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Physics. — "*On the* HALL *effect and the change in the resistance in a magnetic field at low temperatures.* I. *Measurements on the* HALL-*effect and the change in the resistance of metals and alloys in a magnetic field at the boiling point of hydrogen and at lower temperatures*". By H. KAMERLINGH ONNES and BENGT BECKMAN. Communication N°. 129[a] from the Physical Laboratory at Leiden.

(Communicated in the meeting of June 29, 1912).

§ 1. *Introduction.* An investigation of the HALL effect and of the change of resistance produced by a magnetic field was carried out by VAN EVERDINGEN at Leiden some time ago down to liquid air temperatures [1]), but the fundamental importance of these phenomena in the theory of electrical conduction has long made it desirable to extend this investigation to the much lower temperatures which have been freely available since the successful development of methods of obtaining accurate series of observations at liquid hydrogen temperatures. The problem, however, has been forced aside by other researches which could not be delayed, until the study of it and of allied problems for various metals at the lowest possible temperatures has been rendered essential to the further development of the theory of electrons by the discovery of the fact that the resistance of pure mercury disappears at liquid helium temperatures. We have therefore been occupied for some time with various aspects of the investigation of these problems at hydrogen temperatures, and, while we propose to continue this investigation systematically and, if possible, to make some measurements on the more important points at those temperatures which are obtainable with liquid helium, we give in the present paper some results which have already been obtained, and which may be considered to be themselves of some importance.

The investigation has been extended by one of us (B. BECKMAN) with the same experimental material to temperatures obtainable with liquid ethylene, liquid oxygen, and liquid nitrogen, and these results will be discussed in a later paper.

We wish to record our heartiest thanks to Mrs. A. BECKMAN for her assistance in the course of the measurements.

[1]) The results for bismuth (and antimony) given in the dissertations of LEBRET (Leiden 1895) and VAN EVERDINGEN (Leiden 1897) and in Communications Nos. 19, 26, 37, 40, 53, 58, 61 have been confirmed by BLAKE, Ann. d. Physik. **28**, 449, 1909 and LOWNDS, Ann. d. Physik **9**, 677, 1902. LOWNDS investigated rods cut in different directions from bismuth crystals, and extended his investigation for one direction down to liquid air temperatures. He found that with the crystalline axis perpendicular to the field the HALL coefficient is negative at higher temperatures, while as the temperature is lowered it vanishes and then becomes positive.

I. *Bismuth.*

§ 2. *Change in the resistance of a wire of electrolytic bismuth.*

This part of the investigation was made with a wire of electrolytic bismuth provided by HARTMANN and BRAUN 0,3 mm. thick, and identically the same as that used by KAMERLINGH ONNES and CLAY in their determination of the change of resistance (Comm. N°. 99). The KOHLRAUSCH method of overlapping shunts was used. At ordinary temperature and at the boiling point of hydrogen the main current was 4 milliamps, but at — 259° C. it had to be reduced to 0.1 à 0.2 milliamps on account of the effect of heating upon the resistance. In the following Table w' represents the value of the resistance in ohms in a magnetic field of strength H, w_T is the resistance with no field on, and w_0 is the resistance at 0° C. with no field.

We may notice that we have not obtained the maximum in the isopedals observed by BLAKE. It will be seen from the forthcoming paper on the change of resistance with magnetic field at liquid air temperatures that BLAKE's bismuth wires which showed the maximum exhibited a smaller change in the resistance than ours and were therefore probably not so pure. It is possible that as the purity increases the maximum in the isopedals is displaced towards the lower temperatures.

TABLE I.
Resistance of $B_{id\,I}$ as a function of the temperature and of the field strength.

H Gauss	$T = 290°$		$T = 20°,3$		$T = 15°$	
	w'	$\frac{w'}{w_0}$	w'	$\frac{w'}{w_0}$	w'	$\frac{w}{w_0}$
0	2.570	1.057	0.588	0.242	0.526	0.216
2760	2.770	1.140	11.5	4.73		
3850	—	—	—	—	19.9	8.185
5540	3.110	1.280	32.8	13.50	34.9	14.35
7370	3.473	1.388	54.7	22.50	55.9	23.00
9200	3.635	1.495	76.7	31.55	80.8	33.25
11850	4.002	1.646	113.2	46.55	116.4	47.90
13600	4.248	1.746	141.5	58.20	143.1	58.85
15670	4.540	1.868	172	70.75	175.6	72.25
17080			196.5	80.85	199.3	82.00

The general character of the isotherms is also conserved at hydrogen temperatures; the field at which the resistance begins to increase practically proportionally to the field itself is about 12000 gauss just as at liquid air temperatures. The gradual transformation from the change at small fields to the practically linear change in strong fields takes place in the same way at each temperature.

§ 3. *The* HALL-*effect and the increase of resistance for plates of compressed electrolytic bismuth. Experimental method.*

The method adopted was that developed and applied by LEBRET and VAN EVERDINGEN in their dissertations (see Suppl. N°. 2); in it all disturbing influences are eliminated. A diagram is given in Plate 3 of the Supplement quoted, and for all matters concerning the arrangements for measuring we may refer to Chapter I of that paper. Circular plates were used to which were soldered with WOOD's alloy the primary and HALL electrodes as well as two auxiliary electrodes (placed on the diameter in the direction of the main current). All were point electrodes [1]).

Choosing our notation to correspond with that of the Supplement quoted let us write e for the potential difference between the HALL electrodes, I for the main current and d for the thickness of the plate. The HALL constant R is given by

$$R = \frac{ed}{HI}.$$

Let us also write R_s for the resistance of the secondary circuit outside the plate, r for the resistance of the shunt of the compensating circuit, q for a constant determined by the differential galvanometer employed, and R_d for the resistance determined by

$$\frac{1}{R_d} = \frac{1}{2}\left(\frac{1}{R_A} + \frac{1}{R_B}\right)$$

in which R_A and R_B are magnitudes obtained from the resistances of the compensating circuit with reversal of the main current when the field commutator stands in each experiment in the positions A and B respectively; we then obtain

$$R = rdq\,\frac{R_s}{HR_d}.$$

The change in the resistance was also measured as well as the HALL effect.

At ordinary temperature the bismuth plates showed no asymmetry

[1]) VAN EVERDINGEN has solved the problem theoretically for point electrodes with circular plates.

in the HALL-effect, but they showed it very clearly and sometimes very strongly at hydrogen temperatures, giving considerable differences between R_A and R_B. In the following tables twice the asymmetry is given by the side of the mean HALL-constant; for the method of evaluating the asymmetry we may again refer to Chapter I of Suppl. N°. 2. All quantities except w are expressed in C.G.S.

The current in the main circuit was $I = 0.15$ amp. A WIEDEMANN galvanometer was used. The bath of liquid gas in the magnetic field was obtained in a silvered vacuum vessel by the method of Comm. N°. 114.

§ 4. *Results of the measurements.*

Bi_{pI}, Bi_{pII}, Bi_{pIII}, represent three plates of 10 mm. diameter prepared from the same HARTMANN and BRAUN electrolytic bismuth. Bi_{pI} was compressed from a thin rod in a steel mould. Bi_{pII} and Bi_{pIII} were prepared by first grinding the bismuth to a fine powder in an agate mortar and then compressing in the same mould as Bi_{pI}. In the preparation of Bi_{pIII}, which was otherwise the same as that of Bi_{pI} and Bi_{pII}, the grinding operation took place in an atmosphere of carbon dioxide.

TABLE II

The HALL constant, asymmetry and resistance change for Bi_{pI}.

H	$T = 289°$				$T = 20°,3$			
	RH	2 × Asym.	$-R$	$\left[\frac{w'}{w}\right]_T$	RH	2 × Asym.	$-R$	$\left[\frac{w'}{w}\right]_T$
2060	13.9×10^3	0.4×10^3	6.75	1.06	91.4×10^3	39.6×10^3	44.35	10.1
3450	20.9	0.2	6.06	1.12	166.5	48	48.25	21.7
5660	29.1	1.1	5.14	1.21	308	54	54.40	39.3
7160	33.2	0	4.64	1.29	385.5	114.5	53.90	52.0
9880	40.3	1.8	4.08	1.45	563	199	57.00	78.2
11090	42.6	2.3	3.84	1.50	640	243	57.70	89.5
0	$w_T = 0.00209\ \Omega$				$w_T = 0.00044\ \Omega$ $\frac{w_{20K}}{w_{289K}} = 0.22$			

With no field the ratio of the resistance of Bi_{pI} at hydrogen temperature to that at ordinary temperature is almost the same as the same ratio for the bismuth wire Bi_{dI}; but in a magnetic field

TABLE III. HALL-constant, asymmetry and change of resistance for $Bi_{p\,II}$

H	$T = 289°$				$T = 20°.3$				$T = 14°.6$			
	RH	2 × Asym.	$-R$	$\left[\frac{w'}{w}\right]_T$	RH	2×Asym.	$-R$	$\left[\frac{w'}{w}\right]_T$	RH	2×Asym.	$-R$	$\left[\frac{w'}{w}\right]_T$
2060	18.7×10^3	0.2×10^3	9.08	1.023	152.5×10^3	24×10^3	74.1	1.71	167.5×10^3	3×10^3	81.3	1.788
3450	28.2	0.2	8.17	1.057	230	29	66.7	2.31	262	5.5	75.9	2.42
5660	39.85	0.4	7.05	1.108	349.7	37.5	61.8	3.28	—	—	—	—
7160	46.1	0.3	6.44	1.148	431.2	50	60.3	3.93	509	14	71.1	4.14
8520	52.15	0.8	6.12	1.186	503	76	59.0	4.58	—	—	—	—
9880	55.9	0.2	5.66	1.222	583	79	59.0	5.21	687	20	69.5	5.47
11090	59.95	0.8	5.41	1.26	647.5	91	58.4	5.76	755	11	68.1	6.07
12090	62.8	1.4	5.19	1.287	700	100	57.9	6.25	818	20	67.7	6.51
0	$w_{289} = 0.00389\ \Omega$				$w_{20} = 0.00487\ \Omega$							

the ratio of the resistance at hydrogen temperature to the zero resistance is less for the disc than for the wire Bi_{dI}, so

$$\text{for } H = 11090 \text{ at } T = 20.3 \qquad \overset{\text{disc}}{\frac{w'}{w_0} = 19.7} \qquad \overset{\text{wire}}{\frac{w'}{w_0} = 42.}$$

In the case of Bi_{pII} both the negative temperature coefficient and the smallness of the change of resistance with magnetic field indicate the presence of impurities.

TABLE IV.
The HALL constant, asymmetry and resistance change for Bi_{pIII}.

H	$T = 20°$.			
	RH	$-R$	Asym.	$\left[\frac{w'}{w}\right]_T$
2850	247×10^3	86.6	137×10^3	2.16
4700	426	90.7	210	2.91
6675	624	93.5	280	3.67
8275	814	98.5	346	4.44
10160	1007	99.2	400	5.12
11100	1105	99.4	425	5.49
12220	1216	99.5	460	5.87

With the disc Bi_{pIII} measurements were made only at hydrogen temperatures, but we give the results here as, just as with Bi_{pI}, R increases with H, and approaches a limiting value, approximately **100**; this is the highest HALL coefficient yet obtained for bismuth.

All the coefficients we have obtained for bismuth plates are *negative*. Circumstances which give rise to positive [1]) coefficients occur only in certain positions of the crystalline axis and therefore, since all positions of the axis occur at random, they are obscured by those which give rise to negative coefficients.

II. Other Metals.

§ 5. *Experimental method.* This was just the same as for bismuth. A THOMSON differential galvanometer was used for observing the HALL effect. Now the contacts were not soldered with WOOD's alloy, but with tin.

[1]) Here total coefficients are considered, cf. Comm. N°. 129c. [Note added in the translation].

§ 6. HALL *effect for Gold*. The plate Au_{pI} was prepared from a dutch 10 fl.-coin; this was dissolved in aqua regia, precipitated by SO_2, melted in a porcelain crucible and rolled between steel rollers. During the last operation and afterwards it was treated with various acids. From the decrease with temperature of the resistance with no magnetic field (see Table V) it is seen that this plate was made of purer gold than that which composed the wire Au_0 of Comm. N°. 99, which gave $w_{T=20}/w_{T=273} = 0{,}045$ and was known to contain 0,03 % impurity.

d was 0,101 mm., I approximately 1.2 amp., and $R_s = 0{,}6$ to 0,7 ohms.

We found:

TABLE V.
The HALL effect for Gold Au_{pI}.

H	$T = 290°$		$T = 20.3°$		$T = 14°.5$	
	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$
7730	5.62	7.27	7.57	9 79	7.56	9.78
9500	6.75	7.11	9.32	9.81	9.22	9.71
11080	8.11	7.32	10.91	9.84	11.03	9.96
12220	8.85	7.25	11.98	9.81	12.00	9.82
$w_{T=290} = 202.10^{-6}\ \Omega$			$w_{T=20,3} = 6{,}7.10^{-6}\ \Omega$			

§ 7. HALL *effect for Silver*. The plate Ag_{pI} was prepared from

TABLE VI.
The HALL effect for Silver Ag_{pI}.

H	$T = 290°$ K.		$T = 20°.3$ K.		$T = 14°.5$ K.	
	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$
4940	3.97	8.04	—	—	—	—
726	5.81	8.01	7.39	10.18	7.22	9.95
9065	7.23	7.98	9.22	10.17	8.98	9.91
10270	8.16	7.95	10.34	10.07	10.12	9.85
w	$173.10^{-6}\ \Omega$		$1.47 \times 10^{-5}\ \Omega$		$0.925 \times 10^{-6}\ \Omega$	
w/w_0	1.065		0.00905		0.0057	

silver for which we are indebted to the Master of the Royal Mint, Dr. C. HOITSEMA. The silver was found to be practically the same as that of the wire Ag_I of Comm. N°. 99, which had 0,18 °/₀ impurity, and for which $w_{T=20}/w_{T=273} = 0{,}0089$ (cf. w_T in Table). The thickness of the plate, $d = 0{,}096$ mm.

§ 8. HALL *effect for electrolytic Copper.* The electrolytic copper was supplied by FELTEN and GUILLAUME; d was in this case 0,057 mm. We found:

TABLE VII.
The HALL effect for Copper Cu_{pI}.

H	$T = 290°$ K.		$T = 20°$ K.		$T = 14°.5$ K.	
	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$	RH	$R \cdot 10^4$
7260	3 59	4.95	4.79	6.60	4.79	6.60
9065	4.42	4.87	6.03	6.65	5.94	6.55
10270	5.08	4.95	6.78	6.60	6.71	6.54
w	$312.10^{-6}\ \Omega$		$2.94.10^{-6}\ \Omega$		$2.83.10^{-6}\ \Omega$	
w/w_0	1.065		0.0103		0.00907	

§ 9. HALL *effect for Palladium.* The plate Pd_{pI} was supplied by HERAEUS; $d = 0{,}100$ mm. We found:

TABLE VIII.
The HALL effect for Palladium Pd_{pI}.

H	$T = 290$		$T = 20°.3$		$T = 14°.5$	
	RH	R	RH	R	RH	R
8250	5 61	6.80×10^{-4}	11.42	13.83×10^{-4}	11.54	13.98×10^{-4}
9065	6.04	6.66	—	—	—	—
9360	—	—	12.71	13.58	12.96	13.84
9760	6.64	6.80	—	—	—	—
10270	—	—	14.0	13.63	14.09	13.74
w	$126.2 \times 10^{-5}\ \Omega$		$6.11 \times 10^{-5}\ \Omega$		$5.77 \times 10^{-5}\ \Omega$	
$\frac{w}{w_0}$	1.065		0.0515		0.0485	

The plate was annealed and was kept from contact with the liquid hydrogen in the bath by a coat of celluloid dissolved in amyl acetate. By immersing the same plate unprotected in the bath, so that it absorbed a large quantity of hydrogen it was found that the occlusion of hydrogen constantly diminished the HALL coefficient, as is evident from a comparison of the following data with those of Table VIII. It was observed that the change of resistance with temperature diminished at the same time. We found:

	at $T = 20^\circ$ K.	$R = 12{,}0.10^{-4}$	$w = 5{,}3.10^{-5}$ Ohm.
then	„ $T = 290^\circ$	$R = 6{,}3.10^{-4}$	$w = 107.\ 10^{-5}$
and again	„ $T = 14^\circ.5$	$R = 9{,}4.10^{-4}$	
	„ $T = 20^\circ.3$	$R = 10{,}4.10^{-4}$	$w = 6{,}9.10^{-5}$
finally	„ $T = 290^\circ$		$w = 109.\ 10^{-5}$

§ 10. *Summary of results dealing with the change in the* HALL *coefficient for various metals.* In the two subsequent Tables we give figures for the change in the HALL coefficient when the temperature sinks to hydrogen temperatures and in the region of liquid hydrogen temperatures; R is the mean value taken from the previous tables at each definite temperature for each substance.

TABLE IX.
The HALL coefficient R at hydrogen temperatures.

T	Au_{pl}	Ag_{pl}	Cu_{pl}	Pd_{pl}
290°	7.24×10^{-4}	8.00×10^{-4}	4.92×10^{-4}	6.75×10^{-4}
20°.3	9.81	10.14	6.62	13.68
14.5	9.82	9.91	6.56	13.85

TABLE X.
Change of the HALL coefficient $\frac{R_T}{R_{290^\circ K}}$ on cooling to and in the region of liquid hydrogen temperatures.

T	Au_{pl}	Ag_{pl}	Cu_{pl}	Pd_{pl}
290°	1	1	1	1
20°.3	1.355	1.265	1.345	2.03
14.5	1.355	1.24	1.335	2.05

The change of the HALL coefficient on cooling to the temperature of liquid air $\frac{R_{T=83}}{R_{T=293}}$ has been found by SMITH [1]) to be

1.03 for *Au*, 1.095 for *Ag*, and 1.205 for *Cu*.

It seems to be of great importance that the change in the HALL coefficient for *Ag* and *Au* takes place chiefly below —190° C. and becomes practically constant again in the region of liquid hydrogen temperatures. This is also seen to be the case for palladium on comparison of the results of experiments by BENGT BECKMAN upon palladium at liquid air temperature, which are not in agreement with those given by SMITH, and which will be published in the forthcoming paper by BENGT BECKMAN. In connection with the different behaviour for copper, for which BECKMAN has already found an increase in liquid air although smaller than that given by SMITH, the question arises if this cannot be accounted for principally by the influence of impurity. Experiments which we have already undertaken upon alloys — in § 12 we give one set of results — will enable us to decide the point.

§ 11. *Change of resistance of Au_{pl}, Pd_{pl}, Cu_{pl}, in a magnetic field.*

From the measurements with these plates only approximate results can be obtained for this change on account of the smallness of the change in the already very small resistance. In the following table results which were obtained in fields of from 10000 to 11000 gauss are reduced to a standard field of 10 kilogauss.

TABLE XI

Change of resistance in a magnetic field w'_T / w_T

H	*T*	*Au*	*Cu*	*Pd*
10 Kilogauss	20°.3 K	1.017	1.14	1.0015
10 „	14°.5		1.10	

While at ordinary temperature the change caused in the resistance by the field is extremely small, at hydrogen temperatures it becomes quite appreciable.

[1]) A. W. SMITH, Phys. Review, 30, 1, 1910.

III. *Alloys.*

§ 12. *Gold-Silver.* On account of the usually great influence of admixture upon the HALL effect and upon the magnetic change of resistance it was thought desirable to investigate various kinds of alloys. We are already in a position to communicate details of the behaviour of one *solid solution*, viz. an alloy formed by fusing 2°/₀ by volume of silver with gold. The exact analysis we shall publish later. d was here 0,073 mm.

TABLE XII.
HALL-effect for a gold alloy

H	$T = 290°$		$T = 20°.3$		$T = 14°.5$	
	RH	R	RH	R	RH	R
8250	5 70	6.91×10^{-4}	5.60	6.79×10^{-4}	5.44	6.60×10^{-4}
9065	6.31	6.96	—	—	—	—
9360	—	—	6 46	6.90	—	—
9760	6.75	6.91	—	—	6.44	6.60
10270	7.08	6.90	7.01	6.83	6.80	6.62
0	$w = 3.81 \times 10^{-4}\ \Omega$		$w = 1.083 \times 10^{-4}\ \Omega$		$w = 1.080 \times 10^{-4}\ \Omega$	
	$\frac{w}{w_0} = 1.045$		$\frac{w}{w_0} = 0.298$		$\frac{w}{w_0} = 0.297$	

Here we have

$$\frac{R_{T=20.3}}{R_{T=290}} = 0.985.$$

$$\frac{R_{T=14.5}}{R_{T=290}} = 0.955.$$

The observations show that down to hydrogen temperatures and in that region itself the HALL coefficient decreases slightly ; both changes however are so small that they do not exceed the limits of the probable error.

POSTSCRIPT.

IV. *Bismuth crystals.*

§ 13. HALL-*effect in bismuth crystals.* We were not very successful with some of our measurements upon the rods cut in various directions from a crystal which had been formerly used by VAN EVERDINGEN in his researches, and we had therefore meant to postpone the communication of our results until we had obtained a complete series of determinations for various positions of the axis; just as we go to press, however, the important paper by J. BECQUEREL in the Comptes Rendus for 24th June 1912 reaches us, so that we now publish the result which we had already obtained for the case treated by LOWNDS; it is given in the following Table.

TABLE XIII.

HALL-effect in a Bismuth crystal with the axis perpendicular to the field.

$T = 290°$			$T = 20°.3$		
H	RH	R	H	RH	R
2010	20.0×10^3	-9.95	1850	18.0×10^3	$+9.72$
3740	30.6	-8.18	3700	26.0	$+7.03$
5870	38.6	-6.58	5800	33.6	$+5.79$
8250	42.1	-5.11	8700	43.7	$+5.02$
10270	44.3	-4.31	11080	53.1	$+4.79$

At hydrogen temperatures R is positive and approximates to a constant value; at ordinary temperature it is RH corresponding to negative values of R which approaches a constant value. It is possible that small impurities exert considerable influence upon these changes, and it would therefore be risky to conclude from the fact that the value of R at hydrogen temperature which we have found is not greater than that found by LOWNDS for one direction in liquid air, that no change of any importance takes place between the latter temperature and that of liquid hydrogen. (The resistance measurements show that LOWNDS's bismuth plate was freer from impurity than ours).