## Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

## Citation:

Scheltema, N., Determination of the geographical latitude and longitude of Mecca and Jidda, accomplished in 1910-1911. Part III, in:
KNAW, Proceedings, 15 I, 1912, 1912, pp. 556-572

This PDF was made on 24 September 2010, from the 'Digital Library' of the Dutch History of Science Web Center (www.dwc.knaw.nl)
> 'Digital Library > Proceedings of the Royal Netherlands Academy of Arts and Sciences (KNAW), http://www.digitallibrary.nl'

556

| I | $+2^{\mathrm{m}} 33^{\mathrm{s} 73}$ | $( \pm 1884)$ |
| ---: | ---: | ---: |
| II | 33.80 | $( \pm 1.85)$ |
| III | 33.92 | $( \pm 1.58)$ |
| IV | 34.23 | $( \pm 1.73)$ |
| V | 34.38 | $( \pm 1.47)$ |

The mean errors in brackets have the same meaning as above; in solutions IV and $V$ they refer to observations with weight unity. Of all these solutions the $5^{\text {th }}$ seems to me certainly to be preferable. I have, however, communicated also the other results, since they show the influence of the different ways of treating the observations. On the other hand I shall not give the results of a discussion of 2 successive journeys to Mecca together. The thus obtained formulae do not represent the observations better than the formulae deduced from the 3 journeys together.

The final result for Dent 2527 I should like to deduce as follows:

| The $\mathbf{3}$ journeys J.-M.-J. | $1^{\text {st }}$ meth. <br> $2^{\text {nd }}$ meth. | $\begin{array}{r} +2^{\mathrm{m}} 35 \times 26 \\ 34.72 \end{array}$ |
| :---: | :---: | :---: |
|  | Mean | $+2^{\mathrm{m}} 34.99$ |
| The 2 journeys M.-J.-M |  | +234.73 |
|  | Mean | + $2^{\mathrm{m}} 34^{\text {¢ }} 86$ |
| General solution |  | +234.38 |
| Adopted tinal result | . | + $\mathbf{2 m}^{\text {m }} 34^{\text {m }} 6$ |

(To be continued).

Astronomy. - "Determination of the geographical latitude and longitule of Mecca and Jidda executed in 1910-11." By Mr. N. Scheitema. Part III. (Communicated by Frof. E. F. van de Sande Bakhuyzen.)
(Communicated in the meeting of Seplember 28, 1912).
6. Derivation of the difference of longitule Jidda-Mecca.
(Continued).
b. Watch $N^{\circ} .7$.

Watch $\mathrm{N}^{0} .7$ was taken on the $2^{\text {nd }}$ and $3^{\text {rd }}$ journeys to Mecca. Diring the whole period of the observations it clearly showed a progrossive acceleration. Any direct influence of the transport, however, was not clearly visible; nor was this so much to be feared for our canefilly transported pocket-chronometers as for the box-chronometer of Dent.

From the observations with this watch results for the difference of longitude have again been derived in different ways.

1. From the separate journeys.
a. Journeys Ji.-Me.-Ji.; Jidda-time interpolated between the last time-determination before and the first after the journey.
$2^{\text {nd }}$ journey
$+2^{\mathrm{m}}$
Febr. $24 \quad 35^{\star} 08$
,, $25 \quad 34.92$
,, $26 \quad 33.89$
" $27 \quad 34.46$
$+2^{\mathrm{m}} 34^{\mathrm{s}} 59$
Omitting Febr. $26 \quad 34.82$
$3^{\text {rd }}$ journey
$+2^{m}$
March $11 \quad 36 \times 39$
„ 1237.00
, $\quad 14 \quad 37.79$
,, $15 \quad 35.50$
, $\quad 1634.33$
Mean $\quad \begin{gathered}17 \\ +2^{\text {m }} \\ \\ 35^{6} 93\end{gathered}$
b. Journey Me_Ji-Me; discussed in the same manner.

$$
\begin{array}{rrrr}
\text { March } 2 & +2^{\mathrm{m}} & 32^{\mathrm{s}} 27 \\
" & 3 & & 33.13 \\
" & 7 & & 34.20 \\
" & 8 & & 34.72 \\
\cline { 3 - 4 } & +2^{\mathrm{m}} & 33^{5} 58
\end{array}
$$

Mean
c. Journeys $J i-M e-J i$; all observations (including Febr. 26) represented by linear formulae

$$
\begin{array}{lrl}
2^{\text {nd }} \text { journey } & +2^{\mathrm{m}} 35^{\mathrm{s}} 22 & \left( \pm 2^{\mathrm{s} 48}\right) \\
3^{: \mathrm{d}} & 36.46 & ( \pm 1.05)
\end{array}
$$

As the formulae for the second journey to Mecca represent the observations unsatisfactorily, I adopt as the result of this journey that of the direct interpolation excluding Febr. 26; for the 3rd I adopt the mean of the results $a$ and $c$, hence:

$$
\begin{array}{r}
2^{\text {nd }} \text { journey }+2^{\mathrm{m}} 34^{\mathrm{s} 82} \\
3^{\text {rd }} \\
M \text { Mean }+2^{\frac{\mathrm{m}}{\mathrm{~m}} 36.20}
\end{array}
$$

Combining this result with equal weights with result $b$ we obtain $+2^{\text {m }} 34^{8} 54$.
2. From general solutions by means of quadratic formulae.

Of such solutions based on the whole material four have been executed; I and II respectively excluding and including Febr. 26; III and IV as I and II but giving half weight to the six observations during the $3^{\text {rd }}$ stay at Mecca, for the same reason as in the case of Dent.

In this manner we found

| I $+2^{\mathrm{m}} 34.64$ | $( \pm 0.90)$ |  |
| ---: | ---: | ---: |
| II | 34.59 | $( \pm 0.88)$ |
| III | 34.77 | $( \pm 0.80)$ |
| IV | 34.70 | $( \pm 0.78)$ |

I adopted the mean of the results III and IV, viz. $+2^{m 34} 34^{5}$ and then as final result for watch $\mathrm{N}^{\mathrm{o}} .7$ the mean of the results from $\left(1^{\circ}\right)$ the individual journeys and,$^{\circ}$ ) the whole material together

$$
+2^{n} 3464
$$

c. Watch $\mathrm{N}^{\circ} .77$.

This watch was taken on the $2^{\text {nd }}$ journey to Mecca, but mufortunately it stopped between the observations of Febr. 24 and 25, as it had been forgotten to be wound. For a comparison of the corrections determined at Mecca with corrections to Jidda-time we can therefore only use extrapolated values .

So we found:

| Febr. 24 | $+2^{m 11} 33.41$ |  |
| :---: | ---: | ---: |
| $"$ | 25 | 36.77 |
| $"$ | 26 | 35.10 |
| $"$ | 27 | 34.57 |
| Mean I | $+2^{2 n} 34.96$ |  |
| $"$ | 11 | 34.56 |
| $"$ | III | 34.51 |

The first mean value was obtained by giving equal weights to the 4 days; for the $2^{\text {nd }}$ we adopted weights inversely proportional to the interval of time, for which extrapolation had taken place; for the formation of the $3^{\text {rd }}$ we moreover gave half weight to Febr. 26.

After all it seemed best to ignore the smaller weight of the last-mentioned time-determination, but to take into account the intervals of extrapolation and I therefore adopt as final result:

$$
+2^{\mathrm{m}} 34^{\mathrm{s}} 56
$$

d. Watch $\mathrm{N}^{\circ} .80$.

This watch was taken to Mecea on the $1^{\text {st }}$ and $3^{\text {rd }}$ journeys. It did not seem advisable to immediately connect these two, which are separated by an interval of nearly one month. There is the same objection against forming the combination $\mathrm{Me}-\mathrm{Ji}-\mathrm{Me}$. Hence we can only discuss two journeys Ji -Me-Ji each by itself, but then we meet with the difficulty that with linear interpolation the results are not free from the influence of a progressive variation in the daily
rate of the watch. If e.g. a daily acceleration of $0^{3} .10$ takes place, then a linear interpolation in the middle between two time-determinations with an interval of 12 days will yield a result that is $1^{\text {s }} .8$ in error and from a journey $\mathrm{Ji}-\mathrm{Me}-\mathrm{Ji}$ the difference of longitude will be found so much too great.

Now it appears, however, that the change of rate of $\mathrm{N}^{\circ} .80$ was more complicated. When it was transported after a period of rest, it showed a considerable acceleration and then it continued for some time to show this accelerated rate unaltered. In such a case the error committed by linear interpolation will be much smaller, but it will not be easy to account for. Finally I have deduced results by means of quadratic formulae as well as by linear interpolation.

We thus found :

## $1^{\text {st }}$ Journey.

a. By interpolation between the last preceding and the next following observations at Jidda

$$
\text { Febr. } 14+2^{\mathrm{m}} 35^{\mathrm{s} 75}
$$

$15 \quad 35.83$
$16 \quad 35.48$
Mean $\quad+2^{m} 355^{s} 69$
b. By linear formulae $+2^{\mathrm{m}} 377^{\mathrm{s} 85} \quad\left( \pm 1^{\mathrm{s} 63}\right)$
c. By quadratic formulae $+2^{\mathrm{m}} 35^{\mathrm{s}} 05$
(土 0s72)
As the linear formulae represent the observations very unsatisfactorily, the result thus obtained was rejected and we adopted as the result yielded by the $1^{\text {gt }}$ journey the mean of the results $a$ and $c$

$$
+2^{\mathrm{n}} 35^{\mathrm{s}} .37
$$

## $3^{\text {rd }}$ Journey.

a. By interpolation between the nearest ubservations at Jidda

March $11+2^{\mathrm{m}} 34^{5} 51$
$12 \quad 34.51$
$14 \quad 35.49$
$15 \quad 35.80$
$16 \quad 35.44$
$17 \quad 35.33$
Mean $\quad+2^{\mathrm{m}} 35^{\mathrm{s} 18}$
b. By linear formulae $\quad+2^{\mathrm{m}} 35^{\mathrm{s}} 69 \quad\left( \pm 0^{\mathrm{s}} 59\right)$
c. By quadratic formulae $+2^{\mathrm{m}} 33^{3} 92$ ( $\pm 0^{\mathrm{s}} 49$ )

For the solutions $b$ and $c$ only March 7 and 8 were used as first Jidda-group, as the next preceding observations are 4 days earlier.

I adopted the mean of the 3 results $a, b$ and $c$

$$
+2^{\mathrm{m}} 34^{\mathrm{s} 93}
$$

As the final result yielded by $\mathrm{N}^{0} .80 \mathrm{I}$ adopt the mean of the nesults from the two journeys

$$
+2^{\mathrm{m}} 35^{\mathrm{s}} 15
$$

e. Watch No. 81.

This watch was taken on the first journey to Mecca. About that time it seems to have gone fairly regularly.

The following results were found for the difference of longitude.
First we obtain by means of comparison of the results obtained at Mecca with those interpolated between the last preceding and the next following observations at Jidda:

| Febr. 14 | $+2^{\mathrm{m}} 36^{\mathrm{s} 74}$ |
| ---: | ---: |
| 15 | 36.17 |
| 16 | 33.99 |
| Mean | $+2^{\mathrm{m}} 35^{8} 63$ |

Further 4 general solutions have been executed, 1 and 111 by quadratic, II and IV by linear formulae. Only for III and IV the deviating result of Febr. 21 at Jidda was excluded.

| I $+2^{\mathrm{m}} 355^{81}$ | $( \pm 0 \mathrm{O} 88)$ |
| ---: | ---: |
| II | 35.39 |
| III | 35.78 |
| IV | 35.22 |

It appears here, as before, that the exclusion of a deviating timedetermination between others has but little influence. As results from the quadratic and from the linear formulae I adopt the mean of I and III and that of II and IV

$$
\begin{array}{ll} 
& +2^{\mathrm{m}} 35^{\mathrm{s}} 80 \\
\text { and } & +235.30
\end{array}
$$

and as final result the mean of the results obtained by the three methods, direct interpolation, linear and quadratic formulae

$$
+2^{\mathrm{m}} 35^{\mathrm{s}} 58
$$

$f$. Watch $\mathrm{N}^{0} .84$.
This was taken on the $3^{\text {rd }}$ journey. According to the investigation of the results at Jidda its rate was very regular; it showed no progressive variation and only small accidental deviations. During the stay at

Mecca, however, it had a daily rate, in the mean +0.78 , which differed much from that at Jidda and was moreover very irregular, while during the journers themselves the rate seems to have had about the same value as at Jidda.

The following results were derived from this watch:
$1^{\text {st }}$ by interpolation hetween the nearest determinatons of time at Jidda.

| March | 11 | $+2^{\mathrm{m}} 32 \mathrm{~s} 10$ |
| :---: | ---: | ---: |
| $"$, | 12 | 32.38 |
| $"$ | 14 | 34.52 |
| $"$ | 15 | 35.33 |
| $"$ | 16 | 33.77 |
| Mean |  | 17 |

$2^{\text {nd }}$ by means of quadratic formulae. Using also the observations of March 2 and 3 , which are furtier away from the time of the journey, or learing them out, we obtained

$$
\begin{aligned}
+2^{\mathrm{mi}} 34^{\mathrm{s} 72} & \left( \pm 1^{\mathrm{s}} 03\right) \\
32.83 & ( \pm 1.08)
\end{aligned}
$$

or
$3^{\text {rd }}$ We obtained in the same two manners by means of linear formulae:

|  | $+2^{\text {mi }}$ | $33^{\circ} 08$ |
| :--- | ---: | :--- |
| 33.64 | $( \pm 1 * 27)$ |  |
| or | $( \pm 1.02)$ |  |

Of the results by the quadratic formulae the mean of the two has been adopted: of those by the linear formulae it seemed best to adopt the second.

Giving finally equal weight to the results of the 3 methods, the final result becomes;

$$
+2^{\mathrm{m}} 33^{\mathrm{s}} 70
$$

differing rather much from those yielded by the other watches.
g. General result from the 6 watches.

For the derivation of the difference of longitude according to each of the employed watches, given in the preceding paragraphs, different methods of calculation were followed, which had all of them special advantages and disadvantages, and in most cases the mean of the results found by these different methods was adopted. Naturally in this procedure some arbitrariness could not be avoided; its influence on our final results, however, will not be great.

Proceedings Royal Acad. Amsterdam. Vol. XV.

Now there remains to be decided what weight must be given to each of the 6 results. Although it seemed at first that for each watch we should have to adopt a different aceuracy peculiar to it, it appeared after all very difficult to determine this intrinsic accuracy. So e.g. for $\mathrm{N}^{\circ} .84$ we should have had to adopt a rather high weight according to the observations at Jidda and yet it went very irregnlarly during the journey to Mecca. So ultimately I adopted the same intrinsic accuracy for each of the watches; nor could greater weight be given to the chronometer of Dent, as a travelling instrument, than to Leroy's watches.

Hence I have given weights to the 6 results proportional to the number of journeys in which each watch had been used, and besides only a weight of 0.5 to watch 77 owing to the discontinuity during the stay at Mecea.
so 1 obtained :
Difference of longitcde Jidda-Mecca.

| Dent | 2527 | $+2^{\mathrm{m}} 34^{8} 62$ | Weight 3 |  |
| :---: | ---: | ---: | ---: | :--- |
| Watch | 7 | 34.64 | $"$ | 2 |
| $"$ | 77 | 34.56 | $"$ | 0.5 |
| $"$ | 80 | 35.15 | $"$ | 2 |
| $"$ | 81 | 35.58 | $"$ | 1 |
| $"$ | 84 | 33.70 | $"$ | 1 |

From the agreement of these six values inter se there follows as mean error for weight unity $\pm 0 \approx .66$ and the final result from the 6 watches is found to be

## Difference of longitude $+2^{\mathrm{m}} 34{ }^{\mathbf{n} 74} \pm 0^{\circ} 22$.

It is clear from the foregoing that a derivation of the difference of longitude from partial results for each journey must lead to a less advantageous combination of the observations. Yet I want to show that a final result obtained in this way does not differ much from the above given.

We then obtain, indicating the journeys Ji.-Me.-Ji. and Me.-Ji.-Me. respectively by M. and J.

|  | M I | M II | M III | J I | J Il |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Dent | $36 \times 40$ | 35s30 | $33 \times 28$ | $35{ }^{88}$ | $33{ }^{5} 57$ |
| 7 |  | 34.82 | 36.20 |  | 33.58 |
| 77 |  | 34.56 |  |  |  |
| 80 | 35.37 |  | 34.93 |  |  |
| 81 | 35.58 |  |  |  |  |
| 84 |  |  | 33.70 |  |  |
| Mean | $35^{578}$ | $34 \times 96$ | $34{ }^{553}$ | 35*89 | 3358 |

Combining these 5 results with equal weights we should find

$$
+2^{\mathrm{m}} 34^{〔} .95
$$

which agrees with the final result adopted by us just within the limit of its mean error.
7. Reduction of the results to known points in the two cities.

Longitude of Mecca from the meridian of Greenwich.
The situation of the observation-station at Jidda, (the Dutch consulate) relative to the Mecca-gate has been measured four times by Mr. Salam by means of the determination of the direction and the length of the 4 parts of the road, the first by the boussole, the last by counting the steps, the length of which was found to be equal to $0^{m} .768$. To the directions counted from magnetic North to East first of all must be added, to reduce them into astronomical azimuths, the magnetic declination for which, according to the English admiralty chart, - $2^{\circ} .4$ was adopted. The thus corrected results, however, appear to be still in need of a correction of $+1^{\circ} .6^{\prime}$ ) according to the results obtained with the same instrument about the road from Jidda to Mecra, of which we shall treat hereafter. So the total correction of the lectures of the boussole was - $0^{\circ} .8$.

From the directions and distances the rectangular co-ordinates have been derived taking as axes the parallel and the meridian. The mean results of the 4 measurements expressed in meters were as follows (see Plate II, tig. I, the scale of which has been given in hectometers):

|  | $\Delta x$ | $\Delta y$ |
| :---: | :---: | :---: |
| $b-a$ | $-94^{\mathrm{m}}$ | $+350^{\mathrm{m}}$ |
| $c-b$ | -313 | +92 |
| $d-c$ | 0 | -14 |
| $e-d$ | -58 | -27 |
| Sum | $-465^{\mathrm{m}}$ | $+401^{\mathrm{m}}$ |

The sums of the two first $\Delta x$ and $\Delta y$ give as co-ordinates of the Medina-gate relative to the Mecca-gate - $407^{m}$ and $+442^{m}$, for which is found $-368^{\mathrm{m}}$ and $+349^{\mathrm{m}}$ according to the English ad-miralty-chart Sidda with its approaches. I adopted the mean of the two results and thus found as co-ordinates of the Dutch consulate relative to the Mecca-gate:
${ }^{1}$ ) The accurate result is $+2 \cdot .0$, but this difference is here immaterial.

$$
\left.\Delta x=-446^{\mathrm{m}} \quad \triangle y=+355^{\mathrm{m}}\right)
$$

or expressed in seconds of longitude and latitude :

$$
\angle \lambda=15^{\prime \prime} .5 \text { West } \quad \Delta y=11^{\prime \prime} .5 \text { North. }
$$

For the determination of the relative sittation of the observationstation Mecca and the Kabah we have a report of Mr. Salm that the latter is at 187 steps S.S.W. of the former. After this indication it has heen tried to identify the observation-spot in the plan of Mecca by Bercehardt, as revised by Snocek Htrgronse, of which the scale has been given in steps. As the most likely place we have found the spot indicated on Plate II, Fig. 2, as point III, which is situated at a distance of 192 steps from the Kabah, indicated as point I, in a direction $13^{\circ}$ East of the North.

The rectangular co-ordinates of the observation-station reiative to the Kabah thus are:
or

$$
\begin{array}{ll}
\triangle x=+35^{\mathrm{m}} & \triangle y=+144^{\mathrm{m}} \\
\Delta \lambda=1^{\prime \prime} .2 \text { East } & \triangle y=4^{\prime \prime} .7 \text { North }
\end{array}
$$

As regards the mean errors of the values for $\Delta \lambda$ and $\Delta y$, I believe that they are not undervalued if we adopt $\pm 2$ ".5 for both co-ordinates for Jidda and $\pm 1$ ". 5 for Mecca.

So we obtain:

```
Latitude Jidda Mecca-gate \(21^{\circ} 29^{\prime \prime} 5 \pm 27\)
Latitude Mecca \(\mathrm{Ka}^{*}\) bah \(\quad 21^{\circ} 25^{\prime} 18^{\prime \prime} 4 \pm \mathbf{2}^{\prime \prime} 1\)
Difference of longitude \(38^{\prime} 24^{\prime \prime} 4 \pm 4^{\prime \prime} 4=2^{m} 33^{s 63} \pm 0 \cdot 29\)
```

The thus obtained latitude may be compared with the result found in 1876 by Comm. Wharton, on which the Adm. chart is based.

The point determined by him is situated on Geziret el Mifsaka and the latitude he found was $+21^{\circ} 28^{\prime} 0^{\prime \prime}$. Further we find by measurement on the chart that the latitude of the Merca-gate is $2197^{\mathrm{m}}=$ $=1^{\prime} 11^{\prime \prime} .4$ greater. Hence it becomes $21^{\circ} 29^{\prime} 11^{\prime \prime} .4$ i. e. $6^{\prime \prime}$ greater than the value found by us, an agreement which, if we take into account the reductions that had to be added to both results before their comparison, may be considered satisfactory.

In the second place Whartos's result for the longitude of Jidda may be employed to determine the longitude of Mecca from the meridian of Greenwich. He found as longitude of his observationspot $39^{\circ} 11^{\prime} 25^{\prime \prime}$; according to the chart the Mecca-gate is situated

[^0]$2010 \mathrm{~m}=19^{\prime \prime} 8$ further eastward, hence:
Longitude Jidda. Mecea-gate $39^{\circ} 12^{\prime} 34^{\prime \prime} .8=2^{\text {h }} 36^{\mathrm{m}} 50^{\prime} .32$
and consequently :
Longitude Mecca Ka"bah $39^{\circ} 50^{\prime} 59^{\prime \prime} .2=2^{h} 39^{\mathrm{n}} 233^{3} .95$.
We have still attempted to find out the basis of Wharton's longitude and whether the telegraphic determinations of the longitude of Aden and Suez have been employed for it. But although Rearadmiral (. J. de Jong, Chief of the Dutch Dept. of Hydrography has with the greatest kindness put all the available charts and other data at our disposal, we have not succeeded in obtaining certainty.

Direct data as to the basis of Whartos's longitude were not to be found. Then we tried to attain our end by consulting the charts of Aden, Suez and Alexandria and by comparing the longitudes given there with the results of the telegraphic determinations. These had been executed principally in connection with the transit of Venus in 1874, and have been discussed by Airy ${ }^{1}$ ), Coreland ${ }^{2}$ ) and Auwers ${ }^{3}$ ). We did not find any certainty in this way either, as in the first place it is not sare that the bases for the longitude in the different charts agree inter se, and moreover uncertainty as to the exact situation of the observation-spots prevented an accurate determination of the longitude errors of the charts.

Therefore it seemed impossible to find out with any probability the correction needed for the longitude of Jidda adopted in the Adm. Chart. Consequently I must regard the longitude of Mecea as deduced above, as the most reliable value for the present.

In order to find the total mean error of the longitude of Mecca from the meridian of Greenwich, we should have to know that of the adopted longitude of Jidda, and it is impossible even to estimate this. In the total mean error an manown value $m_{. J}$ has therefore been included.

Thas our final results for the geographical position of Mecca Kabah become

Latitude $+21^{\circ} 25^{\prime} 18^{\prime \prime} .4 \pm 2^{\prime \prime} .1$
Longitude $\quad 39^{\circ} 50^{\prime} 59^{\prime \prime} .2 \pm V^{\prime}\left(4^{\prime \prime} .4\right)^{2}+m . r^{2}$. or $2^{h} 39^{\mathrm{m}} 23^{\mathrm{s}} .95 \pm \sqrt{\left(0^{2} .29\right)^{2}+m .7^{2}}$ East of Greenw.

[^1]The results obtained by J. Hess from the itinerary Jidda-Mecea were : ${ }^{1}$ )
Latitude $\quad 21^{\circ} 21^{\prime} .7 \pm 31.8$
Longitude $39^{\circ} 52^{\prime} .5 \pm 3.2$
i. e. agreeing with our result within the given mean errors, which are still rather considerable. So we may conclude that by our work, the accuracy attained has been a great deal increased.

Finally we shall compare our results for Jidda and Mecea with those of Al.i Ber. The following are the corrections needed by the latter

|  | $\Delta y$ | $\Delta \lambda$ |
| :--- | :---: | :---: |
| Jidda | $-3^{\prime} 36^{\prime \prime}$ | $+6^{\prime} 35^{\prime \prime}$ |
| Mecca | $-2^{\prime} 51^{\prime \prime}$ | $-24^{\prime} 1^{\prime \prime}$ |

Aln Bey's errors in latitude are for both places about - $3^{\prime}$; the errors in longitude are great and irregular, which need not surprise us, as they are the resilts from observed lumar distances (for the longitude of Jidda also 2 observations of eclipses of Jupiter-satellites have been taken into account. Aur Bey also determined the latitude of a number of other places in Arabia. Perhaps these also need a correction of about - $3^{\prime}$ and may then be fairly reliable. I dare not, however, decide this question here.

## 8. Road from Jidda to Mecca.

In this last paragraph I shall discuss the results obtained by Mr. Saim in surveying the road between the two places, on a journey on foot from Mecca to Jidda, undertaken for this special purpose. This survey was made by means of observations with the boussole and by counting the steps.
The boussole, of which mention has been made before, was a very handy little instrument by Casella, belonging to the Leyden observatory. It has a little telescope in which also the divisions of the azimuthcircle (full degrees) are made visible by reflexion.
The following table contains the results of these observations and their further reduction. The road was divided into 92 parts, for each of which the direction and the length were determined. For the two terminal and 13 of the 91 intermediate points special names could be indicated in the first column. The $2^{\mu d}$ and $3^{d}$ columns contain for each of the parts of the road the direction $a$ read on the boussole and the length $l$ expressed in steps.

The directions a are counted from magnetic North to East, South

[^2]|  | $a$ |  | $\stackrel{x}{=l^{\prime} \sin a^{\prime}}$ | $\Sigma \stackrel{y}{\stackrel{y}{=}}$ | $\begin{array}{c\|} x \\ \text { corrected } \end{array}$ | corrected |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 Entrance Mecca |  |  |  |  |  |  |
| 1-2 | $337^{\circ}$ | 496 | - 166 | $+350$ | - 130 | + 299 |
| 2 Kahwat al-mu'allim |  |  |  |  |  |  |
| 2-3 | 245 | 606 | - 586 | + 132 | - 489 | + 128 |
| 3-4 | 291 | 760 | - 1148 | + 321 | - 956 | + 304 |
| 4-5 | 240 | 320 | - 1359 | + 187 | - 1138 | + 197 |
| 5-6 | 264 | 230 | - 1536 | + 161 | - 1287 | + 180 |
| 6-7 | 307 | 595 | - 1918 | + 425 | - 1601 | $+413$ |
| 78 | 306 | 2090 | - 3276 | + 1327 | - 2716 | $+1212$ |
| 8-9 | 288 | 1418 | - 4341 | + 1624 | - 3602 | + 1493 |
| 9-10 | 265 | 2100 | - 5966 | + 1413 | - 4976 | + 1363 |
| 10-11 | 275 | 484 | - 6344 | + 1430 | - 5292 | + 1388 |
| 11 Umm ed-dûd |  |  |  |  |  |  |
| 11-12 | 275 | 699 | - 6888 | $+1454$ | - 5749 | + 1425 |
| 12-13 | 227 | 959 | -7413 | + 921 | - 6206 | + 991 |
| 13-14 | 243 | 370 | - 7664 | + 779 | - 6421 | + 880 |
| 14-15 | 252 | 611 | - 8110 | + 613 | - 6801 | + 753 |
| 15-16 | 236 | 818 | - 8623 | + 235 | - 7244 | + 451 |
| 16-17 | 267 | 1284 | - 9621 | + 141 | - 8086 | + 401 |
| 17 Maktala |  |  |  |  |  |  |
| 17-18 | 267 | 1623 | - 10881 | + 22 | - 9148 | + 337 |
| 18-19 | 235 | 1278 | $-11672$ | - 583 | - 9832 | - 148 |
| 19-20 | 249 | 2907 | $-13754$. | - 1483 | - 11608 | - 844 |
| 20-21 | 246 | 1254 | $-14631$ | $-1918$ | $-12359$ | - 1184 |
| 21 Kahwat Sâlim |  |  |  |  |  |  |
| 21-22 | 255 | 1400 | $-15673$ | - 2245 | $-13244$ | - 1427 |
| 22-23 | 289 | 2531 | - 17565 | - 1680 | $-14819$ | - 898 |
| 23 Alameyn |  |  |  |  |  |  |
| 23-24 | 298 | 4710 | $-20878$ | - 93 | $-17559$ | $+534$ |
| 24-25 | 314 | 1150 | $-21549$ | + 503 | $-18105$ | $+1055$ |
|  |  |  |  |  |  |  |


|  | $\%$ |  | $\leq l \stackrel{x}{\stackrel{x}{\sin } x}$ | $\stackrel{y}{\stackrel{y}{=}}$ | $x$ corrected | corrected |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 25 Shumesi |  |  |  |  |  |  |
| 25-26 | $315^{\circ}$ | 1427 | - 22368 | $+1256$ | $-18772$ | $+1712$ |
| 26-27 | 319 | 1102 | - 22958 | + 1881 | - 19250 | $+2255$ |
| 27-28 | 280 | 402 | - 23268 | $+1922$ | - 19509 | $+2298$ |
| 28-29 | 252 | 350 | - 23524 | +1827 | - 19727 | $+2226$ |
| 29-30 | 269 | 806 | -24151 | $+1790$ | - 20256 | +2213 |
| 30-31 | 258 | 922 | - 24848 | $+1611$ | - 20847 | + 2084 |
| 31-32 | 250 | 687 | - 25343 | $+1407$ | - 21269 | + 1926 |
| 32-33 | 246 | 614 | - 25772 | $+1194$ | - 21636 | + 1760 |
| 33-34 | 255 | 30 | - 26018 | + 1117 | $-21845$ | $+1702$ |
| 34-35 | 265 | 970 | - 26768 | +1019 | - 22478 | $+1642$ |
| 35 small kahwah |  |  |  |  |  |  |
| 35-36 | 250 | 567 | $-27177$ | + 851 | $-22827$ | $+1513$ |
| 36-37 | 272 | 2189 | - 28884 | + 839 | - 24263 | $+1552$ |
| 37 Hadda |  |  |  |  |  |  |
| 37-38 | 268 | 1202 | - 29818 | + 766 | $-25051$ | $+1518$ |
| 38-39 | 239 | 443 | - 30107 | + 576 | - 25299 | $+1367$ |
| 39-40 | 250 | 845 | - 30717 | + 325 | - 25819 | $+1174$ |
| 40-41 | 244 | 772 | - 31247 | + 39 | - 26274 | + 949 |
| 41-42 | 234 | 336 | - 31452 | - 124 | - 26451 | + 818 |
| 4243 | 250 | 906 | - 32106 | - 393 | - 27009 | + 611 |
| 43-44 | 257 | 1994 | $-33605$ | - 806 | - 28281 | + 307 |
| 44-45 | 235 | 706 | - 34043 | - 1141 | - 28659 | $+\quad 39$ |
| 45-46 | 230 | 785 | -- 34495 | - 1554 | - 29052 | - 296 |
| 46-47 | 225 | 1930 | -35514 | - 2662 | - 29941 | - 1197 |
| 47-48 | 223 | 506 | - 35771 | - 2962 | - 30166 | - 1442 |
| 48-49 | 217 | 491 | $-35989$ | - 3277 | - 30358 | $-1701$ |
| 49-50 | 220 | 2782 | $-37313$ | -4995 | - 31521 | -3116 |
| 50 Bahra |  |  |  |  |  |  |
| 50-51 | 240 | 277 | - 37495 | $-5110$ | -31679 | $-3198$ |
| 51-52 | 225 |  | $-37663$ | - 5293 | -31825 | -3347 |


|  | 2 | $l$ | $\stackrel{x}{=}$ | $\stackrel{y}{\sum l^{\prime} \cos x^{\prime}}$ | $x$ corrected | $y$ corrected |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 52-53 | $240^{\circ}$ | 250 | $-37828$ | - 5398 | -- 31966 | $-3430$ |
| $53 \quad 54$ | 238 | 755 | $-38314$ | - 5731 | - 32385 | - 3696 |
| 54-55 | 250 | 1153 | - 39146 | - 6073 | - 33095 | - 3959 |
| 55-56 | 261 | 1205 | - 40067 | -6259 | - 33875 | -4088 |
| 56-57 | 280 | 4594 | - 43618 | - 5786 | - 36847 | - 3586 |
| 57-58 | 270 | 150 | - 43735 | - 5791 | - 36946 | - 3587 |
| 58--59 | 281 | 519 | $-44135$ | - 5730 | - 37280 | - 3524 |
| 59 Kahwat al-abd |  |  |  |  |  |  |
| 59-60 | 281 | 210 | - 44297 | - 5705 | - 37415 | $-3498$ |
| 60-61 | 253 | 356 | -- 44559 | - 5797 | - 37638 | -- 3569 |
| 61-62 | 317 | 1056 | - 45146 | - 5219 | - 38115 | - 3065 |
| 62-63 | 312 | 583 | - 45496 | --4930 | - 38401 | - 2811 |
| 63-64 | 297 | 502 | - 45852 | -4767 | - 38695 | - 2664 |
| 64-65 | 305 | 2296 | - 47360 | - 3801 | - 39935 | - 1807 |
| 65-66 | 319 | 1392 | $-48106$ | - 3012 | - 40539 | $-1123$ |
| 66-67 | 295 | 776 | - 48664 | $-2780$ | - 41002 | - 911 |
| 67-68 | 250 | 1743 | - 49921 | $-3298$ | - 42074 | $-1309$ |
| 68-69 | 267 | 780 | - 50527 | - 3355 | - 42585 | $-1340$ |
| 69-70 | 295 | 1733 | - 51775 | - 2836 | -43619 | - 867 |
| 70-71 | 292 | 610 | $-52223$ | - 2677 | - 43991 | $-720$ |
| 71 Kattâna |  |  |  |  |  |  |
| 71-72 | 306 | 438 | - 52508 | $-2488$ | -44226 | - 552 |
| $72-73$ | 290 | 1575 | - 53679 | $-2117$ | - 45199 | - 206 |
| 73-74 | 275 | 317 | - 53926 | $-2106$ | - 45407 | - 190 |
| 74-75 | 295 | 453 | - 54252 | - 1970 | - 45677 | - 66 |
| 75-76 | 296 | 390 | $-54530$ | - 1848 | - 45907 | + 45 |
| 76-77 | 290 | 1490 | - 55638 | $-1497$ | - 46828 | + 372 |
| 77-78 | 310 | 1084 | - 56308 | - 981 | -47377 | + 826 |
| 78 Jarâda |  |  |  |  |  |  |
| 78-79 | 310 | 1930 | $-57501$ | - 63 | - 48353 | $+1633$ |
| 79-80 | 280 |  | $-57822$ | - 20 | 1-48622 | $+1678$ |


|  | $a$ | $l$ | $\leq \frac{x}{\stackrel{x}{\sin } a^{\prime}}$ | $\stackrel{\stackrel{y}{=}}{\Sigma l^{\prime} \cos z^{\prime}}$ | $\begin{gathered} x \\ \text { corrected } \end{gathered}$ | $\begin{gathered} y \\ \text { corrected } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 80-81 | $265^{\circ}$ | 1470 | $-58959$ | - 167 | $-49582$ | + 1589 |
| 81-82 | 256 | 1225 | - 59875 | - 438 | - 50360 | $+1388$ |
| 82-83 | 280 | 3400 | $-62503$ | - 88 | - 52560 | +1759 |
| 83 Raghàma |  |  |  |  |  |  |
| 83-84 | 288 | 7024 | -67779 | + 1385 | - 56952 | $+3152$ |
| 84-85 | 299 | 5866 | - 71869 | $+3435$ | - 60332 | $+4995$ |
| 85-86 | 290 | 1233 | - 72785 | $+3725$ | - 61094 | +5266 |
| 86-87 | 289 | 4440 | - 76103 | $+4716$ | -63855 | +6197 |
| 87-88 | 284 | 460 | - 76455 | $+4788$ | -64149 | + 6268 |
| 88-89 | 287 | 366 | -76731. | + 4860 | -64378 | $+6337$ |
| 89-90 | 293 | 780 | -77300 | $+5074$ | -64850 | +6533 |
| 90-91 | 281 | 424 | - 77627 | + 5124 | - 65124 | + 6585 |
| 91-92 | 285 | 583 | - 78071 | $+5223$ | - 65495 | $+6681$ |
| 92-93 | 283 | 330 | - 78324 | + 5271 | -65705 | +6729 |

93 Jidda, Mecca-gate
and West, from $0^{\circ}$ to $360^{\circ}$. They had first to be reduced to astronomical azimuths counted also from North to East and for this purpose the magnetic declination was taken from the Adm. Chart of Jidda. For 1911 it was assumed to be $2^{\circ} 55^{\prime}$ West $-11 \times 3^{\prime}=2^{\circ} .4$ West, and this value was considered to hold good for the whole of the road. As length of the step $0^{n .78}$ was adopted as given by Mr. Salim, and the length of the parts of the road expressed in meters shall be designated by $l^{\prime}$. The $4^{\text {th }}$ and $5^{\text {th }}$ columns contain the co-ordinates $x$ and $y$ in meters of points $2-93$ relative to point 1 , taking as axes the parallel and the meridian, so that we have

$$
x=\Sigma l^{\prime} \sin \alpha^{\prime} \quad y=\Sigma l^{\prime} \cos \alpha^{\prime}
$$

in which for point $n$ the summation has to be extended over all the parts between point 1 and $n$.

The two values in the last line of the $4^{\text {th }}$ and $5^{\text {th }}$ columns of the table are the co-ordinates of the Mecca-gate at Jidda relative to the Entrance of Mecca. These may be compared with the corresponding differences deduced from the astronomical determinations. For this
N. SCHELTEMA. "Determination of the geographical latitude and longitude of Mecca and Jidda executed in 1910-11".


Fig. 1.
Observation-station at Jidda.
a Mecca•gate,
$c$ Medina-gate.
$a b c$ Part of the rampart.
$e \quad$ Observation-station, Dutch Consulate. Scale in hectom.


Fig. 2.

## Observation-station at Mecca.

I Ka'bah.
II Entrance to the town on the Jidda side.
III Observation-station. Scale in hectom.

Proceedings Royal Acad. Amstcrdam. Vol. XV.


Proceedings Royal Acad. Amsterdam. Vol. XV.
purpose the situation of the point designated as "Entrance Mecca" had first to be determined. We adopted as such point II in the plan of Mecca, plate II fig. 2, and according to this its co-ordinates relative to the Ka'bah are

$$
\begin{aligned}
& \Delta x=638 \mathrm{~m} .=22^{\prime \prime} .2 \text { West } \\
& \Delta y=253 \mathrm{~m} .=88^{\prime \prime} .2 \text { North }
\end{aligned}
$$

Using these values we obtain as relative co-ordinates of points 93 and 1 according to the astronomical determinations

$$
\begin{aligned}
& X=-38^{\prime} 2^{\prime \prime} .2=-65705 \mathrm{~m} \\
& Y=+3^{\prime} 38^{\prime \prime} .9=+6729 \mathrm{~m}
\end{aligned}
$$

while the results from the observations on the journey were

$$
x=-78324 \mathrm{~m} \quad y=+5271 \mathrm{~m}
$$

As the errors in the astronomical results may be regarded as small compared with the accumulated errors of the observations on the journey, we may conclude that the latter results need corrections $\Delta x=+12619 \mathrm{~m} \Delta y=+1458 \mathrm{~m}$. We may regard these corrections as owing to an error in the accepted value for the length of a step and to a constant error in the deduced azimuths.

We then obtain, designating the true length of a step in meters by $0.78(1-p)$ and the constant error of the azimuths by $\delta$, the two equations

$$
\begin{aligned}
& \Delta x=-(p \cos \delta+1-\cos d) x+(1-p) y \sin \delta=+12619 \\
& \Delta y=-(p \cos \delta+1-\cos \delta) y-(1-p) x \sin \delta=+.1458
\end{aligned}
$$

The solution of the two equations $y$ ields

$$
\begin{array}{r}
p \cos \delta+1-\cos \delta=a=+0.15913 \\
(1-p) \sin \delta=b=+0.02932
\end{array}
$$

So the true length of a step and the constant error of the azimuths are found to be

$$
(1-\mathrm{p}) 0^{\mathrm{m}} .78=0^{\mathrm{m}} .6563 \quad \delta=+2^{\circ} 0^{\prime}
$$

while the values found for $a$ and $b$ may be used to correct the co-ordinates of cur 92 points. These corrected co-ordinates are found in the two last columns of our table.

At the same time they have also been drawn, and with them the whole of the road Mecca-Jidda, ini our plate III. No scale has been appended, but in the map itself lines have been drawn at distances of $2^{\prime}$ in longitude and latitude of each other. These have been drawn perpendicular to each other, and as length of a second of latitude and longitude in meters we have accepted after Bussun's dimensions of the earth $30^{m} .753$ and $28 \% .789$, the latter value holding rigorously for $\varphi=21^{\circ} 27^{\prime}$. As the absolute longitude from the meridian of

Greenwich may still need a correction, we have reckoned the longitude and also the latitude on the map from Jidda, Mecca-gate.

At the conclusion of this paper, which has proved that much advice and help has come to me from many sides, the only thing left for me to do, is to express my sincere gratefulness to all those from whom I have received this help.

## Postcript.

(November 1912).
In order to investigate the accuracy of our time-determinations we uad compared the results from the eastern and the western star, but in doing this no attention had been paid to the fact, that in most cases the times of observation of the two stars lie too far apart to neglect the rate of the chronometer in the interval.

Therefore this comparison has been made anew after correcting the differences $\mathrm{E}-\mathbf{W}$; the results, however, have not been materially changed. We now obtained

$$
\begin{array}{lll}
\text { Jidda } 23 \text { nights } & E-W=+0^{*} .12 \\
\text { Mecca } 13 & , & \\
\text { Together } & & E-W=+0.23 \\
& +0.16 \pm 0.10
\end{array}
$$

against before $+0^{*} .11 \pm 0.10$. The constant erior in the zenithdistances would be found now $\angle z=+1^{\prime \prime} .2$ against before $+0^{\prime \prime} .8$, but just as before it is small.

As mean error of the difference from one night we now found $\pm 0$ s. 58 and therefore as mean error of a time-determination from two stars $\pm 0^{*} .29$ against before $\pm 0^{\circ} .32$. The accordance of the two stars was somewhat improved.

Chemistry. - "On a new modification of sulphur". By Dr. A. H. W. Aten. (Communicated by Prof. Holleman). Communicated in the Meeting of September 28, 1912.

This investigation originated in an observation by Aronstein and Melhulzen ${ }^{1}$ ), who noticed that when a solution of sulphur in sulphur chloride $\left(\mathrm{S}_{2} \mathrm{Cl}_{2}\right)$, supersaturated at the temperature of the room, is heated to $170^{\circ}$, no sulphur crystallises on cooling. I have afterwards repeated this experiment and demonstrated that the solution of $S$ in $\mathrm{S}_{2} \mathrm{Cl}_{2}$, which has been heated to $170^{\circ}$ not only fails to deposit sulphur at the temperature of the room, but is even capable of dissolving a

[^3]
[^0]:    ${ }^{1}$ ) These results may be compared witi the co ordinates of the English consulate, which is next to the Dutch, measured on the admiralty-chart, as accurately as was possible: : $\Delta x=-542 \mathrm{~m} \quad \Delta y=+344{ }^{\mathrm{n}}$.
    Our plate II, fig. 1 is based solely on Mr Salim's measurements.

[^1]:    ${ }^{1}$ ) G. B. Airy : Account of observations of the transit of Venus, 1874, Dec. 8. London 1881.
    ${ }^{2}$ ) Dun Echt observatory publications. Vol. III. Dun Echt 1885.
    ") A. Auwers: Die Venus-Durchgänge 1874 und 1882. Bd. 6. Berlin 1896.

[^2]:    1) Hess started from the following co-ordinates of Jilda, Mecca gate: Longitude $39^{\circ} 11^{\prime} 47^{\prime \prime} \mathrm{E}$; Latitude $+21^{\circ} 29^{\prime} 11^{\prime \prime}$, while the Adm. Chart Ed. 1905 gives $39^{\circ} 12^{\prime} 35^{\prime \prime} ;+21^{\circ} 29^{\prime} 11^{\prime \prime}$, i.e. a longitude of $48^{\prime \prime}$ greater.
[^3]:    ${ }^{1}$ ) Verhandelingen Kon. Akad. Wel. Amsterdam, 1898. 1.

