action of potassium hydrogen sulphate as well as of phthalic anhydride, which, judging from provisional experiments consists of hexatriene.

In consequence of the fact noticed by Dr. C. J. ENKLAAR (loc. cit.) that the homologue of hexatriene which he prepared can be obtained in a crystalline condition by strong cooling, Mr. MULLER has cooled a freshly prepared and carefully fractioned specimen of hexatriene in a mixture of solid carbon dioxide and alcohol and obtained it also in the crystalline form [1]), so that this fact may be utilised for the purification of this hydrocarbon.

Finally it may be mentioned here that Mr. LE HEUX, by reduction of the chloroacetine of *s.* divinylglycol with a copper-zinc couple in ethereal solution with addition of hydrochloric acid, obtained a liquid boiling at 77°—81° which on strong cooling became crystalline and consists very probably of hexatriene 1, 3, 5. At any rate it yields with bromine a dibromide identical with the dibromide from the said hydrocarbon.

Utrecht. *Org. Chem. Lab. Univ.*

Physics. — *"On Einstein's theory of the stationary gravitation field."* By Prof. P. EHRENFEST. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of Febr. 22, 1913).

§ 1. Let a "laboratory" L with the observers in it have some accelerated motion with regard to a system of coordinates x, y, z, which is not accelerated. Let it e.g. move parallel to the z-axis with some positive acceleration or other. Then the observers will find that all the inert masses which are at rest with regard to the laboratory, exert a pressure on the bodies which are in contact with their bottom side. There are two ways for these observers to explain this pressure: *a.* "Our laboratory has an acceleration upwards, hence all inert masses press on the bodies under them." *b.* "Our laboratory is at rest. A field of force acts in it, which pulls the masses down."

Observations on the course of the rays of light seem to make it possible to decide experimentally between the suppositions a and b: with regard to the system of coordinates x, y, z the light travels rectilinearly. Hence with regard to an accelerated laboratory *curvilinearly*. By means of this curvilinear propagation of the rays of light the observers might therefore ascertain that their laboratory has an accelerated motion.

1) Preparations which have been kept for some time and then contain polymerides do not solidify even at this low temperature.

The possibility of such an experimental decision disappears immediately when also in a stationary laboratory, in which there is a field of force, the rays of light are admitted to have a corresponding curvature.

The "hypothesis of equivalence" on which EINSTEIN bases his attempt at a theory of gravitation [1]), really requires such a curvature of the rays of light in a field of attraction.

The hypothesis of equivalence, namely, demands that a laboratory L', which *rests in a field of attraction,* is equivalent with respect to *all* physical phenomena with a laboratory L *without gravitation, but accelerated.*

It is therefore required that the observers which are in L, cannot ascertain in any way by experiments, whether their laboratory has an accelerated motion, or whether it is at rest (in a corresponding field of attraction). So we are here concerned in the first place with an attempt to extend the theory of relativity of the case of *uniform* motion of a laboratory to that of non-*uniform* motion.

The physical significance of EINSTEIN's hypothesis of equivalence would, however, chiefly lie in this that it requires a certain functional relation between the field of attraction and other physical quantities (e.g. the velocity of light).

When working out the hypothesis somewhat more closely, EINSTEIN is confronted by certain difficulties. These led him to pronounce the supposition [2]) that the theory of equivalence would possibly only be valid for infinitely small regions of space and time, and not for finite ones.

EINSTEIN confined himself here to a mere supposition, as the said difficulties only presented themselves in the consideration of the dynamic phenomena in the laboratory L', and he had to do there with derivations from so great a number of suppositions, that it becomes difficult to see, *where* the difficulties arise from : the hypothesis of equivalence, or one of the other more special suppositions (as e.g. concerning the dynamic actions of rigid kinematic connections).

The following considerations try to throw light on this question. They show that similar difficulties already occur in those phenomena which are the most elementary in EINSTEIN's theory : in the propagation of rays of light in a statical field of attraction.

The principal result is : *All the statical fields of attraction with the exception of a very particular class, are in contradiction with Einstein's hypothesis of equivalence. Already the statical field of*

[1]) Ann. d. Phys. Bd. **35** (1211) p. 898 ; Bd. **38** (1912) p. 355 and 443.
[2]) Ann. d. Phys. Bd. **38** (1912) p. 452–456.

attraction brought about by several centres of attraction which are stationary with respect to each other, is not compatible with the hypothesis of equivalence.

§ 2. Let, therefore a laboratory L' be given, in which there is a statical field of attraction. With EINSTEIN we suppose that the rays of light propagate in it curvilinearly in some way or other, but so that the following conditions are satisfied:

When *once* a ray of light may have passed through the points $A, B, \ldots F, G$ of the laboratory L' [1]), then

[**A**] this way $A, B, \ldots F, G$ must *always* be possible for the light ("*Constancy of the ways of light*"),

[**B**] the *reversed* way $G, F, \ldots B, A$ must also be always possible ("*reversibility of the ways of light*").

The hypothesis of equivalence now compares this laboratory L' resting in the field of attraction with a laboratory L which is free from gravitation, but has a corresponding acceleration instead. *How must the points of this laboratory in which there is no gravitation move, so that the observers in it shall observe constancy and reversibility of the ways of light in the sense of the hypothesis of equivalence?*

§ 3. For the sake of simplicity we confine ourselves to a two-dimensional laboratory L. As fundamental system of coordinates, with respect to which L moves in an accelerated way may serve the system of coordinates x, y, which has no acceleration, and the time t measured in it. With respect to this system which is without gravitation, the rays of light move in straight lines and with constant velocity 1. In the corresponding x, y, t-world-space of MINKOWSKI every optical signal travelling in this way is represented by a straight line forming an angle of 45° with the t-axis. Such a line in the x, y, t-space is called "*a line of light*". The motion of the different points $A, B, \ldots F. G$ of the moving laboratory L is represented by the same number of (curved) world lines $a, b \ldots f, g$.

When the observers in the laboratory L state that they have succeeded in making an optical signal S_1 pass through the points $A, B, \ldots F, G$ of their laboratory this means that the corresponding line of light s_1 intersects the world lines $a, b, \ldots f, g$ of these points of the laboratory.

According to condition [**A**] of § 2 the observers in the laboratory L must in this case be able to send light signals $S_2, S_3 \ldots$ through

[1]) These points may be imagined e.g. as apertures in the walls of the laboratory.

the points $A, B, \dots F, G$ of the laboratory at other moments as many times as they like. Geometrical representation in the x, y, t-space: The world-lines $a, b, \dots f, g$ are intersected by all the ∞^1 lines of light $s_1, s_2 \dots$; they all lie on the ruled surface formed by the ∞^1 light lines.

In agreement with condition [**B**] of § 2 the observers of the laboratory L must then moreover as often as they like be able to send optical signals S'_1, $S'_2, \dots$ in *opposite* direction $G, F, \dots B, A$. In the x, y, t-space again ∞^1 light lines $s'_1, s'_2, \dots$ correspond with this, which all intersect the world lines $a, b, \dots f, g$. Hence the world lines $a, b, \dots g, h$ all lie on a surface covered by two systems each of ∞^1 light lines. If we then bear in mind that the light lines all make an angle of 45° C. with the t-axis, it is easy to see that such a surface must necessarily be an equilateral hyperboloid of revolution with the axis of revolution // to the t-axis; i.e. the equation of this surface has the form:

$$A(x^2+y^2-t^2)+Bx+Cy+Dt+E=0 \quad . \quad . \quad . \quad . \quad (1)$$

In particular the case may also present itself that $A=0$, i.e. that the hyperboloid degenerates into a plane.

Such hyperboloids will be briefly called *"light-hyperboloids"*. Accordingly the world lines $a, b, \dots f, g$ of the points $A, B, \dots F, G$ of the laboratory L lie on a common "light hyperboloid" H_{ab}.

Now the observers might just as well have sent a light signal instead of from A to B, from A to any other point B' of the laboratory. In exactly the same way we see then that also the two world lines a and b' must lie on a common light hyperboloid $H_{ab'}$. Let the equation of this be:

$$A'(x^2+y^2-t^2)+B'x+C'y+D't+E'=0 \quad . \quad . \quad . \quad . \quad (2)$$

So the world line a lies at the same time on two different light-hyperboloids H_{ab} and $H_{ab'}$; it is the section of both, and this is necessarily a plane section. (Multiply equation (1) by A' and equation (2) by A, and subtract). If we now bear in mind that the point A of the laboratory must never have a greater velocity than that of light, of all the plane sections of a light-hyperboloid only two types deserve consideration: hyperbolas the two branches of which run from $t=-\infty$ to $t=+\infty$, and as limiting case the light lines of the hyperboloid. (In other words the sections with planes which **1** cut the gorge circle of the hyperboloid, and **2** make an angle of $\leqq 45°$ with the t-axis. As besides, the case may occur that the light hyperboloids which pass through the world line a, degenerate to planes, the world line n may also be a straight line, making an angle with the t-axis, which is smaller than 45°.

A, however, was an arbitrary point of the laboratory L. So we have proved the following:

"If the observers in a moving laboratory L, which is without gravitation are to observe constancy and reversibility of the ways of light, it is necessary that the "world-lines" of the points of the laboratory are a system of ∞^2 branches of hyperbolas, or else straight lines in the x, y, t-space."

Without a new supposition, only in consequence of the circumstance that through every pair of these world-lines — e.g. p and q — can always be brought a light hyperboloid H_{pq} [1]), it can further be proved: that the ∞^2 world line hyperbolas lie in ∞^1 *surfaces*, which pass fanlike through a straight line Γ of the x, y, t-space; they cut Γ in two real or conjugated imaginary points Ω_I and Ω_{II} (which may also coincide). In this way dependent on the situation of the points Ω_I and Ω_{II} ∞^6 fields of world lines originate, which are of a very particular nature. [2])

§ 4. The frequency of the static fields of attraction caused by n centres which are stationary with respect to each other, is already greater than ∞^6 for $n \geq 3$. But the "hypothesis of equivalence", cannot be satisfied in any other case than in that of the very special fields of attraction, which correspond to the ∞^6 fields of acceleration of the preceding §.

Remark.

Up to now we have only used the constancy of the *form* of the rays of light. Moreover in every point of the laboratory L' the *velocity* of the light must also be independent of the time. In order to introduce this condition, the measurement of time in L' would have to be taken into account in the considerations, which renders them more intricate.

Possibly the class of fields for which the hypothesis of equivalence is admissible, might then be still further limited.

The field of hyperbolas which in the x, y, t-space represents Born's "motion of hyperbolas" of a two-dimensional laboratory, is contained in the ∞^6 fields of hyperbolas of § 4 as a special case.

Moreover it satisfies (with suitable measurement of time in L') the condition that the velocity of the light is independent of time.

1) Formed by the lines of light of the signals, which may be sent from P to Q, and from Q to P.

2) A proof for these theses and a classification of the above mentioned ∞^6 fields of world lines is found in a paper by Mr. Ch. H. van Os, which will shortly appear.