

*Citation:*

Scheltema, N., Determination of the geographical latitude and longitude of Mecca and Jidda, accomplished in 1910-1911. Part II, in:  
KNAW, Proceedings, 15 I, 1912, 1912, pp. 540-556

**Astronomy.** — "*Determination of the geographical latitude and longitude of Mecca and Jidda executed in 1910—11.*" By Mr. N. SCHELTEMA. Part II. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of June 29, 1912).

4. *Determination of the geographical latitude of Jidda and Mecca.*  
(Continued).

About the results given in the two preceding tables it must still be noted that some of them in the first series at Jidda depend on one pointing only. These are: Nov. 23 North star T. R., Nov. 29 North star T. L. and T. R. and Nov. 26 and Dec. 1 South star T. L. and T. R.

In the first place we shall now see what may be deduced about the accuracy of our observations as regards chance errors, from a comparison of the individual results.

If the mean error of one pointing on a star be . . . .  $m$   
 „ „ „ of one pointing on the signal be . . . .  $M$   
 then we have

m. error of the zenithpoint for the mean of the two threads  $\frac{1}{2} M$  .

(m. error)<sup>2</sup> of a zenithdistance derived from two pointings

on the star in one position of the telescope . . .  $\frac{1}{2} m^2 + \frac{1}{4} M^2$ .

We may now consider the m. error of a latitude  $q$  to be equal to that of the zenithdistance from which it has been deduced and thus we obtain:

$$(\text{m. e.})^2 \text{ of } q \text{ from one posit.} = \text{I} = \frac{1}{2} m^2 + \frac{1}{4} M^2$$

$$\text{„ of } \frac{1}{2} (q_L - q_R) = \text{II} = \frac{1}{4} m^2 + \frac{1}{4} M^2$$

$$\text{„ of } \frac{1}{2} (q_L + q_R) = \text{III} = \frac{1}{4} m^2$$

$$\text{„ of } \frac{1}{2} (q_N - q_S) = \text{IV} = \frac{1}{8} m^2$$

$$\text{„ of } \frac{1}{2} (q_N + q_S) = \text{V} = \frac{1}{8} m^2$$

$$\text{from which: II} + \text{III} = \text{I}$$

$$\text{II} - \text{III} = \frac{1}{4} M^2.$$

We now deduce the values of I, II, and III by comparing the individual results with their mean, first of all for the observations at Jidda and Mecca separately, afterwards for all together. In order to deduce in the latter case the values of II (just as afterwards of IV) the general mean of the  $\varphi_L - \varphi_R$  (and later on of the  $\varphi_N - \varphi_S$ ) has been employed. The result was, however, practically the same when the two separate means were used. The first series of observations at Jidda has been left out of account throughout this investigation, as it was less homogeneous and besides contained Mr. SALIM's first observations, when he had had little practice as yet.

<i>Jidda</i>	<i>Mecca</i>	<i>Together</i>
I $(\pm 10''.86)^2 = 117.96$	$(\pm 10''.91)^2 = 119.08$	$(\pm 10''.88)^2 = 118.44$
II $(\pm 9''.67)^2 = 93.59$	$(\pm 8''.33)^2 = 69.40$	$(\pm 8''.99)^2 = 80.74$
III $(\pm 5''.29)^2 = 27.99$	$(\pm 7''.05)^2 = 49.71$	$(\pm 6''.09)^2 = 37.13$

From this appears very satisfactorily that  $II + III = I$ , while we find in the three cases:

$$II - III (\pm 8''.10)^2 = 65.60 \quad (\pm 4''.44)^2 = 19.69 \quad (\pm 6''.60)^2 = 43.61.$$

We can now compare *inter se* the values of  $m$  and  $M$ . As the signals at Jidda and Mecca were of a different kind the two values of  $M$  must not a priori be accepted as equal. The differences found between the  $m$  and  $M$  for the two places are, however, evidently not real, and we may only conclude from the general results that  $m$  and  $M$  are about equal, only possibly  $M$  slightly greater than  $m$ , which would also a priori be probable.

This investigation raises the question whether it would have been better to employ for the zenithpoint mean values from longer periods instead of the individual results, and although the value of the zenithpoint is generally eliminated, I still wanted to examine this. Therefore the observations have also been reduced with the zenithpoint from the whole of the period in which the instrument remained at one station, and then the squares of the mean error I and II have again been determined. As the last 3 isolated nights of observation at Jidda have not been used here, the values of I and II were also deduced again after the first way of calculation.

Thus we found:

<i>Jidda</i>	<i>Mecca</i>	<i>Together</i>
With individual zenithpoints		
I $(\pm 11''.00)^2 = 121.02$	$(\pm 10''.91)^2 = 119.08$	$(\pm 10''.96)^2 = 120.06$
II $(\pm 10''.02)^2 = 100.45$	$(\pm 8''.33)^2 = 69.40$	$(\pm 9''.22)^2 = 84.93$
With mean zenithpoints		
I $(\pm 9''.99)^2 = 99.88$	$(\pm 11''.68)^2 = 136.54$	$(\pm 10''.87)^2 = 118.21$
II $(\pm 8''.91)^2 = 79.31$	$(\pm 9''.32)^2 = 86.95$	$(\pm 9''.12)^2 = 83.13$

So no improvement is found for all the observations together; and although this is indeed the case for those at Jidda, the value of II remains still considerably higher than the one found for III, which shows that even when mean values are used the mean error of the zenithpoint has not yet become really small.

We shall now consider the values of IV and V, which, not taking into account the influences of flexure and division-errors, must be equal to  $\frac{1}{8}m^2$ . Now these two errors must have been almost eliminated in the  $\frac{1}{2}(q_N + q_S)$  owing to the nearly equal zenithdistance of North- and Southstar, but they may be considerable in the  $\frac{1}{2}(q_N - q_S)$ , and as on different nights couples of different zenithdistance were observed, the value of IV must also have been increased by that influence.

We now find, adding for comparison the values of  $\frac{1}{2} \cdot \text{III}$

	<i>Jidda</i>	<i>Mecca</i>	<i>Together</i>
IV	$(\pm 3''.36)^2 = 11.26$	$(\pm 4''.85)^2 = 23.51$	$(\pm 4''.14)^2 = 17.15$
V	$(\pm 4''.14)^2 = 17.12$	$(\pm 5''.06)^2 = 25.60$	$(\pm 4''.56)^2 = 20.76$
$\frac{1}{2} \cdot \text{III}$	$(\pm 3''.74)^2 = 14.00$	$(\pm 4''.99)^2 = 24.86$	$(\pm 4''.31)^2 = 18.56$

So we see that the values found for IV are not only not higher but on the contrary somewhat lower than those of V and that both are almost equal to  $\frac{1}{2} \cdot \text{III}$ , on which flexure and division-errors must have had some influence too. From this we may conclude that the two influences cannot have been great.

Coming now to a consideration of the mean results for  $q$  in the different positions, we shall first compare those with the telescope left and right.

Denominating the correction of the employed zenithpoint  $\Delta Z$  then we see that

$$\begin{array}{lll}
 \text{Northstar } T. L. & \Delta q = - \Delta Z & \\
 & T. R. & \Delta q = + \Delta Z \quad q_L - q_R = + 2 \Delta Z \\
 \text{Southstar } T. L. & \Delta q = + \Delta Z & \\
 & T. R. & \Delta q = - \Delta Z \quad q_L - q_R = - 2 \Delta Z \\
 \text{Thus } . . . . . & & (q_L - q_R)_{\frac{1}{2}(N-S)} = + 2 \Delta Z
 \end{array}$$

In this way we find

$$\begin{aligned} &\text{from all observations} && \Delta Z = +1''.5 \pm 1''.2 \\ &\text{from those of 1911 only} && +1''.0 \pm 1''.4 \end{aligned}$$

The value of  $\Delta Z$  is fairly small and almost equal to its mean error. The 3 partial results Jidda 1910, Jidda 1911 and Mecca have, however, the same sign. In order to correct one-sided observations we have employed the value deduced from 1911, Jidda and Mecca together,  $+1''.0$ .

In the second place we shall consider the differences between the results from the North and the Southstar. Except on one night in 1910 the zenith-distances of the observed stars lie between  $10^\circ$  and  $45^\circ$  and the mean  $z$  is about  $30^\circ$ . The  $q_N - q_S$  therefore contain twice the flexure for a zenith-distance of about  $30^\circ$  and the influence of the systematic division-errors on an arc of about  $60^\circ$ .

We now find:

$$\begin{aligned} \text{Jidda 1910 } q_N - q_S &= +1''.7 \text{ weight 5.5} \\ \text{,, 1911 ,,} &+5''.6 \text{ ,, 11} \\ \text{Mecca ,,} &+0''.4 \text{ ,, 8} \end{aligned}$$

from which follows for

$$\begin{aligned} \text{all observations together} &+3''.0 \pm 1''.7 \\ \text{for the observations in 1911} &+3''.4 \pm 1''.9 \end{aligned}$$

So the differences are not great. That the flexure of the telescope would be small was to be expected, but our results prove also that the systematic division-errors of the circle cannot be great. For the reduction of the incomplete observations we always employed (even in 1910), according to the results for 1911

$$\frac{1}{2}(q_N - q_S) = +1''.7$$

In this manner we deduced for all observation-nights values for  $\frac{1}{2}(q_N + q_S)$ , and the means taken from these, giving half weight to the nights on which only one star had been observed, were considered our final results. Moreover mean values have been formed from the results in the separate positions and from the separate stars, again giving half weight to incomplete observations.

So we found:

<i>Northstar</i>			<i>Southstar</i>			$\frac{Nth + Sth}{2}$	
<i>T.L.</i>	<i>T.R.</i>	Mean	<i>T.L.</i>	<i>T.R.</i>	Mean		
Jidda						21°	
1° S.	17''7	11'5	15''1	13''0	16''3	14''7	29' 14''5
2° S.	23.3	19.2	20.9	15.7	16.3	16.0	29 18.5

	Mecca	21°
22"1   24"5   23"3   20"9   24"2   22"5		25' 23"1

The results from the two series for Jidda are :

$$21^{\circ} 29' 14''.5 \pm 1''.7$$

$$29 18.5 \pm 1.3$$

The difference between them a little exceeds the sum of their mean errors. Forming for the first series separate results for the two observers we obtain :

$$\text{SCHELTEMA } 21^{\circ} 29' 15''.1$$

$$\text{SALIM } 13.9$$

which are in good accordance.

After full consideration the two series have been united according to their weights and so our final results are :

$$q \text{ JIDDA } 21^{\circ} 29' 17''.0 \pm 1''.0$$

$$q \text{ MECCA } 21^{\circ} 25' 23''.1 \pm 1''.5$$

### 5. *Results of the determinations of time.*

The determinations of time were always made by observing the altitude of a star in the east and of one in the west. Each star was observed in the two positions of the instrument and each time the transits over both the horizontal threads were noted, the instrument remaining clamped. Hence the zenithpoint for the mean of the two threads was employed in deducing the zenith-distance, and for the mean of the two instants the hour angle was then computed after the usual formula

$$\cos t = \frac{\cos z - \sin q \sin \delta}{\cos q \cos \delta}$$

In Nov.—Dec. 1910 the chronometer of Cummins and since the 2<sup>nd</sup> half of January 1911 that of Dent was used for the observations. The rates of Cummins were very great and irregular until it stopped altogether. I therefore omit the communication of the chronometer-corrections and rates for the first period. They were only used for the reduction of the latitude determinations and they were sufficiently accurate for that purpose.

About the determinations of time in the second period I shall first give the necessary data to form a judgment of the accuracy reached as regards systematic and accidental errors. The two following tables contain for this purpose the 4 separate results obtained each night.

As appears from these tables there is only one determination of time at Mecca (Febr. 26) which is not based on an eastern and

a western star, while on another night (Febr. 25) 2 eastern and 2 western stars were observed. Further, on Febr. 14 (Mecca) and Febr. 21 (Jidda) no zenithpoint was determined and this was derived from preceding and following days.

## RESULTS FROM THE DETERMINATIONS OF TIME AT JIDDA.

	Star East			Star West			E.—W.
	T. L.	T. R.	Mean	T. L.	T. R.	Mean	
1911			+ 2 <sup>h</sup>			+ 2 <sup>h</sup>	
Jan. 25	7 <sup>s</sup> 37	9 <sup>s</sup> 89	23 <sup>m</sup> 8 <sup>s</sup> 63	8 <sup>s</sup> 85	8 <sup>s</sup> 30	23 <sup>m</sup> 8 <sup>s</sup> 58	+ 0 <sup>s</sup> 05
" 26	15.77	14.46	15.11	13.86	13.12	13.48	+ 1.63
" 28	24.64	23.56	24.10	25.57	23.91	24.74	— 0.64
" 30	33.36	34.11	33.73	31.42	33.93	32.67	+ 1.06
" 31	38.61	37.24	37.93	36.00	38.05	37.02	+ 0.91
Febr. 1	41.61	40.44	41.02	40.34	41.05	40.70	+ 0.32
" 3	48.04	47.88	47.96	47.41	47.78	47.59	+ 0.37
" 6	0.47	59.27	59.87	59.72	0.14	59.93	— 0.06
" 7	5.08	4.57	24 4.82	5.08	4.80	24 4.94	— 0.12
" 8	10.03	8.16	9.10	7.96	9.15	8.56	+ 0.54
" 12	28.23	27.89	28.06	28.51	29.36	28.94	— 0.88
" 18	2.59	3.06	25 2.83	3.84	2.67	25 3.25	— 0.42
" 20	14.12	14.84	14.48	14.19	15.81	15.00	— 0.52
" 21	20.23	18.35	19.29	18.53	20.72	19.62	— 0.33
" 22	26.11	25.42	25.77	25.14	25.63	25.39	+ 0.38
Mrch 2	10.84	10.14	26 10.50	9.21	9.80	26 9.51	+ 0.99
" 3	13.89	14.17	14.03	14.70	13.83	14.26	— 0.23
" 7	30.96	30.37	30.67	30.23	29.85	30.04	+ 0.63
" 8	36.39	34.11	35.25	34.94	35.47	35.20	+ 0.05
" 19	27.96	27.28	27 27.62	26.29	29.18	27 27.74	— 0.12
" 20	32.53	33.26	32.89	34.41	32.78	33.60	— 0.71
" 21	37.60	37.72	37.66	37.55	38.17	37.86	— 0.20
" 23	47.84	48.75	48.30	48.32	47.84	48.08	+ 0.22

## RESULTS FROM THE DETERMINATIONS OF TIME AT MECCA.

	Star East			Star West			E.-W.
	T. L.	T. R.	Mean	T. L.	T. R.	Mean	
1911			+ 2 <sup>h</sup>			+ 2 <sup>h</sup>	
Febr. 14	20 <sup>s</sup> 13	15 <sup>s</sup> 53	27 <sup>m</sup> 17 <sup>s</sup> 83	16 <sup>s</sup> 06	19 <sup>s</sup> 38	27 <sup>m</sup> 17 <sup>s</sup> 72	+ 0 <sup>s</sup> 11
" 15	24.72	22.43	23.57	22.04	23.87	22.96	+ 0.61
" 16	28.24	26.49	27.37	28.73	28.86	28.80	- 1.43
" 24	11.89	13.10	28 12.49	12.32	11.26	28 11.79	+ 0.70
" 25	17.40	17.40	17.40	18.30	17.10	17.70	- 0.30
	17.69	18.34	18.02	18.55	18.09	18.32	- 0.30
" 26				21.33	22.88	22.10	
" 27	28.61	29.46	29.04	28.85	27.86	28.35	+ 0.69
Mrch 11	22.53	24.28	29 23.41	24.60	23.63	29 24.11	- 0.70
" 12	27.27	28.05	27.66	27.00	26.99	27.00	+ 0.66
" 14	38.92	38.22	38.57	38.71	38.44	38.57	0.00
" 15	43.49	42.58	43.04	42.41	42.30	42.36	+ 0.68
" 16	45.88	46.47	46.18	47.51	45.42	46.46	- 0.28
" 17	50.40	50.29	50.34	50.32	49.17	49.75	+ 0.59

We must now first compare the results obtained in the two positions of the instrument. If the observed corrections of the chronometer are  $\Delta t$ , and the correction of the employed zenithpoint is designated by  $\Delta Z$ , then we find:

$$\text{Eastern star } \Delta Z = + a \frac{\Delta t_L - \Delta t_R}{2}$$

$$\text{Western star } \Delta Z = - a \frac{\Delta t_L - \Delta t_R}{2}$$

in which, if  $\Delta t$  is expressed in seconds of time and  $\Delta Z$  in seconds of arc, the mean value of the factor  $a$  is 13.8.

Leaving out of account the two days on which the zenithpoint had not been determined and reversing the signs for the western stars, we find as mean result:

$$\frac{\Delta t_L - \Delta t_R}{2} = + 0^s.07$$



from which follows  $\Delta Z = +1''.0$ , i.e. the same value as was found from the determinations of latitude

Secondly the results from the eastern and the western star have been compared *inter se* and the mean values obtained were:

Jidda	23	nights	E—W = + 0 <sup>s</sup> .13
Mecca	13	„	+ 0 .08
Together	36	„	E—W = + 0 <sup>s</sup> .11

If this difference is produced by a constant error in the measured zenithdistances, then we find for its amount  $\Delta z = +0''.8$ , while  $+1''.7$  had been found from the determinations of latitude in which the average zenithdistance was somewhat smaller. From a comparison of the separate values for E.—W. with their general mean we find, however, as mean error of the difference found in a single night  $\pm 0^s.63$ , hence of the result from 36 nights  $\pm 0^s.10$ , which is equal to the mean difference itself. The obtained results are, however, satisfactory, as we may conclude that no great unknown sources of error have been at work.

Disregarding a possible systematic personal error, we may further consider the mean error of  $\frac{1}{2}(E+W)$  as equal to that of  $\frac{1}{2}(E-W)$ , and we thus obtain as mean error of a chronometer-correction from an eastern and a western star  $\pm 0^s.32$ .

At each time-determination the Leroy watches were compared with Dent. In the meantime Leroy 5180 = Dutch navy 3 had stopped and on the journeys to Mecca only 2 or 3 watches were taken (2 on the first and second journeys, 3 on the third) for fear of a possible mishap. Prudence demanded this, although now that everything went off well, I regret that all the watches were not taken each time. Naturally the mean errors of the observed corrections of the watches will be somewhat greater than in the case of Dent, owing to the errors of comparison.

The following tables contain the observed corrections for Dent and the Leroy-watches and the thence derived daily rates; the first two tables according to the observations at Jidda, the next two according to those at Mecca. On Febr. 25 Leroy 4129 = Dutch navy 77 was wound up too late after it had already stopped (see the tables on p. 548—550).

It is clearly visible from the daily rates contained in the preceding tables that the time-determination of Febr. 26 at Mecca, based on one star only, has been less accurate. The same appears with even greater force for the one of Febr. 21 at Jidda, although the observations of that night are apparently irreproachable.

For a closer investigation of the regularity of the watches we shall use the rates which have been obtained during the stay at

36\*

## CORRECTIONS DETERMINED AT JIDDA.

	DENT 2527			LEROY 5192 D. N. 7			LEROY 4129 D. N. 77	
	M.Time	Corr.	D. R.	M.Time	Corr.	D. R.	Corr.	D. R.
1911		+ 2h			- 0h		- 2h	
Jan. 25	7h 4m	23m 8s60	+ 5s53	9h31m	2m13s86	+ 0s24	9m 3s82	+ 2s10
" 26	7 49	14.30	+ 4.98	8 3	13.63	+ 2.47	1.85	+ 3.88
" 28	8 34	24.42	+ 4.51	7 58	8.70	+ 1.02	8 54.11	+ 3.42
" 30	7 17	33.20	+ 4.33	8 7	6.66	+ 1.80	47.25	+ 4.46
" 31	6 58	37.47	+ 3.30	9 10	4.78	+ 2.51	42.59	+ 3.85
Febr. 1	7 38	40.86	+ 3.48	9 39	2.22	+ 2.68	38.66	+ 4.12
" 3	7 20	47.77	+ 4.07	9 13	1 56.90	+ 2.47	30.50	+ 3.34
" 6	6 56	59.90	+ 4.78	9 29	49.45	+ 2.41	20.44	+ 4.84
" 7	7 56	24 4.88	+ 4.11	7 7	47.28	+ 1.59	16.08	+ 3.20
" 8	6 58	8.83	+ 4.85	10 6	45.49	+ 2.26	12.48	+ 3.56
" 12	8 16	28.50	+ 5.68	10 11	36.45	+ 1.87	7 58.24	+ 4.09
" 18	10 6	25 3.04	+ 5.92	10 48	25.21	+ 1.94	33.60	+ 4.83
" 20	9 30	14.74	+ 4.71	10 59	21.31	+ 0.74	23.91	+ 3.03
" 21	9 32	19.45	+ 6.61	10 9	20.60	+ 2.74	20.99	+ 5.41
" 22	7 48	25.58	+ 5.48	9 35	17.93	+ 1.13	15.71	
March 2	10 26	26 10.00	+ 4.25	11 22	8.84	- 0.54	51 20.24	+ 4.68
" 3	9 52	14.15	+ 4.12	10 22	9.36	+ 0.09	15.76	+ 4.52
" 7	8 20	30.36	+ 4.53	9 38	9.00	- 0.12	50 57.80	+ 4.35
" 8	10 6	35.23	+ 4.75	11 7	9.13	- 0.21	53.18	+ 5.12
" 19	11 1	27 27.68	+ 5.90	11 27	11.43	- 0.66	49 56.82	+ 5.64
" 20	9 38	33.24	+ 5.04	11 1	12.08	- 0.72	51.28	+ 4.83
" 21	7 8	37.76	+ 5.19	7 48	12.70		47.10	
" 23	7 23	48.19						

		LEROY 4127 D. N. 80			LEROY 3565 D. N. 81		LEROY 4128 D. N. 84	
		M. Time	Corr.	D. R.	Corr.	D. R.	Corr.	D. R.
1911			+ 2h		- 3h		- 2h	
Jan.	25	9h31m	18m34s56		26m44s61		57m42s23	
			+ 1s25			- 6s66		- 0s46
"	26	8 3	35.73	+ 2.49	50.86	- 5.83	42.66	+ 0.78
"	28	7 58	40.70	+ 1.97	27 2.50	- 5.60	41.10	- 0.27
"	30	8 7	44.65	+ 1.90	13.74	- 4.06	41.64	+ 0.44
"	31	9 10	46.63	+ 1.87	17.98	- 6.35	41.18	- 0.08
Febr.	1	9 39	48.54	+ 2.61	24.46	- 5.16	41.26	+ 0.19
"	3	9 13	53.71	+ 2.01	34.69	- 5.76	40.88	+ 0.28
"	6	9 29	59.76	+ 2.42	52.02	- 5.38	40.04	+ 0.42
"	7	7 7	19 1.94	+ 1.58	56.87	- 5.88	39.66	- 0.36
"	8	10 6	3.72	+ 1.81	28 3.48	- 5.39	40.07	+ 0.06
"	12	10 11	10.96		25.04		39.83	
			+ 0.47			- 5.02		- 0.09
"	18	10 48	13.80	+ 0.65	55.30	- 5.18	40.40	+ 0.25
"	20	10 59	15.10	- 0.72	29 5.70	- 6.69	39.89	- 1.54
"	21	10 9	14.41	+ 1.51	12.16	- 3.85	41.38	+ 1.10
"	22	9 35	15.88		15.92		40.31	
			+ 1.33			- 4.57		+ 0.08
March	2	11 22	26.58	+ 1.21	52.83	- 5.35	39.63	- 0.33
"	3	10 22	27.74	+ 1.68	57.96	- 5.19	39.95	- 0.06
"	7	9 38	34.41	+ 1.89	30 18.58	- 5.27	40.18	- 0.19
"	8	11 7	36.42		24.18		40.38	
			- 0.02			- 4.71		+ 0.47
"	19	11 27	36.18	+ 0.15	31 16.02	- 4.52	35.21	+ 0.35
"	20	11 1	36.33	- 0.48	20.46	- 4.64	34.87	+ 0.22
"	21	7 48	35.91		24.48		34.68	

## CORRECTIONS DETERMINED AT MECCA.

	DENT 2527			LEROY 5192 D. N. 7			LEROY 4129 D. N. 77	
	M. Time	Corr.	D. R.	M. Time	Corr.	D. R.	Corr.	D. R.
1911		+ 2 <sup>h</sup>			+ 0 <sup>h</sup>		— 2 <sup>h</sup>	
Febr. 14	10 <sup>h</sup> 40 <sup>m</sup>	27 <sup>m</sup> 17 <sup>s</sup> 78						
" 15	8 55	23.26	+ 5 <sup>s</sup> 91					
" 16	9 22	28.08	+ 4.73					
			+ 5.54					
" 24	8 32	28 12.14		12 <sup>h</sup> 47 <sup>m</sup>	1 <sup>m</sup> 19 <sup>s</sup> 56		4 <sup>m</sup> 32 <sup>s</sup> 64	
			+ 5.70			+ 0 <sup>s</sup> 94	— 0 <sup>h</sup>	
" 25	8 38	17.86	+ 4.30	8 16	20.32	+ 0.27	49 6.80	+ 3 <sup>s</sup> 15
" 26	8 16	22.10	+ 5.40	12 58	20.64	+ 1.67	3.03	+ 4.03
" 27	13 35	28.69	+ 4.68	14 6	22.39	+ 0.36	48 58.81	
Mrch 11	7 44	29 23.76	+ 3.53	8 17	26.65	+ 0.40		
" 12	8 0	27.33	+ 5.23	8 35	27.05	+ 0.15		
" 14	11 36	38.57	+ 4.01	13 0	27.38	— 2.45		
" 15	12 21	42.70	+ 3.63	13 28	24.88	— 1.39		
" 16	12 16	46.32	+ 4.62	13 15	23.50	+ 0.09		
" 17	7 39	50.05		8 18	23.57			

  

	LEROY 4127 D. N. 80			LEROY 3565 D. N. 81		LEROY 4128 D. N. 84	
	M. Time	Corr.	D. R.	Corr.	D. R.	Corr.	D. R.
1911		+ 2 <sup>h</sup>		— 3 <sup>h</sup>		— 2 <sup>h</sup>	
Febr. 14	11 <sup>h</sup> 5 <sup>m</sup>	21 <sup>m</sup> 47 <sup>s</sup> 67		25 <sup>m</sup> 58 <sup>s</sup> 53			
" 15	11 47	48.23	+ 0 <sup>s</sup> 54	26 4.27	— 5 <sup>s</sup> 58		
" 16	13 56	48.39	+ 0.15	11.91	— 7.01		
			+ 0.99				
March 11	8 17	22 10.87	— 0.02			55 <sup>m</sup> 6 <sup>s</sup> 93	+ 0 <sup>s</sup> 75
" 12	8 35	10.85	+ 0.43			6.17	+ 1.45
" 14	13 0	11.79	+ 0.28			3.00	+ 1.26
" 15	13 28	12.08	— 0.38			1.71	— 1.11
" 16	13 15	11.70	— 0.16			2.81	+ 0.73
" 17	8 18	11.57				2.23	

Jidda. First we find as the mean daily rates during 4 periods of from 4 to 6 days each separated by journeys to Mecca:

	DENT	D.N. 7	D.N. 77	D.N. 80	D.N. 81	D.N. 84
Febr. 6—12	+ 4.72	+ 2.16	+ 3.68	+ 1.85	— 5.48	+ 0.03
„ 18—22	+ 5.77	+ 1.84	+ 4.53	+ 0.53	— 5.22	+ 0.02
March 2—8	+ 4.21	— 0.05	+ 4.51	+ 1.64	— 5.23	— 0.13
„ 19—23	+ 5.33	— 0.69	+ 5.26	— 0.15	— 4.58	+ 0.29

Secondly the accidental deviations have been examined, first by forming the mean value  $\frac{1}{n} \sqrt{\Sigma \Delta \Delta}$  of the differences  $\Delta$  between two subsequent daily rates, and afterwards by comparing the rates between Febr. 6 and March 23 themselves with their mean value for the whole period and deducing the mean deviation  $\frac{1}{n'} \sqrt{\Sigma \Delta' \Delta'}$ .

Both these mean deviations I and II follow here.

	DENT	D.N. 7	D.N. 77	D.N. 80	D.N. 81	D.N. 84
I	$\pm 0.61$	$\pm 0.91$	$\pm 0.97$	$\pm 0.60$	$\pm 0.95$	$\pm 0.61$
II	$\pm 0.67$	$\pm 1.29$	$\pm 0.69$	$\pm 0.91$	$\pm 0.39$	$\pm 0.29$

For D.N. 81 the mean deviation I becomes  $\pm 0.57$ , if one time-determination is excluded.

The striking things in these comparisons are in particular the considerable acceleration of D.N. 7, owing to which also the mean deviation II is very great; and secondly the regularity of D.N. 84.

#### 6. *Derivation of the difference of longitude Jidda—Mecca.*

From the corrections and rates of our watches given in the preceding paragraph we must now deduce the most probable value for the difference of longitude between Jidda and Mecca. Apart from the desirability of knowing the result yielded by each of the watches an immediate combination of the results of all would be impossible, because of the fact that on the different journeys different watches were taken and only Dent 2527 was used throughout. We shall therefore derive separately the results to which the 6 employed watches have led, and only afterwards we shall endeavour to derive from the whole of this material the most reliable final result.

Whereas each group of observations at Jidda or at Mecca usually includes time-determinations on 4 nights, determinations on 11 nights at Jidda, viz. from Jan. 25 to Febr. 12 immediately precede the

first journey to Mecca. But of however much value this long series is for the investigation of the watches and of the observations themselves, it cannot be of any immediate use for the derivation of the longitude. The longer the periods that are discussed the greater does the uncertainty become in the calculated rates and corrections of the watches, and soon its influence surpasses that of the errors of the observation. The great difficulty lying here in the answer to the question at what distance from the journey determinations of time may still be used to advantage, this will certainly not be the case for the observations in January. Finally only the observations of Febr. 6--12 have been used as a first group.

In the following we shall indicate Leroy's watches with the numbers they have in the Dutch Navy.

a. *Chronometer DENT 2527.*

This was taken by Mr. SALIM on all his journeys to Mecca and we have therefore at our disposal 4 groups of observations at Jidda, each including 4 nights, and between these 3 groups at Mecca with resp. 3, 3 and 6 determinations of time. Hence the discussion of the results obtained with this chronometer offers the best opportunity for comparing the different methods that may be followed for the deduction of the difference of longitude.

This deduction must be based on the comparison of observed chronometer-corrections at one place with interpolated corrections with regard to the local time of the other, whether that interpolation is made directly or in such a way, that we represent the corrections found for both stations by formulae differing only in the value of the constant term

An exhaustive criticism of these methods of calculation has been given by W. STRUVE on the occasion of his discussion of the results of the chronometer-expeditions<sup>1)</sup> executed between Pulkowa and Altona. He arrived at the conclusion that for observations made during a long period with a great number of journeys in both directions, as in his case, the representation by one formula, which must then contain a rather great number of powers of the time, would be unpractical. Our case, however, is somewhat different. The number of journeys and the duration of each was much less, and, whereas our determinations of time were much less accurate, we had attempted to make up for this inferiority by observing on several nights each time at each station.

<sup>1)</sup> F. G. W. STRUVE. *Expéditions chronométriques entre Poulkova et Altona*. St.-Petersbourg 1844, p. 117—128.

It therefore was difficult for us to decide whether the different journeys would have to be discussed each by itself, or whether it would be preferable to take two or three together. And so finally it seemed best to follow both ways or rather try a number of different methods of calculation.

As the smallest group of observations discussed together we have always taken those obtained during the stay at one station combined with those from the preceding and the following visit to the other station. Then only a real interpolation is possible, and there is besides another circumstance demanding this. The rate of a chronometer may not only be subject to chance perturbations during the transport, but there may also take place a systematic retardation or acceleration, which continues throughout the duration of the transport. So a chronometer-correction calculated by means of extrapolation would be subject to systematic errors. On the other hand it is easy to see that in the calculation of a chronometer-correction for instance during a stay at Mecca from preceding and following observations at Jidda, the above mentioned error will be altogether eliminated for a moment exactly between those of the observations and that it would be small for other moments.

In this respect therefore such a group of observations can yield accurate results. A uniform retardation or acceleration, however, cannot be taken account of in this way but very imperfectly. This will become clear when we represent the chronometer-corrections by formulae. These will then contain terms with the square of the time, and it will be easily seen that in a combination Jidda—Mecca—Jidda the influence of such a term and that of an error in the difference of longitude will not differ greatly. If, however, a combination Mecca—Jidda—Mecca is also discussed then the influence of a quadratic term on the difference of longitude will have the reverse sign. Hence it will be possible to eliminate that influence by forming combinations of the two kinds and taking the mean of their results. This approaches already the calculation of a quadratic formula from a longer period.

I shall now communicate the numerical results obtained by means of Dent 2527 using the different methods of calculation.

#### 1. *Results from the separate journeys.*

*Journeys to Mecca (J.—M.—J.).* Determinations of time in Mecca compared with interpolated values between the observations at Jidda immediately before and after the journey.

1 <sup>st</sup> journey	2 <sup>nd</sup> journey	3 <sup>rd</sup> journey
+ 2 <sup>m</sup>	+ 2 <sup>m</sup>	+ 2 <sup>m</sup>
Febr. 14 37 <sup>s</sup> 35	Febr. 24 35 <sup>s</sup> 44	Mrch 11 34 <sup>s</sup> 75
„ 15 37.57	„ 25 35.65	„ 12 33.52
„ 16 36.60	„ 26 34.49	„ 14 34.55
Mean 37 <sup>s</sup> 17	„ 27 34.39	„ 15 33.77
	Mean 34 <sup>s</sup> 99	„ 16 32.66
		„ 17 32.55
		Mean 33 <sup>s</sup> 63

Mean of the 3 journeys + 2<sup>m</sup> 35<sup>s</sup>26.

*Journeys to Jidda (Me—J—Me).* Treated in exactly the same way they gave the following results.

1 <sup>st</sup> journey	2 <sup>nd</sup> journey
+ 2 <sup>m</sup>	+ 2 <sup>m</sup>
Febr. 18 36 <sup>s</sup> 29	March 2 32 <sup>s</sup> 12
„ 20 35.53	„ 3 32.53
„ 21 36.37	„ 7 34.75
„ 22 35.38	„ 8 34.90
Mean 35 <sup>s</sup> 89	Mean 33 <sup>s</sup> 57

Mean of the 2 journeys + 2<sup>m</sup> 34<sup>s</sup>73.

The combinations Jidda—Mecca—Jidda have also been calculated by means of linear formulae, i.e. the corrections of the chronometer determined at Jidda and at Mecca have been represented resp. by formulae  $a + b(t-t_0)$  and  $a' + b(t-t_0)$ , from which the unknown quantities were solved after the method of least squares. The difference  $a' - a$  gives us the difference of longitude, or when a provisory value for this difference had been applied, the correction needed by that value. Of the 3<sup>rd</sup> group of observations at Jidda March 2 and 3 have only been used for the 2<sup>nd</sup> journey to Mecca, March 7 and 8 only for the 3<sup>rd</sup>.

So we found:

1 <sup>st</sup> journey	+ 2 <sup>m</sup> 35 <sup>s</sup> 62	(± 1 <sup>s</sup> 17)
2 <sup>nd</sup> „	35.62	(± 0.65)
3 <sup>rd</sup> „	32.93	(± 0.91)
Mean	+ 2 <sup>m</sup> 34 <sup>s</sup> 72	

The values in brackets are the mean residual errors in the observed chronometer-corrections, when they are represented by the calculated formulae.

## 2. Results from the whole of the material.

We have represented the observations by formulae of the second



and third degree

$$\frac{a}{a'} + b(t-t_0) + c(t-t_0)^2$$

and

$$\frac{a}{a'} + b(t-t_0) + c(t-t_0)^2 + d(t-t_0)^3$$

from which the values of the unknown quantities have been deduced by the method of least squares.

Five solutions have been found.

I by means of quadratic formulae

II by means of formulae of the 3<sup>rd</sup> degree

III by means of quadratic formulae, correcting the data beforehand for the *supplementary "transport-rate"*.

IV Like I, but giving half weight to the 6 observations of the 3<sup>rd</sup> series at Mecca.

V Like III, but giving half weight to the 6 observations of the 3<sup>rd</sup> series at Mecca.

Defining the supplementary "transport rate" E. as the excess of the daily rate during transport on that of the stationary chronometer and putting  $\tau$  for the duration of a transport, we have as supplementary correction of the chronometer after each journey

$$\Delta \text{ corr.} - \Delta_{\text{stat}} \text{ corr.} = \text{suppl. corr.} = \tau. E.$$

Now  $\Delta \text{ corr.}$  could be determined from the time-determination next preceding and next following the transport, and yet be found, for the mean of two journeys to and fro, independent of an assumed value of the difference of longitude, while  $\Delta_{\text{stat}} \text{ corr.}$  could be derived from the daily rates in the intervals next preceding and next following the transport.

In this way we found for the suppl. corr. after each transport:

1 <sup>st</sup> journey to M. and back	+ 2 <sup>s</sup> 22
2 <sup>nd</sup> " " " " "	+ 1.54
3 <sup>rd</sup> " " " " "	+ 1.60

Mean influence of one single journey + 1<sup>s</sup>79

i. e. the transport caused a retardation. This value was employed to correct the data for solutions III and V.

The solutions IV and V were executed not to give undue weight to the 3<sup>rd</sup> stay at Mecca with 6 observation-nights, overagainst the 1<sup>st</sup> and 2<sup>nd</sup> with 3 and 4 nights, since for each stay there are clearly left systematic errors. Febr. 26 was left out in all solutions. The 3 solutions gave for the difference of longitude.

I	+ 2 <sup>m</sup> 33 <sup>s</sup> .73	(± 1.84)
II	33.80	(± 1.85)
III	33.92	(± 1.58)
IV	34.23	(± 1.73)
V	34.38	(± 1.47)

The mean errors in brackets have the same meaning as above; in solutions IV and V they refer to observations with weight unity. Of all these solutions the 5<sup>th</sup> seems to me certainly to be preferable. I have, however, communicated also the other results, since they show the influence of the different ways of treating the observations. On the other hand I shall not give the results of a discussion of 2 successive journeys to Mecca together. The thus obtained formulae do not represent the observations better than the formulae deduced from the 3 journeys together.

The final result for Dent 2527 I should like to deduce as follows:

The 3 journeys J.—M.—J.	1 <sup>st</sup> meth.	+ 2 <sup>m</sup> 35 <sup>s</sup> .26
	2 <sup>nd</sup> meth.	34.72
	Mean	+ 2 <sup>m</sup> 34 <sup>s</sup> .99
The 2 journeys M.—J.—M.		+ 2 34.73
	Mean	+ 2 <sup>m</sup> 34 <sup>s</sup> .86
General solution		+ 2 34.38
Adopted final result		+ 2 <sup>m</sup> 34 <sup>s</sup> .62

(To be continued).

**Astronomy.** — “*Determination of the geographical latitude and longitude of Mecca and Jidda executed in 1910—11.*” By Mr. N. SCHELTEMA. Part III. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN.)

(Communicated in the meeting of September 28, 1912).

#### 6. *Derivation of the difference of longitude Jidda-Mecca.*

(Continued).

##### b. *Watch N<sup>o</sup>. 7.*

Watch N<sup>o</sup>. 7 was taken on the 2<sup>nd</sup> and 3<sup>rd</sup> journeys to Mecca. During the whole period of the observations it clearly showed a progressive acceleration. Any direct influence of the transport, however, was not clearly visible; nor was this so much to be feared for our carefully transported pocket-chronometers as for the box-chronometer of Dent.