

Citation:

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Physics. — “On the magnetization of ferromagnetic substances considered in connection with the assumption of zero-point energy. II. On the susceptibility in the excited ferromagnetic state”. By Dr. W. H. KEESOM. Supplement N^o. 32b to the Communications from the Physical Laboratory at Leiden. Communicated by Prof. H. KAMERLINGH ONNES.

§ 5¹⁾ *The susceptibility of ferromagnetic substances above their CURIE-point.* According to WEISS's theory ferromagnetic substances above their CURIE-point, before eventually adopting the paramagnetic state, pass into a state (eventually a succession of different states) in which just as below the CURIE-point the magnetization produces a molecular field. In that case, however, spontaneous magnetization does not occur any more; hence the molecular field only comes into action when it is excited by the external field. We assume in this section that the condition (15) for the existence of a CURIE-point is satisfied. For states, in which saturation is not yet appreciable²⁾, follows from equation (7), which then simplifies to $M_m = \frac{1}{3} n\mu a$, with (8), (10), (11) and (16) for the specific susceptibility (for unit of mass):

$$\chi = \frac{n\mu^2}{3(u_1 - u_{1c})}, \dots \dots \dots (27)$$

hence

$$\chi^{-1} \sim (u_1 - u_{1c}) \dots \dots \dots (28)$$

In the first approximation, *viz.* for equipartition, as well as in the second approximation, *viz.* when in the development according to positive powers of x of the second member of the first of the equations (5) only the first two terms need to be considered³⁾, (28) passes into WEISS's law for the susceptibility above the CURIE-point:

$$\chi^{-1} \sim (T - T_c) \dots \dots \dots (29)$$

In fig. (2) is shown how the curve which gives χ^{-1} as a function of temperature, is derived from u_1 as a function of T . According to

1) The §§, equations and figures of this paper are numbered as continuations of those in Suppl. No. 32a.

2) For these states no account has to be taken of the change of u_1 under the influence of the field, which has been treated in § 3b.

3) With an accuracy of 1% in u_1 this is the case for $T > 1.2 \Theta_0$, with one of 1% for $T > 4 \Theta_0$.

(5) $\Delta = \frac{2}{15} \Theta_0$. For sufficiently high temperatures that curve may be regarded as straight. The assumptions of the former paper involve,

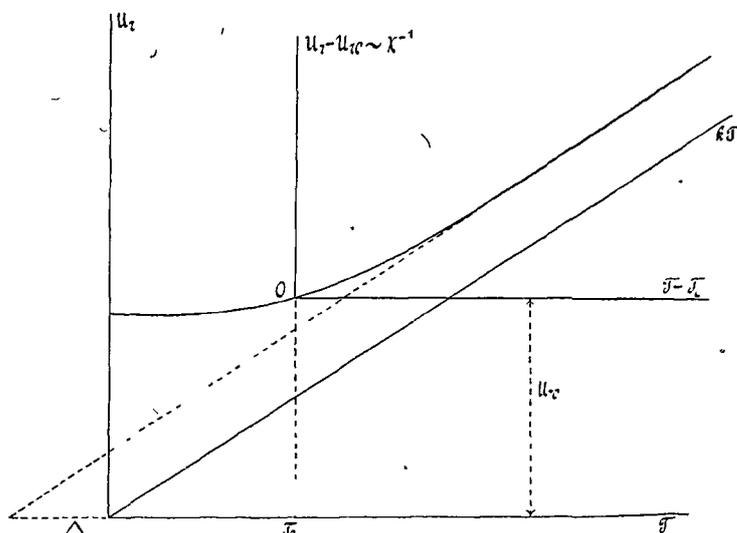


Fig. 2.

however, that in the neighbourhood of $\Theta \chi^{-1}$ as a function of T is curved more or less according to the ratio of T_c to Θ_0 . In an appreciable measure this curvature may only be expected for high values of Θ_0 , such as for the monatomic ferromagnetic elements.

The observations by WEISS and FOEX¹⁾ concerning nickel, iron²⁾ and cobalt actually point to a change of χ^{-1} as indicated by Fig. 2. It is true, that WEISS and FOEX represent χ^{-1} approximately by two portions of straight lines, at least for a certain region of temperatures from the CURIE-point upwards, but they remark expressly, that for instance³⁾ for nickel the inclination of the portion of the graph which is nearest to O has no determinate value.

In accordance with this HONDA and TAKAGI⁴⁾ find the χ^{-1}, T -graph for nickel between over 400°C. and 550°C. gently curved. Further according to HONDA and TAKAGI this portion passes continuously into the nearly straight portion for higher temperatures. If we abstract

¹⁾ P. WEISS and G. FOEX, Arch. sc. phys. et nat. (4) 31 (1911), p. 89.

²⁾ Cf. for this, however, A. PREUSS, Diss. Zürich (Techn. Hochsch.) 1912, p. 64

³⁾ Cf. p. 95 *l.c.*, for iron p. 97, cf. also in particular for cobalt p. 101.

⁴⁾ KÔTARÔ HONDA and HIROMU TAKAGI, Science Reports Tôhoku University Sendai (1) 1 (1913), p. 229.

from the point at the highest temperature the χ^{-1}, T -curve graphed by them in Fig. I, pl. 1, i.e., for nickel¹⁾ has just the form which would be expected from Fig. 2²⁾,

§ 6. *The possibility of the existence of ferromagnetic substances without CURIE-point.* It was pointed out in § 3a that on the assumption of a zero-point energy for the molecular rotations the existence must be considered possible of ferromagnetic substances (substances with a molecular field), which do not show the specific ferromagnetic properties. The condition for it was given in (14). Such a substance will have to be looked for in the first instance among those which have a large zero-point energy of rotation (a small moment of inertia) and a small molecular magnetic moment, and, also a weak molecular field.

From (7) follows, for states in which saturation has not yet become appreciable, with (8), (10) and (11) for the susceptibility

$$\chi^{-1} \sim u_r \sim v \text{ with } v = \frac{N_m n \mu^2}{3} \dots \dots (30)$$

In Fig. 3 are united the different cases which are possible for different values of the constant v , compared with u_{10} . In this figure for the different values of v χ^{-1} has to be measured from the corresponding horizontal line.

If at constant molecular magnetic moment and constant zero-point energy the molecular field increases from 0, we subsequently meet with the following cases:

$v = v_1$. At sufficiently high temperatures $\chi (T + \Delta_1) = \text{const.}$

¹⁾ Similar results they find for cobalt and iron.

²⁾ The observations by WEISS and FOEX concerning the susceptibility of nickel above of the CURIE-point can up to 870° C. be reasonably well represented with the value $\Theta_0 = 2100$ of § 4 and with the magnetic moment of the gramme-atom = 8552. This latter value differs from that derived in § 4 for the state below the CURIE-point. On account of provisional calculations it seems, however, probable to me that for nickel the spontaneous magnetization *below the CURIE-point as well as* the susceptibility in the excited ferromagnetic state *above the CURIE-point* up to the temperature mentioned above, according to the measurements by BLOCH, Thesis Zürich (Techn. Hochsch.) 1912, presumably even up to about 1200° C., can be represented with *the same value of Θ_0 and with the same value of the magnetic moment per atom*, and likewise with *the same value of the molecular field*. In a following paper I hope to communicate the results of further calculations regarding this point. If these calculations lead to a confirmation of that expectation, the significance of WEISS's theory of the molecular field (completed by the assumption made in this communication regarding the thermal equilibrium of these molecular rotations) will become even considerably greater.

(with positive Δ_1). This class of substances could be called *quasi-paramagnetic*: the presence of the molecular field only becomes

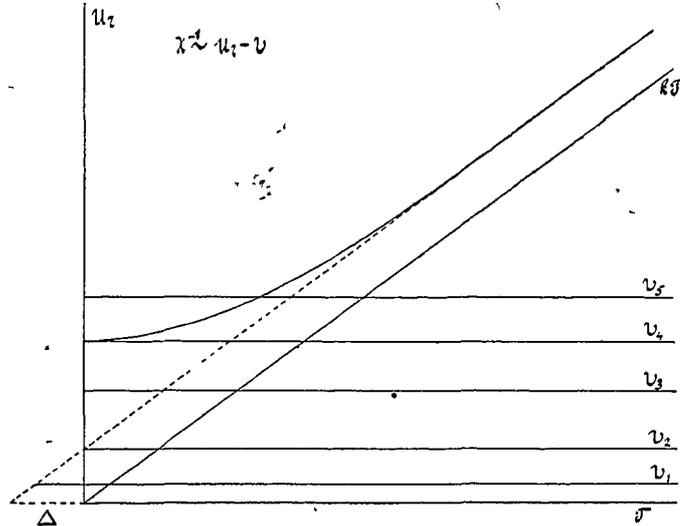


Fig. 3.

apparent, as far as the susceptibility is concerned, in this that these substances deviate from the law of corresponding states, which in accordance with the supposition made by KAMERLINGH ONNES and PERRIER¹⁾ and found confirmed in OOSTERHUIS' theory²⁾ is valid for purely paramagnetic substances (without molecular field) with constant magnetic moment.

$v = v_2$. The substance follows over a large region CURIE'S law ($\Delta_2 = 0$). At low temperatures the χ^{-1} , T -graph is curved, a region with definite Δ ²⁾ cannot, however, be indicated.

$v = v_3$. State of excited ferromagnetism, characterised by a positive value of Δ_3 in $\chi(T - \Delta_3) = \text{const.}$ for sufficiently high temperatures. The substance has, however, no CURIE-point.

$v = v_4$. Also no CURIE-point. If we abstract from saturation phenomena, χ would continually increase on approaching $T = 0$. It is for such a substance or one which differs little from it (v little less than v_4) that one should in the first instance expect the appearance of saturation phenomena at low temperatures.

$v = v_5$. Ferromagnetic substance with CURIE-point.

¹⁾ H. KAMERLINGH ONNES and ALB. PERRIER, Comm. N^o. 124a (Dec. 1911), § 3. Cf. E. OOSTERHUIS, Suppl. N^o. 31, These Proceedings p. 440, note 4.

²⁾ Cf. E. OOSTERHUIS, l. c.

§ 7. *Increase of the molecular magnetic moment with temperature.*
 In the discussions of this and the former paper the molecular magnetic moment has always been supposed to be constant and the magnetic axis to be a fixed axis in the magnetic molecule. The simplest hypothesis is that the electron or the electrons, whose motion (for instance in a circular orbit) causes the magnetic moment, are bound to a definite place in the molecule. In this manner one is led to the hypothesis, that a magnetic molecule owes its magnetic moment to the fact, that it rotates round a definite axis, which then is the magnetic axis, one or more electrons being situated at a distance from it. As long as the rotational velocity is constant and the distances of these electrons from the axis remain the same, the magnetic moment is constant. The question naturally arises, whether this rotation (round the magnetic axis) also participates in the thermal equilibrium. In the affirmative case the fact that the velocity of rotation does not change appreciably from low temperatures upwards over a large region of temperatures would point to a large value of the constant θ_0 , which corresponds to this rotation, and should be attributed to a particularly small moment of inertia about the corresponding axis.

We may resume this as follows: a magnetic molecule would be a molecule (or part of it) which has a small moment of inertia round a definite axis, so that with zero-point energy it rotates with great velocity round this axis, and contains at least one electron which is bound to a place in the molecule situated at a distance from that axis.

According to this hypothesis about the participation of the rotation which we now consider in the thermal equilibrium, at higher temperatures the magnetic moment will begin to increase with temperature, at first slowly, later on more rapidly. WEISS¹⁾ has actually been led by the observations above the CURIE-point for magnetite, to the hypothesis of a magnetic moment increasing with temperature. A difference with WEISS's views lies in this, that WEISS adopts a discontinuous increase of the molecular magnetic moment in rational proportions, whereas from the above hypothesis one would rather expect a continuous increase of the molecular magnetic moment with temperature. A further investigation will have to show, whether the measurements by WEISS and FOËX regarding this matter cannot be interpreted in this way, in particular for instance with application of the developments of § 3a to the rotational energy which here comes into play.

¹⁾ P. WEISS, Arch. sc. phys. et natur. (4) 31 (1911) p. 402, § 12.

If we keep in mind that for other substances the moments of inertia about different axes possibly do not differ from each other to such a degree as this seems to be the case with ferromagnetic substances, and that with different ratios of the moments of inertia very different cases can occur for paramagnetic substances also, and if in addition we take into consideration the possibility that according to § 6 there exist substances which possess a molecular field, but not a CURIE-point, there seems room for the further interpretation of different cases met with by DU BOIS, HONDA, and OWEN¹⁾ in their investigations regarding the susceptibility of elements, and which cannot according to OOSTERHUIS be represented with the assumption of a constant magnetic moment without molecular field.

A P P E N D I X.

The following proof of equation (18) of this Communication I was kindly communicated to me by Prof. LORENTZ.

In this appendix the references to sections and equations relate to the article V 14, "Elektronentheorie", in the Math. Encyklopädie. Some notations in this appendix differ from those used in the foregoing communication.

In considering the energy of a magnetized body one can start from two different points of view, viz. one can assume:

A. that the molecular magnets really are permanent magnets (hence that the magnetic moment is not due to rotations or to the circulation of electric charges), or

B. as in § 48e, that the magnetic moment is due to a rotation or to the circulation of electric charges round a definite axis in the molecule.

In both cases the magnetic energy may be put equal to $\frac{1}{2} \int H^2 dS$, the integration being taken over the whole space, including the space within the magnets. H is not the same for the hypotheses A and B. On the assumption A we have to imagine "magnetism" distributed over the magnets, and to calculate the magnetic force due to this magnetism. To this eventually the magnetic force due to electric currents is to be added. On the assumption B we have to deal with the force H , which, according to the equations of the electron theory, is due to the circulating electricity.

¹⁾ H. DU BOIS and K. HONDA. Proceedings. Jan. 1910, p. 547. K. HONDA. Ann. d. Phys. (4) 32 (1910), p. 1027. Science Reports Tôhoku University Sendai 1 (1912), p. 1. M. OWEN. Proceedings. Dec. 1911, p. 637. Ann. d. Phys. (4) 37 (1912), p. 657.

Concerning the molecular field we will assume that it consists of couples due to mutual actions of the molecules which are not magnetic actions, and that on the assumption B it is out of the question that the molecular field could change the moment of a magnet by induction. The energy per unit of volume due to the molecular field may be written as

$$-\frac{1}{2} kM^2,$$

if kM represents the molecular field.

We now follow the reasoning of § 52*a*. Hence we consider a physically infinitely small sphere of volume B within the magnetized body. The field within it, due to what lies outside it, is indicated by 2, the field due to what lies inside the sphere by 1.

The energy within the sphere due to 2 may be called: Ba ,
 that „ „ „ „ „ the superposition of 2 and 1: Bb ,
 „ „ „ „ „ „ 1 : Bc .

In calculating the latter we distinguish:

the own energy of each elementary magnet contained in B individually; the sum of these may be called Bp ;

the energy due to the superposition of the fields of the different elementary magnets: Bq .

We calculate p and q for the whole space (outside B only ether being assumed) and then have to subtract from this sum the energy in the outside space furnished by the elementary magnets that are inside the sphere. Let this be Br .

Then

$$c = p + q - r.$$

Let further BU be the energy of the (rotatory) heat motion, which opposes the directing of the particles (U is reckoned here per unit of volume).

Then the whole energy per unit of volume is:

$$U = a + b + p + q - r - \frac{1}{2} kM^2 + U,$$

By the own energy p is to be understood that which is due to the constitution of the elementary magnet and to its own field. As on the assumption A nothing ever changes of the elementary magnet, p is an absolute constant which need not be further considered.

On the assumption B, by p is understood the energy (magnetic and eventually ordinary kinetic) which corresponds to the rotation round the axis a (§ 48*e*). This energy is supposed not to be included in U . As the velocity of that rotation changes by induction, p is not an absolute constant on this assumption.

According to A as well as to B (cf. § 52*a*):

$$a = \frac{1}{2} \left(H + \frac{1}{3} M \right)^2, \quad r = \frac{1}{9} M^2.$$

In calculating b and q we have to distinguish between A and B: Assumption A. In this case formulae are valid which correspond to those which are valid for electrically polarized particles, viz.:

$$b = -\frac{1}{3} M \left(H + \frac{1}{3} M \right), \quad [\text{cf. (150)}]$$

whereas in calculating q we have to take for each particle [cf. (152)]

$$-\frac{1}{2} (\mathbf{m} \cdot \mathbf{h}_{11}),$$

here h_{11} is the magnetic force which at the place occupied by one particle is brought about by the remaining particles situated inside the sphere B , \mathbf{m} the magnetic moment of that particle, considered as a vector, (\mathbf{a}, \mathbf{b}) representing the scalar product of two vectors \mathbf{a} and \mathbf{b} . If we take the sum for all particles inside B , we can write

$$q = -\frac{1}{2} M h_{11}.$$

Assumption B. Now [cf. (151)]

$$b = \frac{2}{3} M \left(H + \frac{1}{3} M \right).$$

In calculating q we have to take for each particle

$$\frac{1}{2} (\mathbf{m} \cdot \mathbf{h}_{11}). \quad [\text{cf. § 52 } a, \gamma].$$

This gives

$$q = \frac{1}{2} M h_{11}.$$

On the assumption B we still have to consider the term ν .

For the own energy of a magnet we may write (§ 48*e*)

$$\frac{1}{2} Q g^2,$$

but we must take into consideration, that the angular velocity g according to equation (140) differs from g_0 . From (140) follows

$$\frac{1}{2} Q g^2 = \frac{1}{2} Q g_0^2 - \frac{g_0 \bar{k} |\mathbf{h}| \cos \vartheta}{c},$$

if the second power of the last term in (140) is omitted (if we retained this we should take account of a term which is even smaller than the energy of the weak diamagnetism, which always occurs as a consequence of the appearance of the field and which is superposed on the paramagnetism, respectively ferromagnetism).

To distinguish it from the coefficient of the molecular field the quantity k of (140) is indicated here by \bar{k} .

From the formulae of § 15 one finds for the moment of the particle

$$\frac{1}{c} g_0 \bar{k};$$

hence the last term in the above expression becomes

$$-(\mathbf{m} \cdot \mathbf{h}).$$

Here \mathbf{h} is the whole magnetic force acting on the particle, viz. the LANGEVIN force H (see below) with the exception of the molecular field kM . Omitting the constant term $\frac{1}{2} Qg_0^2$ we find

$$p = -M(H - kM) = -HM + kM^2.$$

Collecting the different terms we obtain according to:

Assumption A.

$$\begin{aligned} U &= \frac{1}{2} \left(H + \frac{1}{3} M \right)^2 - \frac{1}{3} M \left(H + \frac{1}{3} M \right) - \frac{1}{2} M h_{i_1} - \frac{1}{9} M^2 - \frac{1}{2} kM^2 + U_r \\ &= \frac{1}{2} H^2 - \frac{1}{6} M^2 - \frac{1}{2} M h_{i_1} - \frac{1}{2} kM^2 + U_r. \end{aligned}$$

If we write $H = H + \frac{1}{3} M + h_{i_1} + kM$ for the total force which is to be put into LANGEVIN'S formula, we can also write

$$U = \frac{1}{2} HB - \frac{1}{2} HM + U_r.$$

Assumption B:

$$\begin{aligned} U &= \frac{1}{2} \left(H + \frac{1}{3} M \right)^2 + \frac{2}{3} M \left(H + \frac{1}{3} M \right) - HM + kM^2 + \frac{1}{2} M h_{i_1} \\ &\quad - \frac{1}{9} M^2 - \frac{1}{2} kM^2 + U_r \end{aligned}$$

With the aid of the above expression for H we find again

$$U = \frac{1}{2} HB - \frac{1}{2} HM + U_r$$

We may combine the different terms of $H-H$ to one and write it in the form $k'M$. There will be no objection to this, if we consider that the term $k'M$ is by far preponderating. Then

$$H = H + k'M.$$

With this we find

$$U = \frac{1}{2} H^2 - \frac{1}{2} k'M^2 + U_r.$$

The work done on the body at an infinitely small change is

$$H dB$$

The heat to be supplied is then

$$dQ = dU - H dB.$$

This gives

$$\begin{aligned} dQ &= H dH - k'M dM + dU_r - H dB \\ &= -H dM - k'M dM + dU_r \\ &= dU_r - H dM, \end{aligned}$$

which relation agrees with equation (18) of the preceding communication.