Physics. — "*Contribution to the knowledge of the string galvanometer.*" Communicated by Prof. J. K. A. WERTHEIM SALOMONSON.

(Communicated in the meeting of October 25, 1913).

I. *Intensity of the magnetic field with prismatic pole-pieces.* STEFAN (Wied. Ann. 38, 1889, p. 440) has shown that in an electro-magnet with a round core the number of the magnetic lines in the interferricum could considerably be increased, by giving the pole-pieces a conical form with a top-angle of about 110°. In the string galvanometer prismatic pole-pieces are used. What must be the top-angle in this case in order to obtain a maximal magnetization in the interferricum?

STEFAN admits in his calculation, which is afterwards also used by P. WEISS (Soc. fr. d. Phys. 1907, p. 132) that the lines of force in the magnetically saturated iron core are parallel to the axis, and that at the surface of the cone free magnetism is extant. We divide the conical surface into a succession of infinitely narrow circular strips standing perpendicularly to the axis. Each point situated on such a strip, exercises on a point placed at the summit of the cone an attraction, inversely proportional to the square of the distance. The axial component of this force may be represented by an expression proportional to $\frac{\cos \alpha}{l^2}$ in which l represents the distance, and α half the topangle. If we put $l = \frac{r}{\sin \alpha}$ in which r indicates the radius of the circular strip, then the axial force becomes proportional to $\frac{\cos \alpha \sin^2 \alpha}{r^2}$.

This expression has a maximum for $\cos \alpha = \sqrt{\frac{1}{3}}$ or $\alpha =$ about 55°.

What holds for one definite point of the circular strip, holds for every point of any other circular strip situated on the conical surface. Consequently the most favourable top-angle for the cone amounts to about 110°.

We can apply a similar reasoning in the case of prismatical pole-pieces. In this case we divide the prismatic surface into infinitely narrow strips parallel to the rib.

We calculate in the first place the attraction, exercised by an infini-

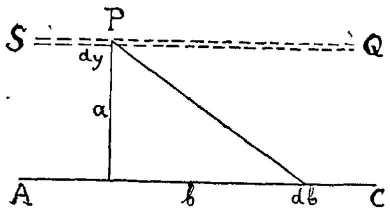

Fig. 1.

tesimal mass dy situated in a point P of the strip SQ on a small mass db situated in the rib AC of the prism. The distance of the strip SQ from the rib AC be a, the distance at which the part db is situated from the projection of P on AC, be b.

As the attraction of P on db is inversely proportional to the square of the mutual distance and proportional to the size of the part db, we may put:

$$K \backsim \frac{db}{a^2 + b^2}.$$

We now obtain the total attraction of the part P on the whole line AC by integrating this expression between the limits $+\infty$ and $-\infty$

$$\int_{-\infty}^{+\infty} \frac{db}{a^2 + b^2} = \frac{1}{a}\, bg\, tg\, \frac{b}{a} \int_{-\infty}^{+\infty} = \frac{\pi}{a}.$$

The attraction of each part of the strip SQ on the whole line AC is consequently inversely proportional to their mutual distance. This holds for every other part of the strip and consequently for the entire strip SQ, and for every other strip running parallel to AC.

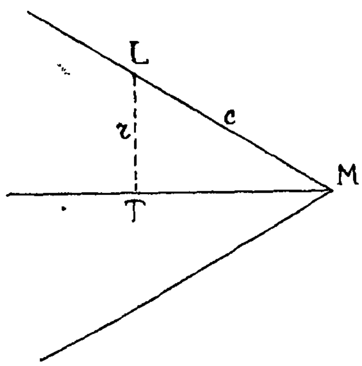

Fig. 2.

We shall now calculate the attractive force exercised in an axial direction by a strip SQ.

Be L a point of the strip SQ, M a point situated in the rib, whilst we suppose that both L and M are situated in a plane, normal to the rib. We found that the attractive force of the strip running through L on the line situated in the rib M, is inversely proportional to their distance $LM = c$, consequently

$$K_1 \backsim \frac{1}{c}.$$

If the distance from L to the line TM be $= r$, the axial component is

$$\frac{\cos\alpha}{c} \text{ and as } \frac{1}{c} = \frac{\sin\alpha}{r}$$

the axial fraction is proportional to $\frac{\sin\alpha\cos\alpha}{r}$.

This expression has a maximum for $\alpha = 45°$.

As the same reasoning may be used for every other strip situated on the planes of the prism, we ought to cut the prism in such a way that the bi-plane-angle at the rib amounts to exactly 90°.

This is only true if the planes of the prisms terminate in the line of intersection. This case however never occurs: the planes are always cut off by a plane parallel to the ribs so that the interferricum is enclosed between two planes parallel to each other and perpendicular to the lines of force. In such a case we can, however, still calculate the maximal field-intensity at the line of intersection of the planes. The field is then formed by two different components, i.e. by the magnetic lines passing through the side-planes and by those issued by the two limiting planes. If these be H_1 and H_2 we have for the total field: $H = H_1 + H_2$.

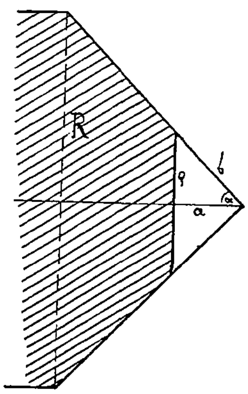

Fig. 3.

We found for each strip H_1 the value of $\frac{\sin\alpha\cos\alpha}{r}$. In order to obtain an expression for the magnetization due to an entire side-plane we suppose that the breadth of the strip is dr, and that consequently its action is proportional to $\sin\alpha\cos\alpha\frac{dr}{r}$. If we integrate this expression between the limits ϱ and R, in which ϱ represents half the depth of the interferricum, R half the thickness of the magnet, we find the value looked for. It amounts to:

$$H_1 = \int_{\rho}^{R} \sin\alpha\cos\alpha\frac{dr}{r} = \sin\alpha\cos\alpha\ lgn.\frac{R}{\varrho}$$

In order to calculate the magnetic field caused by the free magnetism in the parallel boundary plane, we divide it again into length-wise strips.

The attraction exercised on the rib by each strip is inversely proportional to their mutual distance b, consequently $\frac{1}{b}$.

The axial attraction is $\frac{a}{b^2}$, if a represents half the length of the interferricum. If we integrate along b between the limits a and $\sqrt{a^2 + \varrho^2}$, then we find

$$H_2 = 1 - \frac{a}{\sqrt{a^2 + \varrho^2}} = 1 - \cos \alpha$$

Hence: $H_1 + H_2 = 1 - \cos \alpha + \sin \alpha \cos \alpha \, lgn \frac{R}{\varrho}$.

The maximal value of this expression depends on the magnitude $R:\varrho$. If ϱ is infinitely small, which means that the side-planes of the prism terminate in the natural rib, we find again $\alpha = 45°$. With other values of R we find:

$R:r$	α
∞	45°
100	49° 44'
50	51° 38'
25	52° 5'
10	55° 29'

In the string-galvanometer, as a rule, $R:\varrho$ will be somewhere between 25 and 50. By making the top-angle about 51° we obtain the maximal field-intensity. As however the value of the expression for $H_1 + H_2$ does not vary much in the neighbourhood of the maximum, a little deviation in size of the angle will have no prejudicial consequences.

In general we can say that it is better to make the angle a little larger than the theoretical value, as in that case the field will become more homogeneous, whilst with a smaller angle the field-intensity will diminish more rapidly towards the rims. As a maximum perhaps 53°—55° may be taken; i.e. with a core of 5 cm. diameter the angle should be nearly 55° and with a core of 10 cm. 53° must not be surpassed.

II. *The magnetic field and the shape of the string in the* EINTHOVEN-*galvanometer.*

As soon as a constant current passes through the string of the galvanometer of EINTHOVEN the string assumes a curved shape. If we wished to draw this shape, we might use a well-known graphostatical method of construction. It is used e.g. to draw the shape of a chain of a suspension bridge.

Suppose the chain is suspended between A and B and the loads

4_1, 3_1, 2_1, 1_1, 1_0, 2_0, 3_0, 4_0 pulling on the chain in a vertical direction, perpendicularly to AB, then a diagram is drawn in the following

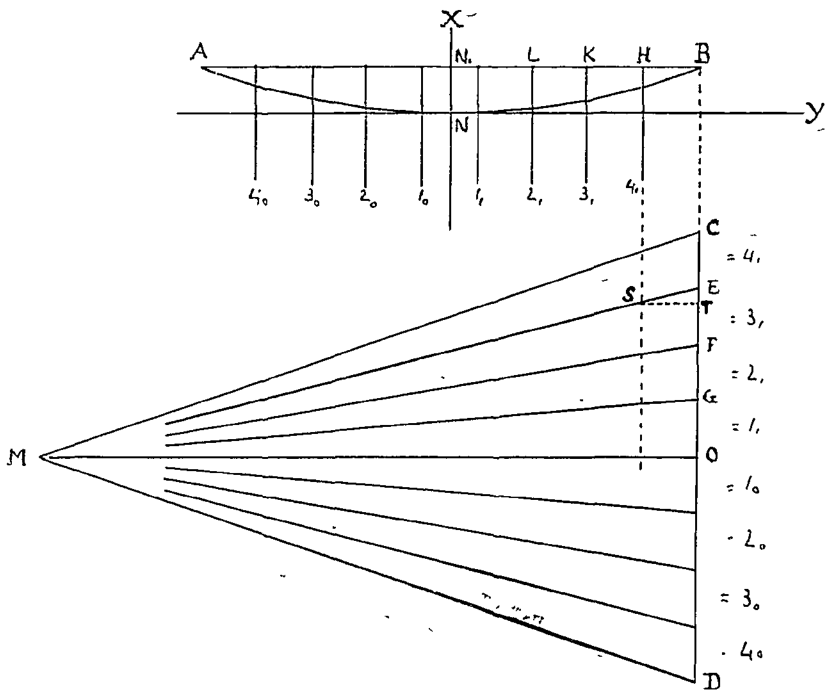

Fig. 4.

manner. Parallel to AB a line OM is drawn representing the longitudinal strain of the chain. Perpendicularly to the end of OM OD is drawn, and on this line parts OG, GF, FE are taken representing the pulls 4_1, 3_1 etc. to 4_0 on the same scale as has been used with OM. The points of division in the line DC are joined with M. From B a line is drawn, parallel to MC, till it intersects with H_4. From there a line is drawn, parallel to ME, till it intersects with $K3_0$ etc. In this way we finally get the line BNA, which represents the shape of the chain.

Theoretically this construction would also be correct, if the number of the lateral loads were infinitely great and their distances infinitely small.

If we regard this method of construction more closely, we immediately see that the line OC represents the sum of all the lateral forces, if we start in the middle of the chain. We may consequently consider the distance from every point of that line to the point O as the integral of the lateral forces. If these be represented by the expression $f(Z)$ and if the forces act at distances dl from each other, then $\int f(Z)dl$ is an expression for the length of each part of the line OC, reckoned from the point O.

If we have drawn the line $\int f(Z)dl$ and we join any divisional point with M, we find the slope of the chain on the place of the load, corresponding to the divisional point. So e.g. in figure 4 the slope at 3_0 will be represented by the line EM. To calculate the slope with regard to the Y-axis, we deduce from the congruency of the triangles SET and MEO, that the increase of height TE which we may indicate as Δh, is $= \frac{EO}{OM} \times ST$.

As EO represents the integral lateral pressure, consequently $\int f(Z)dl$ and OM the total longitudinal tension P, whilst ST represents the length Δl of the chain of which the slope has been calculated, we can write, passing to infinitely small differences

$$dh = \frac{dl}{P}\int f(Z)dl$$

or integrating:

$$h = \frac{1}{P}\int\left(dl\int f(H)dl\right)$$

We may apply this reasoning to the string galvanometer. We find then that the lateral pressure which we called $f(Z)$, is proportional to the intensity of current in the string I and to the field-intensity H at every point which may be written $f(IH)$, or as I is constant over the length of the string, $If(H)$.

For the galvanometer we get the expression:

$$h = \frac{I}{P}\int\left(d\int f(H)dl\right)$$

if the coordinate-system has its origin in the point N. If we take the point N_1 i. e. the middle of the not-deviated string, this expression becomes:

$$h_1 = \frac{I}{P}\left\{\int_0^l dl\left(\int_0^l f(H)dl\right) - \int dl\left(\int f(H)dl\right)\right\}$$

in which the definite integral has simply the meaning of the maximal deflection NN_1 of the string at the existing intensity of current and tension.

If we keep to a coordinate system originating in N, we arrive at the conclusion that the shape of the string is related to the local intensity of the magnetic field in such a way, that we obtain an expression for the shape of the string by integrating twice successively the expression for the magnetic field.

As an example may serve the case that the magnetic field is homogeneous over its entire height. We can then write $f(H) = H$. Then we obtain for the lateral pressure the well-known formula

$$z = I \int H dl = HIl$$

and for the form of the string

$$h = \frac{I}{P} \int H l dl = \frac{1}{2} \frac{I}{P} H l^2.$$

the length of the string being $2l$.

This last expression represents the vertex-equation of a parabola. In a homogeneous field the string takes the form of a parabola.

In these and also in the following considerations we assume that the longitudinal tension in the vertical string is everywhere the same, and that the weight of the string itself may be neglected, compared with its tension P.

The formula given above may also be applied in cases where the field, taken over the height, is not homogeneous. It may be applied, if we can express the local field-intensity either in figures or in a formula.

In the string-galvanometer where the pole-pieces are pierced, we can, as a first approximation, represent the intensity of field by the line I (fig. 5). Over the length l_1 corresponding to half the width of

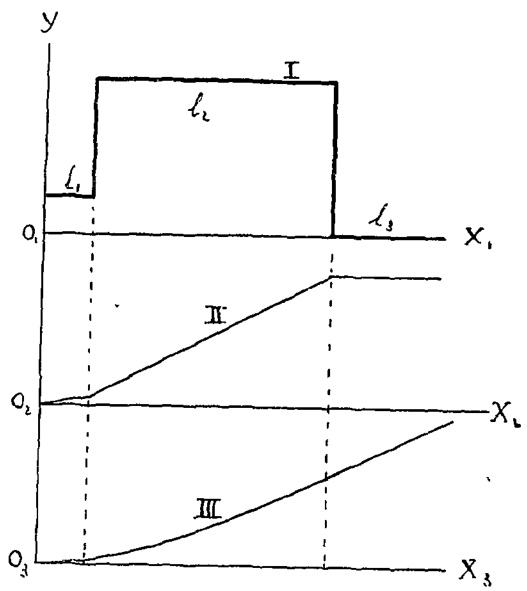

Fig. 5.

the central bore we have a constant field of an intensity H_1; over the length l_2 corresponding to the length of the strongest part, the field is also homogeneous and has a density H_2, whilst the string continues beyond this part over a distance l_3 where the density of the field falls to nought. The entire length of the string is consequently $2(l_1 + l_2 + l_3)$. Now we integrate successively over the parts l_1 l_2 and l_3, and find for the lateral pressure:

$$\text{over } l_1 : z_1 = IH_1l_1$$
$$\text{over } l_2 : z_1z_2 = I(\underline{H_1l_1} + H_2l_2)$$
$$\text{over } l_3 : z_1z_2z_3 = I(\underline{H_1l_1} + \underline{H_2l_2})$$

in which the underscoring indicates that we have no longer to do with a variable, but with a constant. The line answering to these integrals for the lateral pressure is represented by II.

For the form of the string we obtain after a second integration:

$$\text{in part } l_1 : h = \frac{1}{2}\frac{I}{P}H_1l_1^2$$

$$\text{in part } l_2 : h = \frac{I}{P}\left(\frac{1}{2}\underline{H_1l_1^2} + \underline{H_1l_1}l_2 + \frac{1}{2}H_2l_2^2\right)$$

$$\text{and in part } l_3 : h = \frac{I}{P}\left\{\frac{1}{2}\underline{H_1l_1^2} + \underline{H_1l_1l_2} + \frac{1}{2}\underline{H_2l_2^2} + \underline{H_1l_1}l_3 + \underline{H_2l_2}l_3\right\}$$
$$= \frac{I}{P}\left\{H_1\left(\frac{1}{2}\underline{l_1^2} + \underline{l_1l_2} + \underline{l_1}l_3\right) + H_2\left(\frac{1}{2}\underline{l_2^2} + \underline{l_2}l_3\right)\right\}$$

In the points between l_1 and l_2 and between l_2 and l_3 the line given by the last expression shows a gradual change of direction and not an abrupt one. This is proved by the fact that the value of $\frac{dh}{dl}$ at the end of l_1 is equal to $\frac{dh}{dl}$ at the commencement of l_2, if l_2 be made $= 0$. This equality of the differential coefficients holds likewise for the transition of l_2 into l_3.

Though perhaps we might obtain a better approximation for the shape of the string by supposing that the intensity of the magnetic field varies according to a line of the form represented in fig. 6, we can already obtain some practical result with the simple expression, graphically represented in fig. 5.

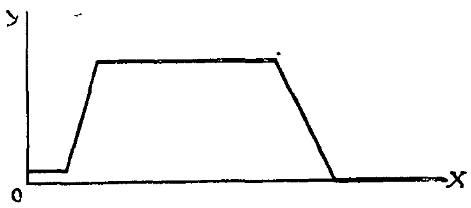

Fig. 6.

Let us first take the case that the string is not longer than the height of the field, in other terms that $l_3 = 0$.

The expression for h remains then:

$$h = \frac{I}{P}\left(\frac{1}{2} H_1 l_1^2 + H_1 l_1 l_2 + \frac{1}{2} H_2 l_2^2\right)$$

$$= \frac{I}{P}\left\{\frac{1}{2} H_1 (l_1 + l_2)^2 + \frac{1}{2} l_1^2 (H_2 - H_1)\right\}$$

If we give to $l_1 + l_2$ the length of 50 millimeters, to H_1 the value of 4000, to H_2 the value of 20000 gausses, we obtain, if the bore possesses a radius $l_1 = 10$ or 7 millimeters, deviations h bearing a proportion as 17.8 to 19.832. This means that the smaller bore of 14 millimeters diameter gives an increase of the sensibility of 11 °/₀ as compared with a perforation of 20 millimeters, the height of the field being in both cases 100 millimeters.

Since 1909 I have used a galvanometer in which the perforation has been reduced to 13.7 millimeters, for which the firm of ZEISS has constructed a special apochomatic objective. The Cambridge Instrument Cy. has of late likewise reduced the bore of their instruments.

With the formulae given above we can also approximately calculate the change in the sensibility if the height of the strongest part of the field is shortened whilst the length of the string remains constant.

Let us write for that purpose:

$$h \sim H_1 \left[\frac{1}{2} l_1^2 + l_1 (l_2 + l_3)\right] + H_2\left(\frac{1}{2} l_1^2 + l_2 l_3\right).$$

If $l_2 + l_3$ be constant, e.g. $= 50$, and if we give to l_2 successively the values 50, 45, 40, 35, 30, whilst l_1 remains $= 7$; if further H_1 be 4000 and $H_2 = 20000$, we obtain for h 26.47, 26.22, 25.47, 24.22 and 22.47. This means that a diminution of the height of the field of 10 °/₀ gives only a diminution of the sensibility of 0.8 °/₀; a diminution of height of 20 °/₀ takes 3.8 °/₀, one of 30 °/₀ 8.2 °/₀ away from the sensibility.

From this we may conclude that it is of no consequence, if the length of the string exceeds the height of the field by 10—15 °/₀. On the other hand with a given length of the string the height of the field may be 10—15 °/₀ less, without causing an appreciable loss of sensitiveness. Now it is possible that with a given diameter of the iron-core a diminution of the height of the field might cause a slight increase of the density, by which even the slight loss caused by an excess of length of the string over the height of the field would be entirely compensated. I must add, however, that I have not given any further consideration to this question.

From our formulae we can obtain a better insight into the significance of the average *active* intensity, in a not homogeneous field. Active intensity means the intensity of a perfectly homogeneous field causing the same deviation h as can be obtained in the not homogeneous field. We get the expression:

$$H_w = \frac{H_1 l_1^2 + 2\, H_1 l_1 l_2 + H_2 l_2^2}{(l_1 + l_2)^2} = H_1 + (H_2 - H_1)\left(\frac{l_2}{l_1 + l_2}\right)^2$$

Be again $H_1 = 4000\ H_2 = 20000,\ l_1 = 7$ mm. and $l_2 = 43$ mm., then H_w, the active density, is 15843, whilst the arithmetic average, calculated as $H_1 + (H_2 - H_1)\frac{l_1 + l_2}{l_2}$ amounts to 17760.

We may add here, that the average active value can also be obtained with sufficient accuracy by dividing the square of the average intensity by the maximal intensity:

$$H_w = \frac{H^2_{gem.}}{H_{max.}}$$

In the example chosen, this empirical formula results in $H_w =$ 15771 instead of 15843, as first calculated, consequently an amount differing by less than $^1/_2\,^0/_0$ from the real value.

The mathematical connection between the shape of the string and the local field-intensity enables us either to calculate or to construct the shape of the string if the local intensity of the field be known with sufficient accuracy.

I have tried to measure the local field intensity by different methods. Firstly Prof. P. Zeeman had the kindness to try his method depending on the resolution of the spectral lines into doubles in the magnetic field. As in the narrow and high interferricum the spectral tubes filled with Helium or with Hg vapours were destroyed in a few seconds, this method has not given practical results.

Therefore I had to apply other methods, viz. the bismuth-method, and the method with the magnetic balance of Cotton.

For the bismuth-method I used thin wires of pure bismuth furnished by the firm of Hartmann-Braun. The measurements were made by means of a wire of 0.17 mm. diameter and a length of 12 mm. with a current of 1 milliampère. The temperature of the wire was measured repeatedly by measuring its resistance, after the field had been reduced as near as possible to nought. The results were finally calculated by means of the formula:

$$H = 2060 + 8t + (120.9 + 2\,4t)\Delta$$

which formula had been calculated from earlier measurements published by Henderson and later measurements of myself.

This formula in which t represents the temperature in centigrades, Δ the procentual increase of resistance, gives, with an intensity of field greater than 4500 Gauss, results, that are accurate to within 1 %, at least between the limits of temperature of 7°—25° Celcius. I obtained with this method, for the electromagnet of my EINTHOVEN galvanometer N°. II:

With a field-curr. of	0.40 Amp.	H=5360	in the strongest part of the field
„ „ „	„ 1.53 „	18900	„ „ „ „ „ „ „
„ „ „	„ 2.39 „	23850	„ „ „ „ „ „ „
„ „ „	„ 3.49 „	26950	„ „ „ „ „ „ „
„ „ „	„ 8.54 „	31350	„ „ „ „ „ „ „
„ „ „	„ 8.54 „	14150	„ „ weakest „ „ „ „

In these measurements I could not know whether the bismuth-wire had actually been in the strongest part of the field. Also it proved to be very difficult to measure the different parts of the field very accurately.

Therefore I have repeated the measurements with a balance of COTTON, which proved to give more accurate results with less difficulty and in less time.

My balance was provided with a rectilinear current carrying conductor of 19.057 mm. length; the arms of the balance were 304.25 resp. 304.48 mm. long. The constant was calculated at $5151 \frac{P}{I}$ in which P represented the weight placed in the scale, I the current strength in Ampères. The sensitiveness of the balance was varied till I got about a 1 mm. deflection for 1 milligram. The balance was placed on a sliding support, so that it could easily be moved in the direction of the interferricum.

In my first measurements I found with the same magnet with a fieldcurrent of:

1.50 Ampère	$H=19730$
3.48 „	27810
8.30 „	31695

which numbers probably differ less than 0.3 % from the real value. They agree with the values obtained before.

With this magnetic balance I have tried to measure the local intensity at every point of the field, with a given magnetizing current, viz. 1.55 Ampère. The balance was arranged so as to place the current-carrying conductor exactly in the central part of the interferricum, symmetrically with regard to the perforation. After the first measurement of the field-intensity the balance was moved along the field over a distance of 1.24 m m, this being one revolution of the micrometer-screw of the sliding support. The average intensity was measured again, and after each measurement the balance was moved along the field over the same distance, till at last the balance had passed through the entire field, and had come out of the interferricum.

In this series of experiments I found that the greater part of the field might be called absolutely homogeneous. Proceeding from this part the average intensity could now be calculated for each distance of 1.24 mm. Obviously the figures calculated for the weaker part of the field cannot be regarded as very accurate. By graphical interpolation I got figures which seemed to me to be more conform to the real value. In order to prove this, I recalculated from the values, for each part of 1,24 mm., average values over 19.05 mm. In table I we find in column I the calculated values for each part of the field of 1.24 mm. length. The second column gives the recalculated figures for each length of 19.05 mm., whilst in the 3rd column the figures as measured are given. The degree of agreement between the 2nd and the 3rd column indicates the degree of accuracy between the same. In general this correspondence is not unsatisfactory, only the values 2 and 3 show differences of 2 % and 3.5 %. For the rest the difference amounts to less than 1 %. The curves of fig. 7 represent graphically the numbers given in the table.

With the figures obtained for the local intensity of field we can now draw the exact shape of the string, by means of the graphical construction described in the beginning, or we can calculate it. The calculation is made by two additions that serve as means of integration. The first addition produces the series of figures 0, $0+1$, $0+1+2$, $0+1+2+3$ etc., consequently the values 0, 3600, 7250, 11050 etc., which indicate the integral values of the lateral pressure, at each point.

If these values be a, b, c, d, then a second addition in exactly the same way, gives us: a, $a+b$, $a+b+c$, $a+b+c+d$, etc. These last figures are given in column IV and show the relative deviation with regard to a Y-axis tangential to the point of maximal amplitude. They enable us to calculate with the field-current employed the relatively maximal deflections of any part of the string.

If e.g. the maximal deflection of a string of a length of 2×48 parts of 1.24 mm. each with a given current were 191.8, then the deflection with the same current would be 41.5 for a string of the same material and tension, but only half the length. We see that in

	I	II	III	IV		I	II	III	IV
0	3600	8940	8940	0	25	20950	20930	20900	45.7
1	3650	8995	8950	0.036	26	"	20870	20860	50.2
2	3800	9305	9010	0.11	27	"	20660	20600	54.8
3	4000	9900	9570	0.22	28	"	20310	20200	59.7
4	6100	10890	11030	0.37	29	"	19850	19850	64.8
5	12000	12020	12000	0.58	30	"	19280	19200	70.1
6	16100	13165	13050	0.91	31	"	18620	18540	75.6
7	19600	14320	14400	1.40	32	"	17310	17850	81.3
8	20430	15475	15400	2.09	33	19800	16500	16350	87.2
9	20750	16630	16750	2.99	34	17700	15630	15400	93.3
10	20950	17770	17800	4.09	35	15750	14700	14600	99.6
11	"	18900	18600	5.40	36	14000	13720	13700	106.0
12	"	19890	19600	6.92	37	12500	12830	12700	112.6
13	"	20490	20600	8.64	38	11000	11590	11700	119.3
14	"	20810	20860	10.58	39	9900	10450	10530	126.1
15	"	20900	20900	12.7	40	8800	9340	9470	133.7
16	"	20935	20930	15.1	41	7900	8316	8320	140.1
17	"	20950	20930	17.6	42	7000	7010	7060	147.2
18	"	"	20950	20.4	43	6200		6060	154.3
19	"	"	"	23.4	44	5400		4860	161.5
20	"	"	"	26.6	45	4600		4020	169.7
21	"	"	"	30.0	46	3800		3440	176.1
22	"	"	"	33.6	47	3100		2780	183.4
23	"	"	"	37.4	48	2400		2340	191.8
24	"	"	"	41.5	49	1750		1910	198.2

reality the deflections are not proportional to the square of the length of the string, as would be the case in an absolutely homogeneous field. This is caused by the central weaker part of the field, which influence increases if the string is shorter.

We should try to minimise the influence of the bore. We have already considered one way of doing it, i.e. to make the bore as narrow as possible. It is however difficult to make the bore

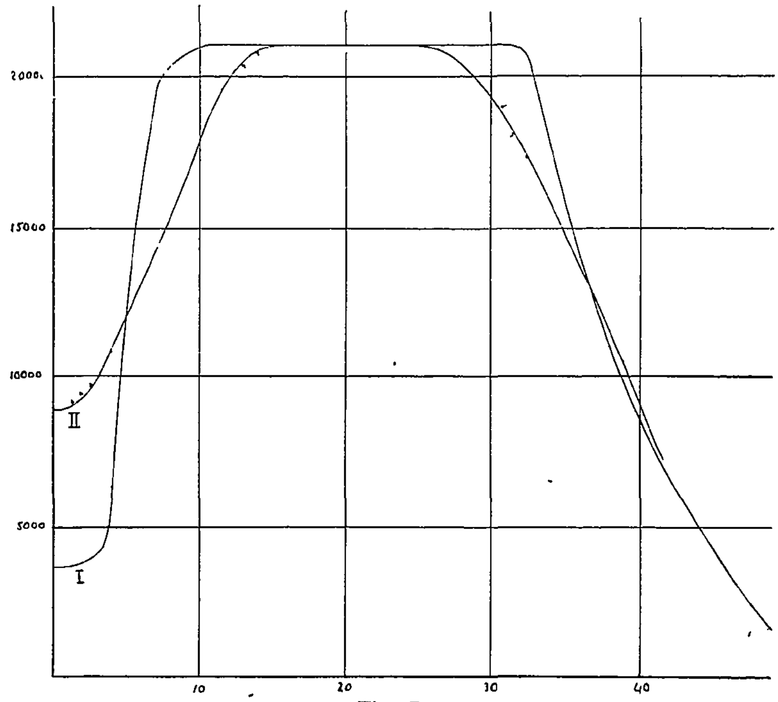

Fig. 7.

narrower than 13 m.m., as there are technical objections against the construction of apochromatical objectives of high aperture with an external diameter less than 12 m.m. Another way consists in increasing the field-current as much as possible. We have then a double advantage. Firstly the maximal field-intensity is increased as much as possible. Secondly with the more perfect magnetic saturation of the iron core, the magnetic leakage near the central bore pushes a relatively greater number of lines of force into the part of the field corresponding to the bore. The central part of the field increases thereby both absolutely and relatively. The direct measurement immediately confirms this.

Using the bismuth-method I found with the electro-magnet of galvanometer II in the weakest part a field with an average intensity of 14150 gauss, when the strongest part reached 31350 gauss. With a maximal field-strength of 20950 gauss the field in the central part fell to 5650 gauss.

With a new electro-magnet of a somewhat different form and size, which is wound for 25 ampère, I found:

at 21	ampère	maximal	39050	minimal :	23200
„ 10	„	„	36250	„	19760
„ 1	„	„	17010	„	7417

I may add here that with 25 ampère the field-strength increased to 40000 gauss. I intend to arm this electro-magnet with pole-pieces made of ferrocobalt, the new alloy of WEISS and hope to obtain with them a maximum of 43000 gauss.

III. *Determination of the active fieldstrength.*

As active field-strength we indicated the intensity of a field homogeneous over the whole length of the string, which causes the same deflection as the not homogeneous field, the tension and the material of the string as well as the current passing through it, being the same.

EINTHOVEN has already given a method to measure the active intensity of the field. His method depends on the measurement of the electro-magnetical damping of the movement of the string.

Here follow two more methods which lead both to about the same result.

First Method.

If a current I passes through the string, it is subjected to a lateral pressure HIl which causes the string to sway, and whereby a certain quantity of potential energy is stored. When the circuit is broken the string generally resumes its former position after a few damped oscillations. With every oscillation potential energy is changed into kinetical energy, and vice versa.

At the moment the string passes for the first time its final position of equilibrium, the total originally stored potential energy has disappeared, and nothing but kinetical energy remains. If the oscillations had been undamped, we might equal these two forms of energy. On account of the damping the value for the kinetical energy must first be corrected. We have then got an equation from which H can be resolved, and which gives an expression for the actual mean intensity of the magnetic field.

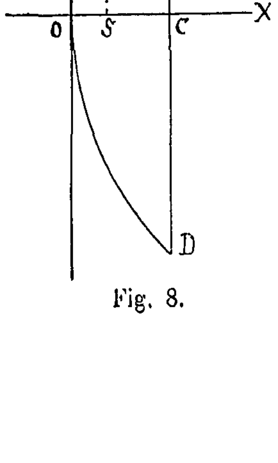

Fig. 8.

In our considerations we assume the field to be homogeneous and we neglect the elasticity and the weight of the string itself with regard to its strain. Under these circumstances the string takes, as we saw, a parabolic shape.

Be $BQOD$ the deflected string, its centre being 0. With regard to a system of coordinates passing through O, where the y-axis touches the parabola, the form of the string is represented by the equation:

$$y^2 = 2px.$$

The potential energy of any small particle dy of the string amounts to:

$$\frac{1}{2} HI\, QR\, dy\,.$$

Now $QR = h - x$ in which h represents the maximal deflection OC of the string, whilst x is the abscissa of the point Q. As $x = \frac{y^2}{2p}$ we obtain for the potential energy of the part dy:

$$\frac{1}{2} HI \left(h - \frac{y}{2p}\right)^2 dy\,.$$

By integrating this expression between the limits O and $\frac{1}{2}l$, l being the length of the string, we get half the potential energy. Hence the total energy amounts to:

$$E_{(pot.)} = 2 \times \frac{1}{2}\int_0^{\frac{1}{2}l} \left(h - \frac{y}{2p}\right)^2 dy = \frac{1}{3}\, lhHI\,.$$

In order to calculate the kinetical energy, each part of the string is assumed to perform a series of damped oscillations, which may be represented by the expression:

$$S = A\, \varepsilon^{-\alpha t} \cos \omega t\,,$$

in which S be the place of the part at any moment with regard to the line BD, A the maximal amplitude, α the damping-constant, ω the number of oscillations in 2π seconds.

If we consider a particle of the string dy situated in Q, we have:

$$A = QR = h - \frac{y^2}{2p}.$$

The velocity at any moment is:

$$v = \frac{ds}{dt} = -\left(h - \frac{y^2}{2p}\right) \{\omega \sin \omega t + \alpha \cos \omega t\}\, E^{-\alpha t}$$

which expression at the moment when the string passes through its final position of rest, becomes:

$$v_{max.} = -\left(h - \frac{y^2}{2p}\right) \omega\, \varepsilon^{-\frac{\pi\alpha}{2\omega}}.$$

Hence in the formula for the kinetical energy

$$E_{(kin)} = \frac{1}{2} mv^2$$

the velocity v is known. The mass m of a part dy of the string is expressed by :

$$m = \pi r^2 g \, dy$$

in which r is half the diameter, g the specific weight of the string. We obtain finally for the kinetical energy of the part dy

$$E_{(kin, \cdot y)} = \frac{1}{2}\left(h - \frac{y^2}{2p}\right)^2 \omega^2 \pi r^2 g \varepsilon^{-\frac{a\pi}{2\omega}} dy.$$

Integrating over half the length of the string, and by multiplying by 2, the total kinetical energy of the string gives at its passage through the position of equilibrium

$$E_{(kin)} = 2\int_0^{\frac{1}{2}l} \frac{1}{2}\left(h - \frac{y^2}{2p}\right)^2 \omega^2 \pi r^2 g \varepsilon^{-\frac{\pi a}{2\omega}} dy = \frac{4}{15} \pi r^2 g l \omega^2 h^2 \varepsilon^{-\frac{\pi a}{2\omega}}$$

This value should have been equal to the value obtained for the potential energy, if the movement had been undamped. The damping makes the amount of the kinetical energy too small. The exact amount is obtained by multiplication by $\varepsilon^{\frac{\pi a}{2\omega}}$.

As the expression of the potential energy is expressed in ergs, this must likewise be done with the kinetical energy, which causes the introduction of the factor 1.0197.

We obtain finally after the introduction of $N = \frac{\omega}{2\pi}$:

$$\frac{1}{3} H\Pi h = 1.0197 \times \frac{4}{15} \pi r^2 g l \omega^2 h^2$$

and hence:

$$H = 32\ 2 \frac{N^2 h \pi r^2 g}{I}$$

2nd method.

We can calculate the lateral pressure on the string in its deviated position in different ways. Above we had already mentioned the expression $H\Pi$ for it.

Proceeding from the graphostatical construction, discussed before, we see that the slope of the string at the point of suspension B corresponds with the inclination of the line MC (vide fig. 4). This slope is given by the tangent at the point, i.e. by the magnitude of

$\frac{dy}{dx}$ and on the other hand by the tangent of the angle CMO, consequently by $\frac{CO}{OM}$.

As the tangent at any point of a parabola is expressed by

$$\frac{dy}{dx} = \frac{y}{2x}$$

we get for the slope at the end of the string, where $y = \frac{1}{2}\, l$ and $x = h$:

$$\frac{dy}{dx}\,(x = h) = \frac{l}{4h}.$$

As CO represents half the total lateral strain, consequently $\frac{1}{2} Z$ and OM the total longitudinal strain P we may write:

$$\frac{l}{4h} = \frac{2P}{Z} \text{ or } Z = \frac{8h}{l} P.$$

The longitudinal strain of a string can be found from the formula for the vibration-frequency of a stretched string, given in KOHLRAUSCH'S Handbuch der prakt. Phys. (11th edition p. 245):

$$N = \frac{1}{2l} \sqrt{\frac{9.81\, P}{p}}$$

in which N represents the frequency per second, l the length in meters and p the weight of 1 m. of wire. We obtain from this for the tension

$$P = \frac{4N^2 l^2 p}{9.81}.$$

By substituting this value of P in the equation for the lateral pressure, we find:

$$Z = \frac{8h}{l} \frac{4N^2 l^2 p}{9\,81}$$

which value may be equalled to the lateral pressure calculated before:

$$HIl = \frac{8h}{l} \frac{4N^2 l^2 p}{9.81}.$$

As HIl indicates the pressure in dynes, p, the weight of 1 m. of string, must likewise be expressed in dynes, thus:

$$p = 9.81\, \pi r^2 g.$$

We obtain finally after this substitution:

$$H = \frac{32\, N^2 h \pi r^2 g}{I}.$$