection with the electron theories of the ordinary conducting state, especially with that of LENARD, is maintained.

1) In so far as we may disregard real microresidual resistance.

²) We will not discuss whether this happens through electrons with migration speed being taken up and electrons without migration speed being given off or by elastic collision of the electrons against the surface of the atoms between which they move backwards and forwards: through energy of ordered motion being transformed into energy of unordered motion. We must remark that for the explanation of the super-conducting state the assumption that in contrast to non elastic collision in ordinary, only elastic collision takes place in the super-conducting state is inadequate. As LORENZ has taught us (comp. REINGANUM, Heidelb. Akad. 1911, 10 p. 7) even with elastic collision the above mentioned transformation must take place and show itself as development of heat.

By the transition from the super-conducting state to the ordinary in proportion as the atoms begin to vibrate separately in larger numbers and room is made for the movement of the electrons between the atoms, the mechanism develops which leads to the approximate relations of WIEDEMANN and FRANZ and of LORENZ. The communication of the movement of the electrons inside the atoms to each other perhaps plays a chief part in the conduction of heath. The continuity of the heat conduction above and below the vanishing point would then be explained by the small change which the process undergoes when the peculiar (connection the atoms which makes super-conduction possible, is destroyed.

The change of the distance between the atoms also clearly plays a part in change of the resistance at the melting point.

whether the hypothesis that restistance is caused by vibrators (the electrons otherwise moving freely through the metal with speeds inaccordance with the kinetic theory of gases 1)) is well adapted to, deduce the change of resistance with temperature, I put the mean free path of the free electrons inversely proportional to the mean amplitude of PLANCK's vibrators, which disturb them in their movements, while this mean amplitude was calculated by the formula which PLANCK at the time gave for the mean energy of the vibrators. The way in which mean values were introduced by this (comp. the reasonings in WIEN's theory, which clearly show the deficiencies of mine) could not allow us to expect more than a qualitative representation. Yet, as is rather remarkable, a close agreement was obtained with the observations between the ordinary temperature and that of liquid hydrogen. It is more difficult to judge of the suitability of the new hypothesis for reproducing the observations with metals above the vanishing point. According to the note at the end of Comm. Nº. 119 the energy of the vibrators would also determine the increase of the volume of the metal from T=0. The mean distance of the surface of the atoms may thus perhaps be taken proportional to the square of the mean amplitude calculated according to PLANCK's just mentioned formula. We may perhaps further conclude that the idea of the condition above the vanishing point at which we arrived starting from the hypothesis concerning the super-conducting state, will appear to be not unsuitable, and in any case gives no ground for objecting to the last named hypothesis.

On both assumptions, however, the assumption that the free path is continuously described by the same electron, and also the other that it is broken by the movement being transferred from one electron to another, a difficulty arises in the explanation of normal resistance, because PLANCK's previous formula has been replaced by a new one. In the discussions at the Conseil SOLVAY²) (Oct. 1911) I pointed out that according to the theory developed in Comm. N^o. 119, if we introduce the new formula, and further calculate in the same way, i.e. with only one frequency, the resistance could not fall below a certain value determined by the "internal temperature"

²) La théorie du rayonnement et des quanta, Rapports et discussions de la réunion à Bruxelles sous les auspices de M. SOLVAY. Paris 1912 p. 129.

¹) KEESOM (Verslag Akademie XXII, p. 108, Suppl. No. 30b) not yet translated in These Proceedings) has come to theimportant conclusion, by the application of the quanta-theory to the free electrons in a metal (considered as a monatomic gas) that at low temperatures the velocity of the free electrons becomes independent of the temperature, and has called this field of temperature the "WIEN field".

(according to $\frac{h\nu}{2} = \frac{1}{2} l_{1} 3\nu$ and $\beta \nu = 200^{\circ}$ K. for silver 100° K.) multiplied by VT, while above T=0 (for various metals above helium temperatures) it seems to become practically nothing. We must therefore adhere to the old formula¹) for calculating the amplitude, or rather, accepting the new formula on account of the more satisfactory representation that it gives in many respects, we must assume that the amplitude of the vibrators that comes into consideration for the determination of free path and of the distance between the atomic surfaces (the part of the path between their old and their new positions, upon which the electrons experience resistance in their movement from one atom to another)²) is only determined by that part of the energy of the vibrators, which is dependent on the temperature. In addition, in order to explain the existence of the super-conducting state one would have to assume that when the excess of the energy above the zero point energy has fallen to the small value which corresponds to the temperature of the vanishing point, the resistance to the motion of the electrons between the atoms suddenly becomes zero.³)

In the reasonings of Comm. N⁰. 119 it was assumed that all vibrators in the metal have the same frequency. As the resistance

is mainly determined by $e^{-\frac{\beta}{T}}$ one, need only assume as the single difference between the super-conducting condition and the normal that the frequency of the vibrators is say four times higher in order to find at the vanishing point a micro-residual resistance 10⁴ smaller than the ordinary resistance at the same temperature and at 2° K. one which is 10° times smaller. But against this explanation it may be adduced that in order to bring the formula of Comm. N°. 119 into agreement with the observations at the lowest temperatures the frequency has to be taken lower as the temperature falls '). WIEN

4) As I pointed out at the discussion of the Conseil Solvay (l. c. p. 298) one might suppose considering that the vibrations take place in the system of mutually connected molecules that there are two kinds of vibrations, a longitudinal and a

¹⁾ As WIEN does in his theory. Sitz. Ber. Ak. d. Wiss. Berlin 1913, p. 200.

²⁾ We may remark that it is not necessary that when an electron jumps over with resistance the whole surplus velocity which it has to propagate should be lost.

⁵) Perhaps the distance of the surfaces of neighbouring atoms has then become equal to that of two neighbouring electrons in the same atom (comp. KEESOM'S paper cited above p. 108 note 1) and the connection of the electrons of two atoms similar to that of the electrons in one and the same atom (comp. the speculations on "atom-fast" compounds in KAMERLINGH ONNES and KEESOM, Encyclop. d. Math. Wissensch. V 10, Suppl. No. 23 Nr. 57.

in taking into account in the calculation of the free path of the electrons all the frequencies which play a part in the specific heat has succeeded in explaining this peculiarity: the resistance according to his theory diminishes at very low temperatures only as T^* or as $T^{5/2}$, (depending on the choice of a subsidiary hypothesis). But then it becomes much more difficult to explain the extremely small value of the possible micro-residual resistance by considering the super-conducting metal simply as a metal with slightly modified properties. It thus seems as if at the vanishing point something occurs by which the small frequencies lose their influence on the resistance although they continue to play a part for the specific heat. The spectrum of the frequencies of the vibrators which are operative in the resistance would thus become limited to a few high frequencies or at least be cut off on the side of the small frequencies, in the same way as this happens according to DEBIJE on the side of the high frequencies ¹).

Astronomy. — "Investigation of the inequalities of approximately monthly period in the longitude of the moon, according to the meridian observations at Greenwich." By J. E. DE VOS VAN STEEN-WIJK. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of April 25, 1913).

It is now about a year ago that Prof. VAN DE SANDE BAKHUYZEN brought under my notice the calculations that he and others had made to determine the corrections needed by HANSEN-NEWCOME's tables of the moon, which still show systematic deviations. I willingly undertook to continue his calculations on the errors of the longitude, and gratefully acknowledge his frequent advice and ready helpfulness. My investigation is confined to the inequalities in the longitude of

transversal kind. Perhaps above the vanishing point only two vibrations play a part in the resistance, a transversal and a longitudinal one, so that according to PLANCK the small frequency becomes prominent at the lower temperatures, and at the vanishing point this frequency changes into a vely high one, so that the original higher one assumes the more important part.

A rotation in opposite senses of two neighbouring atoms with small frequency above the vanishing point, might perhaps, by the atomic surfaces overlapping below the vanishing point, change into a rotation with high frequency. [The caloric investigation of what happens in passing the vanishing point will throw light on this question. As to the specific heat above and below the vanishing point compare the addition to note 1 page 117.

1) This raises the question whether above the vanishing point also the small frequencies do not in some way lose their influence on the resistance all the more the smaller they are.