in taking into account in the calculation of the free path of the electrons all the frequencies which play a part in the specific heat has succeeded in explaining this peculiarity: the resistance according to his theory diminishes at very low temperatures only as T^2 or as $T^{5/2}$, (depending on the choice of a subsidiary hypothesis). But then it becomes much more difficult to explain the extremely small value of the possible micro-residual resistance by considering the super-conducting metal simply as a metal with slightly modified properties. It thus seems as if at the vanishing point something occurs by which the small frequencies lose their influence on the resistance although they continue to play a part for the specific heat. The spectrum of the frequencies of the vibrators which are operative in the resistance would thus become limited to a few high frequencies or at least be cut off on the side of the small frequencies, in the same way as this happens according to DEBIJE on the side of the high frequencies [1]).

Astronomy. — *"Investigation of the inequalities of approximately monthly period in the longitude of the moon, according to the meridian observations at Greenwich."* By J. E. DE VOS VAN STEENWIJK. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of April 25, 1913).

It is now about a year ago that Prof. VAN DE SANDE BAKHUYZEN brought under my notice the calculations that he and others had made to determine the corrections needed by HANSEN-NEWCOMB's tables of the moon, which still show systematic deviations. I willingly undertook to continue his calculations on the errors of the longitude, and gratefully acknowledge his frequent advice and ready helpfulness.

My investigation is confined to the inequalities in the longitude of

transversal kind. Perhaps above the vanishing point only two vibrations play a part in the resistance, a transversal and a longitudinal one, so that according to PLANCK the small frequency becomes prominent at the lower temperatures, and at the vanishing point this frequency changes into a very high one, so that the original higher one assumes the more important part.

A rotation in opposite senses of two neighbouring atoms with small frequency above the vanishing point, might perhaps, by the atomic surfaces overlapping below the vanishing point, change into a rotation with high frequency. [The caloric investigation of what happens in passing the vanishing point will throw light on this question. As to the specific heat above and below the vanishing point compare the addition to note 1 page 117.

1) This raises the question whether above the vanishing point also the small frequencies do not in some way lose their influence on the resistance all the more the smaller they are.

approximately monthly period, or to speak more accurately, periods differing only slightly from the anomalistic time of revolution, which can also be taken as inequalities of long period in the eccentricity and the longitude of the perigee, and to the comparison of them with the values found for the same by E. W. BROWN in his new lunar theory.

My method was exactly the same as that used by Prof. E. F. VAN DE SANDE BAKHUYZEN, in his two papers in 1903, and previously by NEWCOMB in his "Investigation of corrections to HANSEN's tables of the moon". I also used errors in R. A. instead of those in longitude.

NEWCOMB discussed the years 1862—1874, according to the observations made at Greenwich and Washington, and in a less thorough manner the years 1847—1858 according to the Greenwich-observations, while E. F. VAN DE SANDE BAKHUYZEN treated the years 1895—1902 also by using the observations at Greenwich. As their results pointed to terms in the eccentricity and the longitude of the perigee of about an 18 years' period, nearly agreeing with the Jovian evection as found by theory, it was natural for me te extend the material discussed by VAN DE SANDE BAKHUYZEN, so as to cover a period of 20 years.

I began a preliminary investigation with a view to the then approaching solar eclipse of April 17th 1912. I used the observations of the years 1907—1909 and the results of my calculations were published in the Proceedings of this Academy **14**, 1180. On account of the short period discussed, my investigation could yield no result of general bearing.

For this first investigation I had applied beforehand to HANSEN-NEWCOMB's tables precisely the same corrections as Prof. VAN DE SANDE BAKHUYZEN had done and when later on I began to work after the more extended plan, it was first necessary to consider if any change needed be made in this.

Prof. VAN DE SANDE BAKHUYZEN had applied to the differences $\Delta\alpha$, which are given in the Greenwich-results in the sense calculation minus observation, the following corrections: [1])

a. Corrections to the calculated mean longitude viz:

1°. periodic corrections $ndz = +1''.69 \sin D + 0''.16 \sin (D-g) - 0''.24 \sin (D+g') + 0''.09 \sin g' - 0''.33 \sin 2D - 0''.21 \sin (2D-g)$ [2])

[1]) Under the heading: "Comparison of the errors of the Moon from observations by Transit and Altazimuth" corrections have been applied to the $\Delta\alpha$ for the motion of the moon in the interval $\Delta\alpha$ itself, which are not yet taken into account in other parts of the Results.

[2]) In this the solar parallax was taken as $\pi = 8''.796$.

2°. a correction for the slowly varying error (secular term) which was deduced from the annual means of the $\Delta\alpha$.

The sum of these corrections was reduced to corrections of the R.A. by means of NEWCOMB's factors F and $(v.\,a.)$.

b. Corrections for personal errors in the observation of the times of transit of the two limbs of the moon.

After mature consideration Prof. BAKHUYZEN advised me to introduce new corrections deduced from BROWN's lunar theory and corresponding to the solar parallax 8″.80, instead of those given above under 1°.

In this it was kept in view, that inequalities with small amplitude only need to be introduced for our purposes, when their period is approximately commensurable with that of the mean anomaly g, as otherwise their influence on the average almost disappears.

The new inequalities calculated by BROWN, or their differences with those according to HANSEN, were taken from the third paper of BATTERMANN[1]), where they appear on page 16—18, numbered 1—45.

Most of them are perturbations by the planets, some of them are corrections to the perturbations due to the figure of the earth, and a few are solar perturbations. The perturbations 23—29 by the planets, the solar perturbation N°. 39 and the term N°. 44 produced by the figure of the earth, when brought into the form $a \sin(g + \chi)$ all show values of χ, with a period between 9 and 38 years. They were not yet introduced as they stand in immediate relation to the results to be derived from my investigation.

The corrections introduced were the following:

$$\begin{array}{l} +\,1''.37 \sin D \\ \left.\begin{array}{l} +\,0''.20 \\ +\,0''.31 \end{array}\right\} \sin(D-g) \\ -\,0''.20 \sin(D+g') \\ -\,0''.12 \sin(2D-g) \\ +\,0''.25 \sin g' \\ -\,0''.19 \sin 2D. \end{array}$$

The values of these terms were collected in two new tables of the same form as NEWCOMB's tables VII and VIII.

In order not to break the connection with the years 1895—1902 too much, the old corrections were used for the years 1890—1894, so that 1890—1902 form a homogeneous whole. From the results for the years 1907—1909, which have been calculated with the old and with the new corrections, which gave only insignificant differences, it appears however, that the whole period 1890—1910 may also be considered as one whole.

[1]) Beobachtungs-Ergebnisse der Königlichen Sternwarte zu Berlin n°. 13 1910.

The year 1891 has been left out of the calculations, as the transit-circle was not in working order for some months, and the results of the different parts of the year may differ systematically from each other.

Further, up to 1902 only observations taken with the Transit circle are used, after that also Altazimuth observations, in so far as they were taken in the meridian (which did not occur before 1903) and finally, after 1905 I could also use observations of the crater Moesting A.

The secular term was computed for every month or two months from a graphical interpolation between the annual means, given in the paper by Prof. BAKHUYZEN in These Proc. **14**, 691 under the heading *M—NI*, after these values had been diminished by a 28th part for the motion of the moon between the moments of calculated and observed transit [1]).

Finally we come to the corrections to be applied to the observed time of transit of the moon's limb. These depend principally upon

D_t Limb I	Trans. c. 1892—94	Trans. c. 1895—1902	Trans. c. 1903—09	Altaz. 1903—09
3	—0s066 (1)	—	—0s148 (1)	—
4	—0.051 (14)	—0s121 (12)	—0.020 (17)	—
5	—0.035 (12)	—0.069 (32)	—0.036 (29)	+0s245 (2)
6	—0.065 (12)	—0.022 (36)	+0.014 (29)	+0.151 (8)
7	+0.012 (16)	—0.036 (41)	+0.026 (28)	+0.140 (10)
8	+0.014 (17)	+0.022 (50)	+0.038 (45)	+0.077 (23)
9	+0.072 (17)	+0.007 (46)	+0.022 (42)	+0.046 (30)
10	+0.019 (16)	+0.002 (43)	+0.041 (39)	+0.043 (33)
11	+0.012 (20)	+0.011 (50)	—0.019 (44)	—0.027 (36)
12	—0.029 (23)	+0.011 (54)	—0.019 (43)	—0.055 (43)
13	+0.006 (16)	+0.029 (49)	+0.009 (47)	—0.016 (37)
14	+0.012 (18)	+0.007 (49)	—0.051 (31)	—0.082 (28)
15	+0.023 (17)	0.000 (27)	—0.049 (14)	—0.081 (17)

personal errors, and can therefore in the course of years undergo great and irregular variations. On the other hand it is desirable, in order not to be too dependent upon accidental errors, not to determine the corrections for each year separately.

Taking this into consideration, the errors in observing the limbs

[1]) Comp. These Proc. **14**, 992.

were determined for the two periods 1892—94, and 1903—09, while 1890 and 1910, which were added later, were connected to the other years as well as possible.

Like my predecessors I have arranged the $\Delta\alpha$, after applying all the above mentioned corrections according to the days of true age of the moon D_t, in order to investigate whether there may exist a dependence on this age. These results, as well as those found by Prof. Bakhuyzen, I give here, viz. the mean deviation for each day diminished by the total mean for the limb. In the first table I have collected the results for limb I, in the second those for limb II; the numbers in brackets give the weights.

We must remember that the systematic errors of the observers are only correctly represented by these figures if the theoretical corrections depending on D are quite correct.

D_t Limb II	Trans. c. 1892—94	Trans. c. 1895—1902	Trans. c. 1903—1909	Altaz. 1903—09
14	+0s016 (2)	−0s027 (14)	+0s060 (4)	0s000 (1)
15	−0.060 (9)	−0.065 (33)	+0.128 (21)	+0.034 (14)
16	−0.003 (22)	+0.023 (61)	+0.057 (47)	−0.002 (31)
17	−0.011 (16)	+0.030 (49)	+0.056 (37)	+0.002 (26)
18	−0.034 (19)	−0.009 (53)	+0.103 (37)	−0.061 (16)
19	−0.035 (19)	+0.048 (45)	+0.077 (29)	+0.019 (17)
20	−0.044 (12)	+0.008 (34)	+0.104 (32)	+0.080 (22)
21	+0.007 (14)	+0.003 (31)	+0.095 (28)	+0.050 (17)
22	+0.034 (13)	+0.013 (37)	+0.062 (29)	−0.018 (14)
23	+0.085 (12)	+0.017 (38)	+0.033 (20)	−0.048 (6)
24	+0.110 (14)	−0.015 (31)	+0.037 (24)	−0.260 (1)
25	−0.140 (5)	+0.008 (19)	+0.045 (17)	+0.170 (1)
26	+0.099 (3)	+0.126 (8)	+0.047 (3)	—

This condition is certainly not fulfilled by the old set of corrections. Too much importance must therefore not be attached to the variation of these figures and by making the corrections too complicate we run the risk of introducing periodic terms and spoiling our results.

Newcomb, however, already indicated the possibility of one cause of divergence, viz. the tendency of observers to estimate the moon's diameter smaller by day-light, that is at very small and very great values of its age; in the first instance this would cause a divergence in the negative direction, and in the second in the positive.

In the years 1892—94 this divergence seems to show itself, and I therefore considered it desirable to introduce for these years a separate correction of $+0^s.06$ for $D_t = 3, 4, 5, 6$ and one of $-0^s.05$ for $D_t = 23, 24, 25, 26$; after applying these corrections the mean deviation for each limb was determined afresh. For the years 1895—1902 Prof. BAKHUYZEN rejected the observations for $D_t = 4$ and 26, and applied no further separate corrections. The years 1903—09 do not show these particular divergencies so conspicuously and for these I thought it advisable not to apply any separate corrections for a dependence on D_t. The years calculated later, 1890 and 1910, have been brought into connection with the others as well as possible. The correction for the observed limb for 1890 is deduced from 1890—94, and that for 1910 from 1903–10.

I now give the differences between the results from the two limbs, Limb II—Limb I, computed for each year, after the above mentioned corrections had been applied.

With one exception, these differences for the different years agree fairly well with one another.

After the corrections for the errors of observation of the limbs had been applied, a yearly mean was formed for each instrument, both for the observations of the limbs and of the crater, and further corrections were added to bring each class of observations into agreement with the total mean.

The total corrections were finally:

1892—94	$D_t = 3, 4, 5, 6$	$+0^s08$
	$D_t = 23, 24, 25, 26$	-0.08
	Remaining Obs. L. I	$+0.02$
	L. II	-0.03
1895—99	Limb I	$+0.02$
	Limb II	-0.02
1900—02	Limb I	$+0.03$
	Limb II	-0.03
1903—09	Trans. c. Limb I	$+0.05$
	Trans. c. Limb II	-0.07
	Altaz. Limb I	$+0.04$
	Altaz. Limb II	-0.01
	Trans. c. Cr.	0.00
	Altaz. Cr.	$+0.01$

After the above corrections had been applied, the observations

	Limb II—Limb I	
	Trans. c.	Altazim.
1890	— 0s015	
1892	+ 0.054	
1893	+ 0.066	
1894	+ 0.037	
1895	— 0.010	
1896	+ 0.087	
1897	+ 0.008	
1898	+ 0.073	
1899	+ 0.055	
1900	+ 0.047	
1901	+ 0.053	
1902	+ 0.066	
1903	+ 0.213	+ 0s146
1904	+ 0.145	+ 0.073
1905	+ 0.098	+ 0.051
1906	+ 0.104	+ 0.089
1907	+ 0.130	+ 0.019
1908	+ 0.082	+ 0.025
1909	+ 0.107	+ 0.025
1910	+ 0.045	— 0.162

for each year were collected in groups, according to the value of g, so that each group contained a range of 20°; the mean for each group was taken to hold for the mean value for g in that group.

In this way, I got for each year 18 equations of the form $c + h \sin g + k \cos g = r$, in which c is the residual error in the mean longitude for that year and h and k have the same meaning as in NEWCOMB's investigation. The errors are taken in the sense calculation minus observation. These equations were now solved for each year by the method of least squares, taking the weights for each group proportional to the number of observations.

This gave the following results:

	c	h	k
1890	+ 1″07	+ 1″40	+ 0″48
1892	+ 0.76	+ 0.69	+ 0.87
1893	+ 0.88	— 0.45	+ 1 04
1894	+ 0.64	— 0.32	+ 1 34
1895	+ 0.07	+ 0.29	+ 0.44
1896	+ 0.05	+ 0.66	+ 1.16
1897	— 0.36	+ 0.57	+ 1.77
1898	— 0.22	+ 0.51	+ 2.10
1899	+ 0.48	— 0.93	+ 2.83
1900	+ 0.27	— 1.66	+ 1.12
1901	+ 0.46	— 1.46	+ 0.52
1902	— 0.16	— 1.18	+ 0.01
1903	— 0.05	— 0.56	— 1.64
1904	— 0.14	+ 1.32	— 2.32
1905	— 0.58	+ 2.38	— 0.66
1906	— 0.34	+ 2 98	— 0.26
1907	+ 0.52	+ 2.67	+ 0.74
1908	+ 0.18	+ 2.01	+ 1.06
1909	+ 0 09	+ 1.90	+ 2.14
1910	— 0.09	+ 1.65	+ 2.54

For the sake of comparison I here add the values of h and k for 1909, calculated with the old and the new corrections, from which it appears that this makes little difference.

1909 old correction $h = + 1''81$ $k = + 2''17$
new ,, $h = + 1.90$ $k = + 2.14$

The best way to further discuss these results appeared to me to be, that first of all the values found for h and k should be corrected for Brown's inequalities, in which the quantities χ, as defined above, are varying in long periods.

In this way for 1890—1910, and for the 1st and 2nd series of Newcomb, 1847—1858 and 1862—1874, the sum was formed of the following terms:

9*

I	$+0''66 \; sin \; \{g + 298°47 + 0°101075\, t\}$
	$+0.08 \; sin \; \{g + 92.28 - 0.020582\, t\}$
	$+0.07 \; sin \; \{g + 350.40 + 0.062456\, t\}$
	$+0.07 \; sin \; \{g + 179.20 - 0.062456\, t\}$
	$+0.04 \; sin \; \{g + 87.95 + 0.035364\, t\}$
II	$+1.14 \; sin \; \{g + 12.85 + 0.056550\, t\}$
III	$+0.44 \; sin \; \{g + 322.71 - 0.026541\, t\}$
IV	$+0.28 \; sin \; \{g + 2\,\omega + 180°\}$
V	$+0.50 \; sin \; (\Omega - 10°6) \; cos \; g$

in which t is expressed in days counted from 1900.0. For NEWCOMB's first series, on account of their smaller accuracy, only the 5 largest terms, marked here by the numbers I—V were calculated.

As these corrections must be applied to the tabular values and as h and k have been taken in the sense calculation minus observation, I have now, indicating BROWN's terms by Br, formed $-h - Br$ and $-k - Br$, so that these differences represent the corrections which, according to the observations, must be applied to the tables after they have been corrected according to BROWN.

After this the corrected values of $-h$ and $-k$ were freed from their constant parts, which depend upon the corrections, which are still required for the eccentricity and the longitude of the perigee ($-h_c = +2\,\delta e$, $-k_c = -2\,e\,\delta\pi$). This was done in two different ways. The first time I regarded the mean values of $-h - Br$ and $-k - Br$ for each period as the constant parts to be subtracted from the individual values. As the two following tables, Table I and II, show, the results thus found for $-h_c$ and $-k_c$ for the three series are in fairly good accordance with each other.

The tables also contain the results $-h_v$ and $-k_v$ freed from the constant parts.

A second time I have tried to represent the constant parts for the three series together by quantities varying linearly with the time. In order to be able to dispose of 4 periods of about equal length, I divided the last into two parts, and calculated h_c and k_c for each half-series (these values differed not much from those for the whole series). I had therefore for each of the two unknown quantities four equations of the form $-h_c = a + bt$ and $-k_c = a' + b't$. To the first series of NEWCOMB I gave a weight 1, and to each of the other three series a weight 3. The results found were:

$$\left.\begin{array}{ll} a = -0''.62 & b = +0''.0034 \\ a' = -0''.47 & b' = -0''.0090 \end{array}\right\} \text{ epoch } 1894.5.$$

TABLE I. Investigation of the h. 1st calculation.

	$-h$	*Brown*	$-h-Br.$	$-h_c$	$-h_v$
1847.8	+0″08	+0″03	+0″05	—0″24	+0″29
48.9	+0.55	+0.49	+0.06	"	+0.30
50.1	+0.20	+0.34	—0.14	"	+0.10
51.2	+0.32	+0.35	—0.03	"	+0.21
52.4	—0.26	+0.69	—0.95	"	—0.71
53.5	—1.10	—0.14	—0.96	"	—0.72
54.6	—1.45	—0.34	—1.11	"	—0.87
55.8	—0.77	—0.84	+0.07	"	+0.31
56.9	—1.76	—1.46	—0.30	"	—0.06
58.1	+0.17	—0.72	+0.89	"	+1.13
1862.5	—0.04	+1.74	—1.78	—0.75	—1.03
63.5	+0.64	+2.00	—1.36	"	—0.61
64.5	+1.07	+2.00	—0.93	"	—0.18
65.5	+1.03	+0.97	+0.06	"	+0.81
66.5	+0.47	+0.58	—0.11	"	+0.64
67.5	+0.93	+0.28	+0.65	"	+1.40
68.5	—0.34	—0.54	+0.20	"	+0.95
69.5	—1.67	—0.46	—1.21	"	—0.46
70.5	—1.48	—0.25	—1.23	"	—0.48
71.5	—1.65	—0.78	—0.87	"	—0.12
72.5	—2.15	—0.69	—1.46	"	—0.71
73.5	—1.91	—0.71	—1.20	"	—0.45
74.5	—1.92	—1.37	—0.55	"	+0.20
1890.5	—1.40	—0.53	—0.87	—0.66	—0.21
92.5	—0.69	—0.20	—0.49	"	+0.17
93.5	+0.45	+0.13	+0.32	"	+0.98
94.5	+0.32	+0.36	—0.04	"	+0.62
95.5	—0.29	—0.09	—0.20	"	+0.46
96.5	—0.66	+0.35	—1.01	"	—0.35
97.5	—0.57	+0.95	—1.52	"	—0.86
98.5	—0.51	+0.93	—1.44	"	—0.78
99.5	+0.93	+1.64	—0.71	"	—0.05
1900.5	+1.66	+2.10	-0.44	"	+0.22
01.5	+1.46	+1.51	—0.05	"	+0.61
02.5	+1.18	+1.28	—0.10	"	+0.56
03.5	+0.56	+0.54	+0.02	"	+0.68
04.5	—1.32	—0.62	—0.70	"	—0.04
05.5	—2.38	—1.17	—1.21	"	—0.55
06.5	—2.98	—1.43	—1.55	"	—0.89
07.5	—2.67	—2.02	—0.65	"	+0.01
08.5	- 2.01	—1.52	- 0.49	"	+0.17
09.5	—1.90	—0.78	—1.12	"	—0.46
0.5	—1.65	—0.72	—0.93	"	—0.27

TABLE II. Investigation of the k. 1st calculation.

	$-k$	*Brown*	$-k-Br$	$-k_c$	$-k_v$
1847.8	−0″55	+0″80	−1″35	−0″21	−1″14
48.9	+1.38	+0.76	+0.62	"	+0.83
50.1	+1.91	+1.44	+0.47	"	+0.68
51.2	+1.92	+1.20	+0.72	"	+0.93
52.4	+2.45	+1.85	+0.60	"	+0.81
53.5	+1.88	+2.03	−0.15	"	+0.06
54.6	+1.40	+1.44	−0.04	"	+0.17
55.8	−0.31	+1.31	−1.62	"	−1.41
56.9	−1.82	0.00	−1.82	"	−1.61
58.1	−0.66	−1.17	+0.51	"	+0.72
1862.5	−1.23	−1.53	+0.30	−0.17	+0.47
63.5	−1.78	−1.20	−0.58	"	−0.41
64.5	−1.09	−0.06	−1.03	"	−0.86
65.5	+0.15	+0.50	−0.35	"	−0.18
66.5	−0.10	+0.36	−0.46	"	−0.29
67.5	+0.36	+0.83	−0.47	"	−0.30
68.5	+1.46	+0.77	+0.69	"	+0.86
69.5	+1.56	+0.24	+1.32	"	+1.49
70.5	+1.14	+0.62	+0.52	"	+0.69
71.5	+0.36	+0.64	−0.28	"	−0.11
72.5	+0.12	+0.27	−0.15	"	+0.02
73.5	−0.16	+0.60	−0.76	"	−0.59
74.5	−0.60	+0.27	−0.87	"	−0.70
1890.5	−0.48	+0.20	−0.68	−0.58	−0.10
92.5	−0.87	+0.24	−1.11	"	−0.53
93.5	−1.04	−0.40	−0.64	"	−0.06
94.5	−1.34	−0.28	−1.06	"	−0.48
95.5	−0.44	−0.70	+0.26	"	+0.84
96.5	−1.16	−1.56	+0.40	"	+0.98
97.5	−1.77	−1.44	−0.33	"	+0.25
98.5	−2.10	−1.44	−0.66	"	−0.08
99.5	−2.83	−1.53	−1.30	"	−0.72
1900.5	−1.12	−0.45	−0.67	"	−0.09
01.5	−0.52	+0.38	−0.90	"	−0.32
02.5	−0.01	+0.75	−0.76	"	−0.18
03.5	+1.64	+1.70	−0.06	"	+0.52
04.5	+2.32	+1.84	+0.48	"	+1.06
05.5	+0.66	+1.10	−0.44	"	+0.14
06.5	+0.26	+0.92	−0.66	"	−0.08
07.5	−0.74	+0.16	−0.90	"	−0.32
08.5	−1.06	−0.96	−0.10	"	+0.48
09.5	−2.14	−0.98	−1.16	"	−0.58
10.5	−2.54	−1.15	−1.39	"	−0.81

TABLE III. Investigation of the h and k. 2nd Calculation.

	$-h_c$	$-k_c$	$-h_v$	$-k_v$
1847.8	−0″78	−0″05	+0″83	−1″30
48.9	〃	−0.06	+0.84	+0.68
50.1	−0.77	−0.07	+0.63	+0.54
51.2	〃	−0.08	+0.74	+0.80
52.4	〃	−0.09	−0.19	+0.69
53.5	−0.76	−0.10	−0.20	−0.05
54.6	〃	−0.11	−0.35	+0.07
55.8	〃	−0.12	+0.83	−1.50
56.9	−0.75	−0.13	+0.45	−1.69
58.1	〃	−0.14	+1.64	+0.65
1862.5	−0.73	−0.18	−1.05	+0.48
63.5	−0.72	−0.19	−0.64	−0.39
64.5	〃	−0.20	−0.21	−0.83
65.5	〃	−0.21	+0.78	−0.14
66.5	−0.71	−0.22	+0.60	−0.24
67.5	〃	−0.23	+1.36	−0.24
68.5	〃	−0.23	+0.91	+0.92
69.5	−0.70	−0.24	−0.51	+1.56
70.5	〃	−0.25	−0.53	+0.77
71.5	〃	−0.26	−0.17	−0.02
72.5	−0.69	−0.27	−0.77	+0.12
73.5	〃	−0.28	−0.51	−0.48
74.5	〃	−0.29	+0.14	−0.58
1890.5	−0.63	−0.44	−0.24	−0.24
92.5	〃	−0.45	+0.14	−0.66
93.5	〃	−0.46	+0.95	−0.18
94.5	−0.62	−0.47	+0.58	−0.59
95.5	〃	−0.48	+0.42	+0.74
96.5	〃	−0.49	−0.39	+0.89
97.5	−0.61	−0.50	−0.91	+0.17
98.5	〃	−0.50	−0.83	−0.16
99.5	〃	−0.51	−0.10	−0.79
1900.5	−0.60	−0.52	+0.16	−0.15
01.5	〃	−0.53	+0.55	−0.37
02.5	〃	−0.54	+0.50	−0.22
03.5	−0.59	−0.55	+0.61	+0.49
04.5	〃	−0.56	−0.11	+1.04
05.5	〃	−0.57	−0.62	+0.13
06.5	−0.58	−0.58	−0.97	−0.08
07.5	〃	−0.59	−0.07	−0.31
08.5	〃	−0.59	+0.09	+0.49
09.5	−0.57	−0.60	−0.55	−0.56
10.5	〃	−0.61	−0.36	−0.78

By this formula $-h_c$ and $-k_c$ were then calculated for each year and with these values those for $-h_v$ and $-k_v$ were deduced. We thus have two sets of values for $-h_v$ and $-k_v$.

Before submitting these results to a nearer investigation, I tried to show that the introduction of BROWN's terms is justified by the observations, at least as far as the five are concerned, which have perceptible coefficients, and were marked above in roman figures. For this purpose tables were drawn up of $-h_v$ and $-k_v$, according to the first calculation, in which all BROWN's terms were introduced except the term to be investigated. The yearly results were then so combined, that groups were formed with values of the argument of the investigated term between 0° and 10°, 10° and 20° etc., in such a manner that the results for the different quadrants, when necessary, were reduced by a change of sign to those for the first quadrant. The h and k, when we represent BROWN's terms by $a \sin (g+\chi)$, then give nine equations each of the form $-h_v = a \cos \chi$ and $-k_v = a \sin \chi$.

Now $\chi = \chi_0 + \mu (t - 1900.0)$ and, as we take the period or μ as known, we can deduce from the observations the value for a and χ_0 for each term by the formulae

$$tg\,(\chi_0 + \mu\,(t - 1900.0)) = \frac{-k_v}{-h_c} \qquad a^2 = h_v^2 + k_v^2 \text{ [1]}$$

In this solution weights are given to the various equations proportional to the number of years that they are based upon.

The results found, compared with those according to the theory, follow below:

	a Theory	a Obs.	χ_0 Obs.—Theory
BROWN I	+ 0″66	+ 0″61	+ 1°2
II	+ 1.14	+ 1.34	— 2.0
III	+ 0.44	+ 0.58	— 21.9
IV	+ 0.28	+ 0.19	— 26.0
V	+ 0.50	+ 0.35	+ 19.5

This agreement may be considered as satisfactory, it certainly completely justifies the introduction of BROWN's inequalities.

Finally I endeavoured to deduce from the observed h and k for the largest term Br. II, the Jovian evection, also the length of the period. The $-h$ and $-k$, corrected for all other terms, being represented for each year by

[1] For Br. V we have only a coefficient k, so that the deduction for this term becomes somewhat different.

$$-h_v = a \cos \chi \qquad\qquad -k_v = a \sin \chi \text{ [1]}$$

the values of χ were calculated for the separate years and these values were united in 7 groups, as follows:

Period	Mean Epoch	χ	O—C
1847.8—58.1	1852.9	134°	+ 20°
62.5—68.5	1865.5	344	— 35
69.5—74.5	1872.0	523	+ 7
90.5—95.5	1893.3	985	+ 20
1896.5—00.5	1898.5	1112	+ 37
1901.5—05.5	1903.5	1154	— 27
1906.5—10.5	1908.5	1272	— 14

Each group gives a normal value for χ and these are then represented by equations

$$\chi = \chi_0 + \mu\,(t—1900.0).$$

To the first equation I gave the weight 0.7, to the others the weight 1. These being solved by least squares gave the result

$$\chi_0 = 1107°.1 = 27°.1 \qquad \mu = 21°.085$$

The last column of the table gives the differences between observation and calculation. The annual variation of the argument found is thus 0°.43 larger than that which follows from the theory, 20°.65.

NEWCOMB found 21°.6
BAKHUYZEN „ 19°.36.

The argument for 1900.0 is now found 14°.3 larger than according to the theory; for the mean epoch of the observations 1886 the difference, however, is only + 8°.3, while my previous calculation mentioned above gave Obs.—Th. = — 2°.0.

We now proceed to the investigation of the residual values for $-h_v$ and $-k_v$, which after correction for all BROWN's terms, still show a distinct periodicity, though the amplitude is greatly decreased.

By a graphic representation and some preliminary calculations I came to the conclusion that the best agreement would be attained by a term of a period of nine years. The values of h and k seemed to agree completely in this and together to point to the existence of a term of the form $a \sin (g + \chi)$.

[1]) Our χ is connected with the N introduced by NEWCOMB and also used by E. F. v. D. SANDE BAKHUYZEN by $\chi = N - 90°$.
For h_v and k_v the values were taken according to the 2nd calculation.

I then examined the accurate length of the period more closel in the following manner. Proceeding from the form $a\, sin\, (g + \chi$ so that $-h_v = a\, cos\, \chi \quad -k_v = a\, sin\, \chi$ and from the 2nd calculation for h_c and k_c, the value for χ for eacl year was found as follows

1847.8	303°	1862.5	155°	1890.5	225°	1901.5	326°
48.9	39	63.5	212	92.5	282	02.5	336
50.1	40	64.5	257	93.5	349	03.5	39
51.2	48	65.5	350	94.5	314	04.5	96
52.4	106	66.5	339	95.5	60	05.5	168
53.5	194	67.5	350	96.5	116	06.5	185
54.6	169	68.5	45	97.5	170	07.5	257
55.8	299	69.5	107	98.5	191	08.5	80
56.9	285	70.5	124	99.5	263	09.5	226
58 1	22	71.5	188	1900.5	317	10.5	245
		72.5	171				
		73.5	224				
		74.5	286				

These values for χ were now united in 7 groups, and the meai results for these groups represented by equations $\chi = \chi_0 + \mu\, (t - 1900.0)$ the first group received the weight 0.7, the others weight 1.

Period	Mean Epoch	χ	O—C
1847.8—58.1	1852.9	150°	+ 5°
62.5—68.5	1865.5	655	+ 2
69.5—74.5	1872.0	903	— 12
90.5—95.5	1893.3	1758	— 17
1896.5—00.5	1898.5	2011	+ 27
1901.5—05.5	1903.5	2209	+ 23
1906.5—10.5	1908.5	2359	— 29

By solving the 7 equations by least squares I found

$$\chi = 244°.7 + 40°.35\, (t - 1900.0).$$

The differences Obs—Calc. are given above.

For the period I now found 8.92 years, which could, therefore, retain provisionally the assumed value of 9 years, with which the further calculations had already been made. We have

$$-h_v = a \cos \{\chi_0 + 40^\circ (t-t_0)\}$$
$$-k_v = a \sin \{\chi_0 + 40^\circ (t-t_0)\}$$

and putting

$$-h_v = \beta \sin 40^\circ (t-t_0) + \gamma \cos 40^\circ (t-t_0)$$
$$-k_v = \beta' \sin 40^\circ (t-t_0) + \gamma' \cos 40^\circ (t-t_0).$$

we then have

$$\beta = -a \sin \chi_0 \qquad \gamma = a \cos \chi_0$$
$$\beta' = a \cos \chi_0 \qquad \gamma' = a \sin \chi_0.$$

I calculated each of the four coefficients independently. In this NEWCOMB's first series was left out, on account of its smaller accuracy. I found, assuming for t_0 1894.5, from both sets of values A and B obtained by the two methods of calculation.

A

Each series being calculated with its own h_c and k_c

$$\beta = -0''.30 \quad \gamma = +0''.68 \quad \beta' = +0''.60 \quad \gamma' = +0''.16$$

B

The h_c and k_c being calculated by formulae $a + bt$

$$\beta = -0''.29 \quad \gamma = +0''.63 \quad \beta' = +0''.60 \quad \gamma' = +0''.14$$

After all the results of calculation B seem to me to be the most reliable, but the differences between the two sets are very slight. We see further that the relations $\beta' = \gamma$ and $\beta = -\gamma'$ are very satisfactorily fulfilled and may thus assume according to calculation B:

$$a \sin \chi_0 = +0''.22$$
$$a \cos \chi_0 = +0''.62$$

from which

$$\chi_0 = 19^\circ.53$$
$$a = +0''.66$$

The value found for χ_0 must still undergo a small correction, because the annual variation was not assumed quite correctly; considering that the mean epoch of the observations is about 1886, this correction becomes $+2^\circ.98$.

Finally, transferring the zero-epoch to 1900.0 we find for our empirical term

$$+0''.66 \sin \{g + 244^\circ.4 + 40^\circ.35\ (t-1900.0)\}.$$

The value now determined for the argument for 1900.0 thus agrees very nearly with that found above. The period of this term differs comparatively little from that of the term Br I; the difference of 3.°43 between the annual variation for both, however, makes the variation of the two arguments differ 180° in 52 years and so the two terms cannot be combined into one.

The complete formulae for the corrections, which must still be added after BROWN's inequalities have been taken into account, are therefore

$$-h = -0''.60 + 0''.0034\{t-1900.0\} + 0''.66 \cos\{244°.4 + 40°.35\,(t-1900.0)\}$$
$$-k = -0''.52 - 0''.0090\{t-1900.0\} + 0''.66 \sin\{244°.4 + 40°.35\,(t-1900.0)\}$$

The two periodic terms can also be combined, as was done above.

Let us now consider the meaning of the corrections found, first as regards the non-periodic parts. We have:

$$-h_c = +2\delta e$$
$$-k_c = -2e\delta\pi$$

in which δe and $\delta\pi$ represent the corrections to the eccentricity and the longitude of the perigee adopted by HANSEN for his tables.

We find thus

$$\delta e = -0''.30 + 0''.0017\,(t-1900.0)$$
$$\delta\pi = +4''.7 \quad + 0''.082 \quad (t-1900.0).$$

The correction found for the annual variation of the eccentricity is certainly too small to be considered as real. If we assume it to be zero, we find

$$\delta e = -0''.32.$$

The correction found for HANSEN's annual motion of the perigee may be compared with what was found by others.

The correction $+0''.08$ must be applied to HANSEN's tabular value of the sidereal motion in a Julian year for 1850.0, 146435''.23, which was deduced by him from the observations.

We get thus for 1850.0.

Annual motion of π = **146435''.31.**

COWELL found from his discussion of the observations at Greenwich (Monthly Notices Jan. 1905) 146435''.38, in near agreement with the result obtained here.

BROWN (Monthly Notices April 1904) gives as the result of his theoretical calculation of the motion of the perigee two values, holding for two different values of the ellipticity of the earth, viz. 1 : 292.9 and 1 : 296.3. Extrapolating from these for the value which is at present considered the most accurate 1 : 297.5, we find for 1850.0

Annual motion of π = **146435''.05**

for which BROWN gives as "extreme possible error" $\pm$ 0''.10.

Our result 0''.26 larger, is thus in moderate accordance with the theory.

As regards the periodic terms, it is very satisfactory that BROWN's inequalities are so well confirmed by the observations. It is, however, remarkable that after BROWN's thorough investigations the observations still betray an inequality with a coefficient of 0'' 66, which is theoretically unexplained.

Still I think that we are driven to this conclusion, and that the supposition of E. F. BAKHUYZEN (Proc. Akad. Amsterdam **6** 1903, 417), when he could find no trace of the existence of BROWN's term I (= Radau I) in the observations, that in the years considered another term must have neutralized its effect, is fully confirmed. For we now find that with equal coefficients their arguments in 1863 differed by 180°.

Astronomy. — *Investigation of the inequalities of approximately monthly period in the longitude of the moon, according to the meridian observations at Greenwich.* 2nd part. By J. E. DE VOS VAN STEENWIJK. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

In connection with my previous paper on the inequalities in the longitude of the moon, the period of which differs little from the anomalistic period of revolution, I have made some further calculations.

Even after applying the corrections which we have discussed and all the new inequalities determined by BROWN, a discordance still remained between observation and theory, which could be expressed by the following empirical term to be added to the theoretical longitude:

$$+ 0''.66 \sin_{\text{t}} \{g + 244°.4 + 40°.35 \ (t—1900.0)\}$$

I have already pointed out that it is striking that an inequality with such a comparatively large coefficient should exist, which is not explained by theory, and on this account I have tried to establish the reality of this term with greater certainty.

For this purpose first the value of this term and its influence upon h and k were calculated for each year, and my $-h_v$ and $-k_v$ were corrected for this. After this the mean residual discordance in h_v and k_v was determined, and in the second place for comparison the same was done, when h_v and k_v were not corrected for the empirical term. In the third place the same