Physics. — *"A theory of polar armatures."* By H. du Bois. (Communication from the Bosscha-Laboratory).

A well-known partial theory for truncated cones was given by Stefan and applied to the isthmus-method by Sir Alfred Ewing. As a first approximation the magnetisation of the poles is everywhere

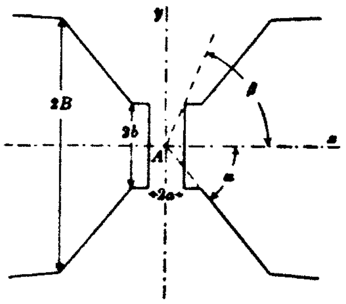

Fig. 1.

assumed parallel to the x-axis (Fig. 1) and thus polar elements have to be dealt with on the terminal surfaces only.

Now the magnetic field due to coils of various shapes has been thoroughly investigated in every detail by various authors, whereas that produced by ferromagnetic pole-pieces is only known for particular points in a few special cases. I believe it is now useful to develop a more general and complete theory for arbitrary points in the field, regard being also paid to protruding frontal surfaces, such as I have been using since 1889 (see fig. 1).

Considering the increasing introduction of prismatic pole-pieces, e. g. for string-galvanometers and other applications, I have also calculated equations for these, generally exhibiting a formal analogy with the conic formulae. Instead of a meridian section, Fig. 1 in this case represents a normal section, the generatrices being directed normally to the plane of figure and parallel to the z-axis.

For the determination of attraction or repulsion the first derivatives of the field with respect to the coordinates have to be considered; e. g. for gradient-methods in measuring weak para- or diamagnetic susceptibilities and also for extraction-magnets, such as those used in ophtalmologic surgery and in ore-separators.

Besides the intensity of the field its topography, especially its more or less uniform distribution appears more and more important in quantitative work and ought to be investigated. Here the second derivatives of the field also come in.

The following equations may occasionally serve as well for certain electrostatic problems showing the same geometrical configuration, on account of the well-known general analogies. The details and proofs are to be given elsewhere.

Round armatures. Considering in the first place surfaces of revolution, more especially cones, the coincident vertices of which both lie in A, the field in this point is known to be

$$\mathfrak{H}^0 = \mathfrak{H}_1 + \mathfrak{H}_2 = 4\pi\,\mathfrak{J}\,\sin vers\,\beta + 4\pi\,\mathfrak{J}\,\sin^2\alpha\cos\alpha\log\frac{B}{b}\,. \quad . \; (I)$$

The notation sufficiently appears from Fig. 1. Both terms are generally of the same order practically; the first corresponds to the truncated frontal planes, the second to the conic surfaces; the latter shows a maximum for $\alpha = \tan^{-1}\sqrt{2} = 54°44'$.

In order to judge of the field's uniformity we now consider the second derivatives, which are related to one another by LAPLACE's equation and the symmetry of the case. The x-component, $\mathfrak{H}_x$, of the field is everywhere meant, though the index x is mostly omitted for simplification. For the centre A, where the first derivatives evidently vanish, the following values are found

$$\frac{\partial^2\mathfrak{H}_1}{\partial x^2} = -2\,\frac{\partial^2\mathfrak{H}_1}{\partial y^2} = -2\,\frac{\partial^2\mathfrak{H}_1}{\partial z^2} = 4\pi\,\mathfrak{J}\,\frac{3\sin^2\beta\cos^3\beta}{a^2} = 4\pi\,\mathfrak{J}\,\frac{3\sin^4\beta\cos\beta}{b^2} \quad (1)$$

Now the term $\mathfrak{H}_1$ always shows a minimum in the centre A, when passing along the longitudinal x-axis, corresponding to a maximum along the equatorial transverse axes, because the numerator $\sin^2\beta\cos^3\beta$ remains positive for $0 < \beta < \pi/2$; in particular this is a maximum, and accordingly the non-uniformity is greatest, for $\beta = \tan^{-1}\sqrt{2/3} = 39°14'$.

The term $\mathfrak{H}_2$ behaves exactly in the opposite way, its second derivative vanishing for that same angle. This well-known result also follows from the general formula, which I now find, viz:

$$\frac{\partial^2\mathfrak{H}_2}{\partial x^2} = -2\,\frac{\partial^2\mathfrak{H}_2}{\partial y^2} = -2\,\frac{\partial^2\mathfrak{H}_2}{\partial z^2} = 4\pi\,\mathfrak{J}\,.\,\frac{3}{2}\sin^4\alpha\cos\alpha\,(5\cos^2\alpha-3)\left(\frac{1}{b^2}-\frac{1}{B^2}\right) (2)$$

As $B > b$ this expression evidently is $\pm$ for $\alpha \lessgtr \cos^{-1}\sqrt{3/5} = 39°14'$; accordingly $\mathfrak{H}_2$ shows a longitudinal minimum and transverse maximum for smaller semi-angles, whereas for larger ones the reverse holds, so as to make the field weaker on the axis than in its lateral surroundings. Finally for the total field

$$\frac{\partial^2(\mathfrak{H}_1+\mathfrak{H}_2)}{\partial x^2} = 4\pi\,\mathfrak{J}\,\frac{3}{2b^2}\left[2\sin^4\beta\cos\beta + \sin^4\alpha\cos\alpha\,(5\cos^2\alpha-3)\left(1-\frac{b^2}{B^2}\right)\right] (3)$$

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Equalizing the contents of the square brackets to zero gives a relation between α and β. In most practical cases b^2/B^2 may be neglected and we find

e. g. for	$\alpha = 39°14'$	$54°44'$	$57°$	$60°$	$63°26'$
the value:	$\beta = 90°$	$79°26'$	$76°52'$	$72°49'$	$63°26'$

as corresponding sets. For the most favourable semi-angle $\alpha = 54°44'$ it is thus possible to combine uniformity and intensity of the field. For $\alpha = 63°26'$ the same value is obtained for β and we have the ordinary non-protruding truncated cones. These results, somewhat at variance with current ideas, were shown to be correct by measurements with a very small test-coil, for which I am indebted to Dr. W. J. DE HAAS.

For excentric axial points, at a distance x from the centre A, the value of the first term is

$$\mathfrak{H}_1(x) = 4\pi\mathfrak{J}\left(1 - \frac{a+x}{2\sqrt{(a+x)^2+b^2}} - \frac{a-x}{2\sqrt{(a-x)^2+b^2}}\right) \quad . \quad . \quad (4)$$

That of the second term for one single cone

$$\mathfrak{H}_2(x) = 2\pi\mathfrak{J}\sin^2\alpha\cos\alpha\left[\log\frac{B - x\sin\alpha\cos\alpha + \sqrt{B^2 - 2Bx\sin\alpha\cos\alpha + x^2\sin^2\alpha}}{b - x\sin\alpha\cos\alpha + \sqrt{b^2 - 2bx\sin\alpha\cos\alpha + x^2\sin^2\alpha}} + \right.$$

$$\left. + \frac{x\,tg\,\alpha - 2B}{\sqrt{B^2 - 2Bx\sin\alpha\cos\alpha + x^2\sin^2\alpha}} - \frac{x\,tg\,\alpha - 2b}{\sqrt{b^2 - 2bx\sin\alpha\cos\alpha + x^2\sin^2\alpha}}\right] \quad . \quad (5)$$

This formula was developed by CZERMAK and HAUSMANINGER in a somewhat different form.

By (4) and (5) the total field for any axial point may be calculated, whether the vertices coincide or not. However a cone is a magnetic "optimum-surface" relatively to its vertex only.

For excentric points on an equatorial y-axis the first term becomes

$$\mathfrak{H}_1(y) = 2\mathfrak{J}\int_0^{2\pi} d\theta \left|\frac{a(ry\cos\theta - a^2 - y^2)}{(a^2 + y^2\sin^2\theta)\sqrt{a^2 + y^2 - 2ry\cos\theta + r^2}}\right|_{r=0}^{r=b}, \quad . \quad (6)$$

which is reducible to elliptic integrals. For the second term a still more complicated integral is found, of which the first part also leads to elliptic integrals of the third kind; whereas the logarithmic term can only be expressed by series of elliptic integrals, a result kindly worked out by Prof. W. KAPTEYN. In fact for two concentric cones we find

$$\mathfrak{H}_2(y)=\mathfrak{J}\sin^2 a\cos a\int_0^{2\pi}d\theta\left[2\frac{b+(y-2b\cos\theta)\sin^2 a\cos\theta}{(1-\sin^2 a\cos^2\theta)\sqrt{b^2-2by\sin^2 a\cos\theta+y^2\sin^2 a}}-\right.$$
$$-2\frac{B+(y-2B\cos\theta)\sin^2 a\cos\theta}{(1-\sin^2 a\cos^2\theta)\sqrt{B^2-2By\sin^2 a\cos\theta+y^2\sin^2 a}}+$$
$$+\log\frac{B-y\sin^2 a\cos\theta+\sqrt{B^2-2By\sin^2 a\cos\theta+y^2\sin^2 a}}{b-y\sin^2 a\cos\theta+\sqrt{b^2-2by\sin^2 a\cos\theta+y^2\sin^2 a}}\times$$
$$\left.\times\frac{b-y\sin^2 a\cos\theta-\sqrt{b^2-2by\sin^2 a\cos\theta+y^2\sin^2 a}}{B-y\sin^2 a\cos\theta-\sqrt{B^2-2By\sin^2 a\cos\theta+y^2\sin^2 a}}\right] \quad (7)$$

If the point considered neither lies on the x-axis nor on the y-axis the equation for $\mathfrak{H}_2(x, y)$ becomes more complicated still.

By applying (4) to pole-shoes having parallel frontal planes only the field for any axial point is easily found; after integration and division by the polar distance the mean value is found to be

$$\overline{\mathfrak{H}}_1 = 4\pi\mathfrak{J}\left(1+\frac{1}{2}\frac{b}{a}-\frac{1}{2}\sqrt{4+\frac{b^2}{a^2}}\right) \quad . \quad . \quad . \quad . \quad (8)$$

As a matter of fact the uniformity in such cases is generally rather satisfactory. It may even be improved within a larger range by hollowing out the front surfaces. If a spherical zone be considered of radius R, perforated in its centre; if the visual angle of the periphery be 2γ, that of the aperture $2\gamma'$ as seen from the sphere's centre, then at a distance x from the latter the field is

$$\mathfrak{H}=\frac{2\pi\mathfrak{J}}{3x^3}\left|\frac{x^4-2R^4+(2R^2-x^2)Rx\cos\theta+R^2x^2\sin^2\theta}{\pm\sqrt{x^2+R^2-2xR\cos\theta}}\right|_{\theta=\gamma'}^{\theta=\gamma} \quad . \quad . \quad (9)$$

The sign depends upon whether the point considered lies on the concave or convex side ($x < R$ or $> R$). By (9) the field in any axial point of a centered pair of spherical zones may be calculated, the interferric space having the shape of a biconvex, biconcave or concave-convex lense; without aperture we have $\gamma' = 0$. The formula for $\partial^2\mathfrak{H}/\partial x^2$ becomes rather complicated; this derivative vanishes for concentric concave hemispheres, for which we find after considerable simplification

$$\mathfrak{H}=\frac{4\pi}{3}\mathfrak{J}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

independent of x, i. e. a perfectly uniform field, a result following moreover from known properties. The same holds more generally for a spheroidal cavity in the midst of a ferromagnetic medium, rigidly magnetised parallel to the axis of symmetry; we then have

$$\mathfrak{H} = \frac{4\pi\mathfrak{J}}{1-m^2}\left(1 - \frac{m}{\sqrt{1-m^2}}\cos^{-1} m\right); \quad . \quad . \quad . \quad (11)$$

here m denotes the ratio of the axis of revolution to a transverse axis of the spheroid; such a case might be approximately realized if the necessity arose.

The attraction exerted upon a small body in an axial point is proportional to $\partial\mathfrak{H}/\partial x$ in case of saturation, or to $\mathfrak{H} \,.\, \partial\mathfrak{H}/\partial x$ if a magnetisation proportional to the field be induced in it. It may therefore be found by differentiation of the expressions (4), (5) or (9), though this generally becomes rather intricate.

Prismatic armatures. If we denote the length at right angles to the normal section (Fig. 1) by $2c$, then we have for $c = \infty$, i e. practically for prisms of sufficient length, if the inclined planes have one mutual bisectrix through A

$$\mathfrak{H}^0 = \mathfrak{H}_1 + \mathfrak{H}_2 = 8\mathfrak{J}\beta + 8\mathfrak{J} \sin a \cos a \log \frac{B}{b} \quad . \quad . \quad . \quad (I^*)$$

For shorter prisms the first term becomes

$$\mathfrak{H}_1 = 8\mathfrak{J} \tan^{-1} \frac{b}{a} \sqrt{\frac{c^2}{a^2+b^2+c^2}}; \quad . \quad . \quad . \quad . \quad (I^*,1)$$

and the second term

$$\mathfrak{H}_1 = 8\mathfrak{J} \sin a \cos a \left[\log \frac{B}{b} - \log \frac{B^2\left(\sqrt{1+\dfrac{b^2}{c^2 \sin^2 a}} - 1\right)}{b^2\left(\sqrt{1+\dfrac{B^2}{c^2 \sin^2 a}} - 1\right)} \right] . \quad (I^*,2)$$

The subtractive term in brackets vanishes for $c = \infty$; then evidently $\partial\mathfrak{H}_2/\partial a$ vanishes for $a = 45°$, which is the most favourable angle in this case, giving the strongest field; for shorter prisms however $a > 45°$.

The uniformity along the z-axis is complete for prisms of sufficient length, i.e. $\partial^2\mathfrak{H}_x/\partial z^2 = 0$; for this case we find

$$\frac{\partial^2\mathfrak{H}_1}{\partial x^2} = -\frac{\partial^2\mathfrak{H}_1}{\partial y^2} = 8\mathfrak{J}\,\frac{\sin 2\beta \cos^2\beta}{a^2} = 8\mathfrak{J}\,\frac{\sin^2\beta \sin 2\beta}{b^2} \quad . \quad (1^*)$$

This expression remains positive and passes through a maximum for $\beta = \tan^{-1} \sqrt{1/_3} = 30°$, the non-uniformity consequently being greatest for this angle.

The term $\mathfrak{H}_2$ again behaves inversely, its second derivative vanishing for this same angle; in fact $\cos 3a$ then vanishes in the formula

$$\frac{\partial^2 \mathfrak{H}_2}{\partial x^2} = -\frac{\partial^2 \mathfrak{H}_2}{\partial y^2} = 8\mathfrak{J} \sin^3 a \cos 3a \left(\frac{1}{b^2} - \frac{1}{B^2}\right) \quad . \quad . \quad . \quad (2^*)$$

As $B > b$ this expression is $\pm$ for $a \lessgtr 30°$. For the total field we finally have

$$\frac{\partial^2 (\mathfrak{H}_1 + \mathfrak{H}_2)}{\partial x^2} = 8\mathfrak{J} \frac{1}{b^2} \left[\sin^2 \beta \sin 2\beta + \sin^3 a \cos 3a \left(1 - \frac{b^2}{B^2}\right)\right] \quad . \quad (3^*)$$

Equalizing the bracketed terms to zero gives a relation between a and β; neglecting b^2/B^2 we find

e.g. for	$a = 30°$	45°	48°	50°46′	54°44′	60°
the value:	$\beta = 90°$	82°38′	79°59′	77°9′	72°26′	60° ’

as corresponding sets. For $a = 60°$ we obtain the same value for β, i. e. non-protruding frontal rectangles.

In excentric axial points at a distance x from the centre A the value of the first term is

$$\mathfrak{H}_1(x) = 4\mathfrak{J} \tan^{-1} \frac{2ab}{a^2 - b^2 - x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4^*)$$

That of the second term for one pair of inclined planes

$$\left.\begin{aligned} \mathfrak{H}_2(x) = 2\mathfrak{J} \sin a \cos a \Bigg[\log \frac{B^2 - 2Bx \sin a \cos a + x^2 \sin^2 a}{b^2 - 2bx \sin a \cos a + x^2 \sin^2 a} + \\ + 2\, tg\, a \left(\tan^{-1} \frac{b - x \sin a \cos a}{x \sin^2 a} - \tan^{-1} \frac{B - x \sin a \cos a}{x \sin^2 a} \right) \Bigg] \end{aligned}\right\} \quad (5^*)$$

By means of (4*) and (5*) the total field may be calculated for any axial point, whether the 4 inclined planes intersect in one line or not; only in the former case do they form an “optimum-surface” with regard to A.

For excentric points on an equatorial axis of y we find as the first term, for $c = \infty$

$$\mathfrak{H}_1(y) = 4\mathfrak{J} \tan^{-1} \frac{2ab}{a^2 - b^2 + y^2}; \quad . \quad . \quad . \quad . \quad . \quad (6^*)$$

and as the second term for two pairs of inclined planes

$$\left.\begin{aligned} \mathfrak{H}_2(y) = 2\mathfrak{J} \sin a \cos a \Bigg[\log \frac{B^2 + 2By \sin^2 a + y^2 \sin^2 a}{b^2 + 2by \sin^2 a + y^2 \sin^2 a} \times \\ \times \frac{B^2 - 2By \sin^2 a + y^2 \sin^2 a}{b^2 - 2by \sin^2 a + y^2 \sin^2 a} + 2\, tg\, a \left(\tan^{-1} \frac{y \sin^2 a + b}{y \sin a \cos a} + \right. \\ \left. + \tan^{-1} \frac{y \sin^2 a - b}{y \sin a \cos a} - \tan^{-1} \frac{y \sin^2 a + B}{y \sin a \cos a} - \tan^{-1} \frac{y \sin^2 a - B}{y \sin a \cos a} \right) \Bigg] \end{aligned}\right\} \quad (7^*)$$

The distribution of the field is thereby completely determined; in the symmetric equatorial plane it is everywhere directed parallel to the x-axis. The most general case of any arbitrary point in the field leads to an expression for $\mathfrak{H}_x(x, y)$, capable of integration but more complicated still than (7*). By differentiation $\partial \mathfrak{H}_x/\partial y$ may also be obtained, though this also turns out rather intricate. In much the same way the distribution of $\mathfrak{H}_x$ along the z-axis may be calculated for prisms of finite length and the integrals.

$$\int_{z_1}^{z_2} \mathfrak{H}_x(z)dz \quad \text{and} \quad \int_{z_1}^{z_2} \mathfrak{H}^2_x(z)dz$$

may be computed, of which the latter is of importance e. g. in the study of transverse magnetic birefringency. The case of an air-space shaped like a cylindric lens is of less practical importance and may here be omitted.

Physiology. — "*Influence of some inorganic salts on the action of the lipase of the pancreas.*" (By Prof. Dr. C. A. PEKELHARING.)

Hydrolytic fat-splitting by the lipase of the pancreas, the only enzyme that will be considered here, may be aided by a number of inorganic salts as well as by bile acids. It does not follow however that this action is always due to the same cause, to the process of activating the enzyme.

It has been proved by RACHFORD as early as 1891 that bile aids the action of the lipase of the pancreas especially on account of the presence of bile salts. The fat-splitting power of rabbit's pancreatic juice was increased by the addition of a solution of glycocholate of soda nearly as much as by the addition of bile [1]). According to the researches of more recent investigators, especially TERROINE [2]), it is highly probable, that the action of bile acids is based on a direct influence on the enzyme, so that here we might speak of an "activator" in the real sense of the word. The fact that various electrolytes also aid the hydrolysis of fat by the lipase, has been demonstrated by POTTEVIN [3]) and more in detail by TERROINE [4]); afterwards also by MINAMI [5]). However, the mode of action of the electrolytes is still unknown, as has been clearly pointed out by TERROINE. The investigators I mentioned used for their experiments pancreatic juice or a

1) Journ. of Physiol. Vol. XII. p. 88.
2) Biochem. Zeitschr. Bd. XXIII. S. 457.
3) Compt. rend. Acad. d. Sciences, T. CXXXVI, p. 767.
4) l. c. S. 440.
5) Bioch. Zeitschr. Bd. XXXIX, S. 392.