Citation:

Zwiers, H.J., Researches on the orbit of the periodic comet Holmes and on the perturbations of its elliptic motion. V, in:
KNAW, Proceedings, 15 I, 1912, 1912, pp. 192-200

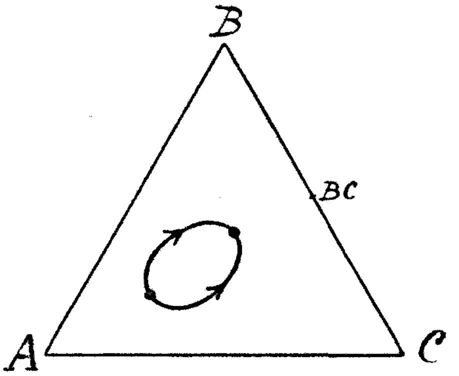

Fig. 9.

tract more rapidly than the liquid-vapour surface, for this surface of nodes entirely disappears at the triple point temperature of *BC*. So if we draw the ternary critical end-point line in this case, we get a closed curve, as is drawn in fig. 9 with a temperature minimum and maximum.

If critical end-points occur also in one of the binary systems *AB* or *AC* or in both, other cases may occur, but they are easy to derive from what precedes. If also ternary compounds are included in our considerations, the cases get somewhat more complicated, as I hope to show on a following occasion.

Anorg. chemical laboratory
Amsterdam, June 27 1912. *of the University.*

Astronomy. — "*Researches on the orbit of the periodic comet Holmes and on the perturbations of its elliptic motion*". V. By Dr. H. J. Zwiers. (Communicated by Prof. E. F. van de Sande Bakhuyzen).

In January 1906 I communicated to this Academy the most probable elements I had derived for the return of the comet Holmes in 1906—07. In a later paper, November 1906, I discussed the then known three photographic observations of the comet by Prof. Max Wolf at Heidelberg, and from these derived corrections to the mean longitude, to the inclination and to the longitude of the ascending node of the orbit. The elements obtained were:

Epoch 1906 January 16.0 M. T. Greenw.

$$\begin{array}{rl} M_0 = & 351^\circ 47' 36''.838 \\ \mu = & 517''.447665 \\ \log a = & 0.5574268 \\ \varphi = & 24^\circ 20' 25''.55 \\ i = & 20\ 49\ \ 0\ .62 \\ \pi = & 346\ \ 2\ 31\ .63 \\ \Omega = & 331\ 44\ 37\ .85 \end{array} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} 1906.0$$

These elements left the following errors O—C in the three observed places:

1906 Aug.	28.55	$\Delta\alpha = +0^s.095$	$\Delta\delta = -0''.33$
Sept.	25.51	$= +\ .099$	$= +1\ .26$
Oct.	10.35	$= -\ .177$	$= -1\ .15$

So the obtained elements very satisfactorily represented the observations, and might therefore be adopted for the apparition in 1906, until by a rigorous calculation of the perturbations this apparition may be exactly combined with the previous ones.

On December 7, however, Prof. Wolf succeeded in taking another observation, this time with the great reflector of 28 inches aperture. It was, however, exceedingly difficult to obtain trustworthy measurements from this last plate. First of all the image of the comet was not sharply defined; "das Bild ist verwaschen, aber deutlich", Prof. Wolf wrote already on Dec. 8. A much greater difficulty arose owing to a peculiarity of the photographic star images, especially on plates taken with reflectors of this shape. The following quotation from a letter of Prof. Wolf of 1906 Dec. 27 may serve to characterize the phenomenon, indicated by him with the name "Verzeichnung".

"Die relative Verzeichnung, ein von mir eingeführtes Wort, ist "der grösste Feind und wichtigste Fehler der photogr. Positions- "bestimmungen. Sie besteht darin, das für jede Sternhelligkeit der zu "messende (Mittel-)punkt des entstehenden Sternscheibchens an anderer "Stelle des Scheibchens zu suchen ist. Also z. B. liegt beim Reflektor "der geometrische Mittelpunkt eines Sternes 9ter Grösse in der Nähe "des Gesichtsfeldrandes um mehrere Bogenminuten, soviel ich bis "jetzt schätzen kann, von dem Punkt entfernt, auf den man die "Position eines schwachen Objektes (Cometen) beziehen muss. Für "jede Sterngrösse ändert sich dies, ebenso für jeden Radius ab "optischem Centrum, also $R = f(r, m)$. Bei Brashear *b* ist die rel. "Verzeichnung erst in 6° radius merkbar. Bei Brashear *a* schon in "3°—4° *r*. Beim Reflector schon in 10′—20′ *r*."

On the 18th of Dec. Prof. Wolf wrote to me:

"Aus A. G. Cambr. 1572 und 1584 erhalte ich für den Kometen

$\alpha_{1906.0}$ $3^h38^m50^s.41$ $\delta_{1906.0}$ $+51^\circ16'52''.7$ 1906 Dez. 7 $7^h8^m.1$ MZ. Kgst.

Grösse 16.

> "Ob die relative Verzeichnung ganz richtig eliminiert ist, weiss "ich aber nicht. Ich bringe es auch vorerst nicht heraus. Mir "scheint deshalb, dass das Gewicht dieser Beobachtung etwas "geringer ist, als das der ersten Beobachtungen."

With the scarcity of the material this observation too demanded the necessary attention but after what has been said above I need hardly mention that I commenced the calculations for it with little hope of success.

For the reduction to apparent place I found:

in α: $+5^{s}.112$ in δ: $+9''.75$

and as correction for parallax:

in α: $-0^{s}.247$ in δ: $+0''.72$

The observed apparent place thus becomes:

$$1906 \text{ Dec. } 7.273046: \alpha = 3^{h}38^{m}55^{s}.275 \quad \delta = +51^{\circ}17'3''.17$$

Time of aberration: 0.011279 day.

This observation has further been treated in exactly the same way as the three preceding ones in my communication of Nov. 1906. As starting-elements I again adopted those given in my paper of January 1906, p. 677, after increasing M with 50″. I obtained as differences Obs.—Comp.:

$$1906 \text{ Dec. } 7.27: \Delta\alpha = +1^{s}.065 \quad \Delta\delta = +15''.53$$

For the derivation of the differential quotients of α and δ with respect to M, i and Ω the computed places were then derived 1. with $\Delta M = +40''$ (instead of $+50''$); 2. with $\Delta i = +10''$; 3. with $\Delta\Omega = +10''$. Thus this fourth place yielded the two following equations of condition:

From α: $+0.2288\,\Delta M - 0.0372\,\Delta i - 0.0114\,\Delta\Omega = +1^{s}.065$

From δ: $+0.426\,\Delta M + 1.374\,\Delta i + 0.033\,\Delta\Omega = +15''.53.$

The first equation was again multiplied by $15 \cos\delta$ and just as before $\frac{\Delta\Omega}{10}$ was introduced as unknown quantity instead of $\Delta\Omega$; moreover I gave half weight to both equations. Thus I obtained 2 new equations, in addition to the former six, given in my paper Researches IV (Nov. 1906):

from the R. A.:

$$0.18128\,\Delta M + 9.39236_{n}\,\Delta i + 9.87872_{n}\,\frac{\Delta\Omega}{10} = 0.84917$$

from the declination:

$$9.47889\,\Delta M + 9.98747\,\Delta i + 9.36799\,\frac{\Delta\Omega}{10} = 1.04067$$

in which all co-efficients are logarithmic.

From the total of 8 equations of condition there follow the normal equations:

$$\begin{array}{rrrrl}
+12.3229\,\Delta M & -0.47796\,\Delta i & -4.9039\,\dfrac{\Delta\text{☊}}{10} & = & -17.461 \\
-0.47796\ \text{,,} & +5.1423\ \text{,,} & -2.3300\ \text{,,} & = & +58.562 \\
-4.9039\ \text{,,} & -2.3300\ \text{,,} & +4.4680\ \text{,,} & = & -26.733.
\end{array}$$

These give the following values for the corrections of the elements:

$$\begin{aligned}
\Delta M &= -\ 2''.6793 \\
\Delta i &= +\ 9\ .29 \\
\Delta\text{☊} &= -40\ .78
\end{aligned}$$

By means of substitution in the equations of condition we find that these corrections leave the following errors O—C in the observations:

		$\Delta\alpha$	$\Delta\delta$
1906	Aug. 28.55	$\Delta\alpha = -0^s.190$	$\Delta\delta = -0''.11$
	Sept. 25.51	$-0\ .207$	$+0\ .72$
	Oct. 10.35	$-0\ .510$	$-2\ .26$
	Dec. 7.27	$+1\ .559$	$+5\ .25$

The now found corrections of the elements do not differ considerably from those determined before, but a comparison of the remaining errors shows that the introduction of the uncertain fourth place in the calculation cannot be said to have improved matters. Therefore I continue to regard the elements given at the beginning of this paper and agreeing absolutely with those from the "Proceedings" of Nov. 1906, as the most accurate for the present moment.

For the approaching return of the comet I have kept these elements unaltered since there was no time to calculate the perturbations. I have only reduced the elements i, π and ☊ to the ecliptic and the aequinox of 1912.0. So the employed elements are:

Epoch 1912 June 15.0 M. T. Greenw.

$$\begin{aligned}
M &= 328^\circ 25' 19''.269 \\
\mu &= 517''.447665 \\
T &= 1913 \text{ January } 20.695 \text{ M. T. Gr.} \\
\log a &= 0.557427 \\
\varphi &= 24^\circ 20' 25''.6 \\
\left.\begin{aligned}
i &= 20\ 49\ \ 3\ .3 \\
\pi &= 346\ \ 7\ 32\ .9 \\
\text{☊} &= 331\ 49\ 42\ .1
\end{aligned}\right\} & \ 1912.0
\end{aligned}$$

According to these elements circumstances are not quite so favourable this time. The perihelion passage occurs shortly before the conjunction with the sun so that the comet is then at a great distance from the earth and its place in the heavens is moreover not far from the sun. The circumstances are more favourable at the opposition in 1912, although the comet then remains invisible for our northern regions owing to its considerable southern declination. In order to calculate an ephemeris for that opposition I have first derived the following expressions for the heliocentric co ordinates:

$$\begin{aligned} x &= [9.99\,3799]\, \sin\, (v + 77^\circ 42' 18''.3) \\ y &= [9.87\,6101]\, \sin\, (v - 20\;52\;48\;.5) \\ z &= [9.83\,2770]\, \sin\, (v - \;\;1\;43\;55\;.6) \end{aligned}$$

The rectangular solar co-ordinates have been taken from the Nautical Almanac and reduced to the mean aequinox of the beginning of the year.

The resulting mean places of the comet were reduced to the aequinox of the date by means of the constants f, g, G of the Naut. Alm.

The following table gives the apparent places of the comet for Greenwich mean noon; column H gives the theoretical brightness according to $H = \frac{1}{r^2 \varrho^2}$. It may be remembered, that the value of H for the time of the photographs by Wolf in 1906 varied between 0.032 and 0.038.

Apparent places of the comet from 1912 June 15 to 1913 Jan. 1 for 0^h mean time at Greenwich.

1912	α	δ	log ρ	H
	h m s	° ′ ″		
June 15	19 23 29.74	— 49 16 38.2	0.24 0106	0.0473
17	21 41.65	20 43.5	0.23 6066	
19	19 46.28	23 59.7	2204	.0497
21	17 44.08	26 21.6	0.22 8530	
23	15 35.55	27 43.9	5051	.0519
25	13 21.29	28 3.2	1779	
27	11 1.95	27 16.6	0.21 8719	.0541
29	8 38.22	25 21.3	5878	
July 1	6 10.91	22 14.7	3263	.0561
3	3 40.82	17 54.9	0878	
5	1 8.82	12 19.3	0.20 8732	.0580
7	18 58 35.69	5 26.4	6828	
9	56 2.35	— 48 57 15.7	5172	.0596
11	53 29.71	47 47.7	3764	
13	50 58.71	37 2.5	2608	.0611
15	48 30.30	25 0.2	1706	
17	46 5.39	11 42.4	1058	.0622
19	43 44.82	— 47 57 11.1	0660	
21	41 29.38	41 30.6	0510	.0631
23	39 19.87	24 42.5	0603	
25	37 16.84	6 50.5	0935	.0637
27	35 20.86	— 46 47 58.3	1504	
29	33 32.44	28 9.8	2302	.0640
31	31 52.05	7 29.1	3322	
Aug. 2	30 20.08	— 45 46 0.3	4558	.0641
4	28 56.87	23 47.6	6004	
6	27 42.67	0 55.1	7651	.0639
8	26 37.69	— 44 37 26.8	9492	
10	25 42.13	13 26.9	0.21 1522	.0635
12	24 56.14	— 43 48 59.5	3728	
14	24 19.80	24 8.9	6103	.0629
16	23 53.11	— 42 58 58.7	8634	
18	23 36.05	33 32.6	0.22 1309	.0620
20	23 28.54	7 53.4	4124	
22	23 30.42	— 41 42 4.6	7065	.0611
24	23 41.53	16 8.9	0.23 0124	
26	24 1.68	— 40 50 8.9	3291	.0600
28	24 30.72	24 6.4	6559	
30	25 8.35	— 39 58 3.5	9918	.0588
Sept. 1	25 54.37	32 1.8	0.24 3360	
3	26 48.58	6 2.7	6879	.0576
5	27 50.86	— 38 40 7.0	0.25 0470	
7	29 0.94	14 16.3	4121	.0563
9	30 18.57	— 37 48 31.3	7825	
11	31 43.54	22 52.8	0.26 1575	.0549
13	33 15.68	— 36 57 21.1	5370	
15	34 54.72	31 56.2	9197	.0536
17	36 40.39	6 38.2	0.27 3049	
19	38 32.44	— 35 41 27.3	6921	.0522
21	40 30.62	16 23.7	0.28 0810	
23	42 34.68	— 34 51 27.2	4709	.0509
25	44 44.37	26 37.3	8613	
27	46 59.46	1 53.7	0.29 2518	.0496
29	49 19.72	— 33 37 15.3	6423	

1912/13	α	δ	log ρ	H
	h m s	° ′ ″		
Oct. 1	18 51 44.96	— 33 12 41.9	0.30 0324	0.0483
3	54 14.99	— 32 48 12.8	4217	
5	56 49.62	23 47.3	8098	.0470
7	59 28.72	— 31 59 24.6	0.31 1966	
9	19 2 12.10	35 4.0	5817	.0458
11	4 59.57	10 45.0	9648	
13	7 50.95	— 30 46 26.7	0.32 3455	.0446
15	10 46.07	22 8.6	7235	
17	13 44.73	— 29 57 49.5	0.33 0986	.0434
19	16 46.75	33 28.8	4707	
21	19 51.95	9 5.6	8397	.0423
23	23 0.18	— 28 44 40.0	0.34 2052	
25	26 11.28	20 10.3	5672	.0412
27	29 25.12	— 27 55 36.0	9257	
29	32 41.57	30 56.3	0.35 2807	.0402
31	36 0.47	6 10.7	6320	
Nov. 2	39 21.75	— 26 41 18.4	9795	.0392
4	42 45.30	16 18.9	0.36 3231	
6	46 11.01	— 25 51 11.4	6626	.0383
8	49 38.74	25 55.3	9981	
10	53 8.41	0 30.1	0.37 3294	.0374
12	56 39.92	— 24 34 55.3	6564	
14	20 0 13.14	9 10.4	9789	.0365
16	3 47.93	— 23 43 14.4	0.38 2970	
18	7 24.20	17 7.6	6107	.0357
20	11 1.85	— 22 50 49.6	9198	
22	14 40.78	24 20.0	0.39 2245	.0349
24	18 20.90	— 21 57 38.1	5248	
26	22 2.13	30 43.8	8206	.0342
28	25 44.40	3 36.9	0.40 1120	
30	29 27.67	— 20 36 17.1	3990	.0334
Dec. 2	33 11.89	8 44.1	6817	
4	36 56.98	— 19 40 57.7	9600	.0327
6	40 42.88	12 57.6	0.41 2337	
8	44 29.53	— 18 44 43.6	5029	.0321
10	48 16.89	16 15.6	7677	
12	52 4.87	— 17 47 33.6	0.42 0278	.0314
14	55 53.40	18 37.7	2833	
16	59 42.42	— 16 49 27.9	5342	.0308
18	21 3 31.90	20 4.2	7808	
20	7 21.77	— 15 50 26.8	0.43 0229	.0302
22	11 11.94	20 35.7	2605	
24	15 2.38	— 14 50 30.9	4937	.0297
26	18 53.14	20 12.6	7227	
28	22 44.16	— 13 49 41.0	9474	.0291
30	26 35.41	18 56.1	0.44 1679	
Jan. 1	30 26.86	— 12 47 58.2	3842	.0286

The following table gives the variations of α and δ in two suppositions: 1st that the comet reaches its perihelion 4 days earlier, 2nd that it reaches it 4 days later.

Variations of α and δ for altered times of perihelion passage.

1912/13	$\Delta T = -4d$		$\Delta T = +4d$	
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
	m s	′ ″	m s	′ ″
June 15	+ 8 43.27	+ 33 58.7	— 8 46.83	— 31 17.0
19	+ 8 56.85	+ 33 20.7	— 8 59.63	— 30 29.0
23	+ 9 8.79	+ 32 27.1	— 9 10.67	— 29 28.8
27	+ 9 18.52	+ 31 21.2	— 9 19.43	— 28 15.7
July 1	+ 9 26.01	+ 30 5.4	— 9 25.48	— 26 55.1
5	+ 9 30.67	+ 28 44.1	— 9 28.94	— 25 28.0
9	+ 9 32.35	+ 27 15.7	— 9 29.29	— 24 1.4
13	+ 9 31.05	+ 25 48.7	— 9 26.57	— 22 34.2
17	+ 9 26.66	+ 24 25.8	— 9 21.08	— 21 13.8
21	+ 9 19.67	+ 23 9.2	— 9 12.89	— 20 1.7
25	+ 9 10.05	+ 22 3.5	— 9 2.62	— 19 0.5
29	+ 8 58.51	+ 21 10.2	— 8 50.35	— 18 13.6
Aug. 2	+ 8 45.41	+ 20 30.5	— 8 36.79	— 17 40.0
6	+ 8 31.09	+ 20 5.8	— 8 22.31	— 17 21.8
10	+ 8 16.05	+ 19 54.1	— 8 7.19	— 17 17.0
14	+ 8 0.57	+ 19 55.7	— 7 51.87	— 17 25.5
18	+ 7 45.05	+ 20 9.8	— 7 36.55	— 17 45.1
22	+ 7 29.69	+ 20 34.6	— 7 21.55	— 18 15.8
26	+ 7 14.73	+ 21 8.7	— 7 6.95	— 18 54.5
30	+ 7 0.13	+ 21 50.0	— 6 52.95	— 19 40.4
Sept. 3	+ 6 46.33	+ 22 37.9	— 6 39.42	— 20 31.9
7	+ 6 32.95	+ 23 29.3	— 6 26.75	— 21 28.1
11	+ 6 20.36	+ 24 24.7	— 6 14.53	— 22 27.1
15	+ 6 8.45	+ 25 23.3	— 6 3.09	— 23 27.8
19	+ 5 57.10	+ 26 21.3	— 5 52.24	— 24 30.5
23	+ 5 46.43	+ 27 20.7	— 5 42.05	— 25 33.3
27	+ 5 36.33	+ 28 21.2	— 5 32.41	— 26 35.1
Oct. 1	+ 5 26.80	+ 29 20.5	— 5 23.28	— 27 36.7
5	+ 5 17.87	+ 30 18.3	— 5 14.67	— 28 37.4
9	+ 5 9.37	+ 31 14.5	— 5 6.56	— 29 37.7
13	+ 5 1.33	+ 32 9.7	— 4 58.86	— 30 35.4
17	+ 4 53.74	+ 33 3.7	— 4 51.57	— 31 31.8
21	+ 4 46.53	+ 33 54.6	— 4 44.61	— 32 25.4
25	+ 4 39.73	+ 34 45.2	— 4 38.02	— 33 18.5
29	+ 4 33.21	+ 35 32.7	— 4 31.80	— 34 9.4
Nov. 2	+ 4 27.07	+ 36 17.9	— 4 25.81	— 34 58.1
6	+ 4 21.13	+ 37 1.6	— 4 20.18	— 35 44.3
10	+ 4 15.58	+ 37 42.5	— 4 14.69	— 36 28.7
14	+ 4 10.28	+ 38 24.1	— 4 9.55	— 37 10.5
18	+ 4 5.19	+ 38 58.8	— 4 4.61	— 37 50.8
22	+ 4 0.37	+ 39 33.7	— 3 59.89	— 38 28.0
26	+ 3 55.77	+ 40 5.8	— 3 55.44	— 39 3.8
30	+ 3 51.43	+ 40 36.0	— 3 51.11	— 39 37.0
Dec. 4	+ 3 47.25	+ 41 3.8	— 3 47.07	— 40 8.1
8	+ 3 43.33	+ 41 29.8	— 3 43.15	— 40 36.5
12	+ 3 39.59	+ 41 52.9	— 3 39.51	— 41 3.4
16	+ 3 36.07	+ 42 14.4	— 3 35.99	— 41 27.4
20	+ 3 32.65	+ 42 33.4	— 3 32.75	— 41 49.5
24	+ 3 29.60	+ 42 50.2	— 3 29.54	— 42 9.4
28	+ 3 26.63	+ 43 4.5	— 3 26.64	— 42 27.0
Jan. 1	+ 3 23.84	+ 43 16.8	— 3 23.88	— 42 41.8

Leyden, June 1912.

Mathematics. — *"The scale of regularity of polytopes"*. By Dr. E. L. ELTE (Meppel). (Communicated by Prof. P. H. SCHOUTE).

In my dissertation[1]) it was my aim to determine the semiregular polytopes, i. e. the polytopes analogous to the semiregular polyhedra. So this investigation had to be based on a definition of the notion "semiregular polytope". Now ordinarily a semiregular polyhedron is defined as follows: "A semiregular polyhedron has *either* congruent (or symmetric) vertices and regular faces *or* congruent faces and regular vertices. So there are two kinds of semiregular polyhedra which we will call with CATALAN[2]) "semiregular of the first kind" and "semiregular of the second kind"; those of the first kind are enumerated in the following table. For any of these polyhedra this table gives the numbers of vertices, edges, faces and indicates which faces pass through each vertex and which couples of faces pass through each kind of edges. Here p_n denotes a regular polygon with n vertices.

Notation	N°.	Vertices	Edges	Faces	Faces through a vertex	Faces through the edges		
tT	1	12	18	8	$1p_3$, $2p_6$	p_6, p_6	p_6, p_3	
tC	2	24	36	14	$1p_3$, $2p_8$	p_8, p_8	p_8, p_3	
tO	3	24	36	14	$1p_4$, $2p_6$	p_6, p_6	p_6, p_4	
tD	4	60	90	32	$1p_3$, $2p_{10}$	p_{10}, p_{10}	p_{10}, p_3	
tI	5	60	90	32	$1p_5$, $2p_6$	p_6, p_6	p_6, p_5	
CO	6	12	24	14	$2p_3$, $2p_4$	p_4, p_3		
ID	7	30	60	32	$2p_3$, $2p_5$	p_5, p_3		
RCO	8	24	48	26	$1p_3$, $3p_4$	p_4, p_4	p_4, p_3	
RID	9	60	120	62	$1p_3$, $2p_4$, $1p_5$	p_4, p_5	p_4, p_3	
tCO	10	48	72	26	$1p_4$, $1p_6$, $1p_8$	p_6, p_8	p_4, p_8	p_4, p_6
tID	11	120	180	62	$1p_4$, $1p_6$, $1p_{10}$	p_{10}, p_6	p_{10}, p_4	p_4, p_6
CS	12	24	60	38	$1p_4$, $4p_3$	p_1, p_4	p_3, p_3	
DS	13	60	150	92	$1p_5$, $4p_3$	p_5, p_3	p_3, p_3	
P_n	14	$2n$	$3n$	$n+2$	$1p_n$, $2p_4$	p_n, p_4	p_4, p_4	
AP_n	15	$2n$	$4n$	$2n+2$	$1p_n$, $3p_3$	p_n, p_3	p_3, p_3	

---

[1]) "The semiregular polytopes of the hyperspaces", Groningen, 1912.

[2]) "Mémoire sur la théorie des polyèdres", *Journal de l'École Polytechnique*, Cahier 47.