ore, around which a mixture of ore and prisms of gold-coloured aegirine-augite columns is formed.

The metamorphoses described above by which gold-coloured pyroxenes with the optical properties of aegirine-augites are formed, appear to be connected with pneumatolytic processes in magmas rich in alkali.

Finally it may be mentioned here, that to the South of the road Panarukan Besuki, quite near to mile-post 13, a loose piece of a leucitite was found with phenocrysts of leucites as large as 4 m.m., which certainly had come down from the northern slope of the Ringgit and consequently may be expected there in greater quantities; hitherto such types of rocks were not recorded from the Ringgit-mountain.

Mathematics. — "*Expansion of a function in series of* ABEL'S *functions* $\varphi_n(x)$". By Prof. W. KAPTEYN.

(Communicated in the meeting of February 22, 1913).

1. In the Oeuvres complètes of ABEL [1]) may be found the following expansion

$$\frac{1}{1-v} e^{-\frac{xv}{1-v}} = \sum_0^\infty \varphi_n(x) v^n$$

where

$$\varphi_n(x) = 1 - C_1^n x + C_2^n \frac{x^2}{2!} - \dots + (-1)^n \frac{x^n}{n!}$$

C_p^n representing the binomial coefficients.

These polynomia form the object of the dissertation of Dr. A. A. NIJLAND (Utrecht 1896) and have been treated afterwards by E. LE ROY in his memoir "Sur les séries divergentes" (Annales de Toulouse 1899).

In this paper I wish to examine when a given function of a real variable may be expanded in a series of this form

$$f(x) = a_0 + a_1\varphi_1(x) + a_2\varphi_2(x) + \dots \quad (1)$$

2. In this article we collect those properties of the polynomia $\varphi_n(x)$ which we want for our investigation and which we take from NIJLAND'S dissertation.

In the first place we have the important relations

[1]) Oeuvres Complètes II p. 284.

$$\left.\begin{aligned}\int_0^\infty e^{-x}\varphi_m(x)\varphi_n(x)dx &= 0 \quad (m \neq n)\\ \int_0^\infty e^{-x}\varphi_n^2(x)dx &= 1\end{aligned}\right\} \quad \ldots\ldots \quad (2)$$

In the second place $\varphi_n(x)$ satisfies the differential equation

$$x\varphi_n''(x) + (1-x)\varphi_n'(x) + n\varphi_n(x) = 0$$

which also may be written

$$\frac{d}{dx}\left[xe^{-x}\varphi_n'(x)\right] + ne^{-x}\varphi_n(x) = 0 \quad \ldots\ldots \quad (3)$$

In the third place we have the following properties, which may be easily obtained

$$\varphi_n(x) = \varphi_n'(x) - \varphi'_{n+1}(x) \quad \ldots\ldots \quad (4)$$

$$\frac{x}{n}\varphi_n'(x) = \varphi_n(x) - \varphi_{n-1}(x) \quad \ldots\ldots \quad (5)$$

$$(n+1)\varphi_{n+1}(x) - (2n+1-x)\varphi_n(x) + n\varphi_{n-1}(x) = 0 \quad \ldots \quad (6)$$

$$\int_0^\infty e^{-x}x^n\varphi_m(x)dx = (-1)^m C_m^n\, n!\ (m \leqq n) \quad \ldots \quad (7)$$
$$= 0 \qquad (m > n)$$

3. If the expansion (1) is possible, the coefficients a_n may be expressed by means of the equations (2)

$$a_n = \int_0^\infty e^{-\alpha}f(\alpha)\varphi_n(\alpha)d\alpha.$$

With these values the second member of (1) reduces to

$$S = \sum_0^\infty \varphi_n(x)\int_0^\infty e^{-\alpha}f(\alpha)\varphi_n(\alpha)d\alpha. \quad \ldots\ldots \quad (8)$$

In order to determine this sum we introduce $\varphi_n(x)$ in the form of a definite integral. This definite integral, which has been given by LE ROY, may be found in the following way.

Denoting by $J_0(t)$ the Besselian function of order zero, MACLAURIN's expansion gives easily

$$e^x J_0(2\sqrt{\alpha x}) = \sum_0^\infty \frac{x^m}{m!}\varphi_m(\alpha) \quad \ldots\ldots \quad (9)$$

Hence, multiplying both members by $\frac{e^{-\alpha}\alpha^n d\alpha}{n!}$ and integrating between the limits 0 and ∞

$$\frac{e^x}{n!}\int_0^\infty e^{-\alpha}\alpha^n J_0(2\sqrt{\alpha x})d\alpha = \frac{1}{n!}\sum_0^\infty \frac{x^m}{m!}\int_0^\infty e^{-\alpha}\alpha^n \varphi_m(\alpha)d\alpha$$

where the second member may be reduced by means of (7) to

$$\sum_0^n (-1)^m C_m^n \frac{x^m}{m!} = \varphi_n(x).$$

Therefore we have

$$\varphi_n(x) = \frac{e^x}{n!}\int_0^\infty e^{-\alpha}\,\alpha^n\, J_0(2\sqrt{\alpha x})\,d\alpha \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and

$$S = \sum_0^\infty \frac{\varphi_n(x)}{n!}\int_0^\infty f(\alpha)\,d\alpha\int_0^\infty e^{-\beta}\,\beta^n\, J_0(2\sqrt{\alpha\beta})\,d\beta\,.$$

Now, from the equation (9) we obtain

$$\sum_0^\infty \frac{\beta^n\varphi_n(x)}{n!} = e^\beta J_0(2\sqrt{\beta x})$$

thus

$$S = \int_0^\infty f(\alpha)\,d\alpha\int_0^\infty J_0(2\sqrt{\alpha\beta})\,J_0(2\sqrt{\beta x})\,d\beta,$$

or, putting β^2 instead of β

$$S = 2\int_0^\infty f(\alpha)\,d\alpha\int_0^\infty J_0(2\beta\sqrt{\alpha})\,J_0(2\beta\sqrt{x})\,\beta\,d\beta \quad . \quad . \quad . \quad (11)$$

3. This double integral may be determined by a theorem of HANKEL (Math. Ann. Bd. 8 p. 481), who proved that

$$\int_0^\infty \gamma\,\varphi(\gamma)\,d\gamma\int_0^\infty J_0(\beta\gamma)\,J_0(\beta\xi)\,\beta d\beta = \varphi(\xi)$$

where ξ represents a positive value and $\varphi(\xi)$ a function which satisfies the conditions of DIRICHLET for all values between 0 and ∞. Putting

$$\gamma = 2\sqrt{\alpha}, \qquad \xi = 2\sqrt{x}, \qquad \varphi(2\sqrt{x}) = f(x)$$

this theorem gives immediately

$$S = 2\int_0^\infty f(\alpha)\,d\alpha\int_0^\infty J_0(2\beta\sqrt{\alpha})\,J_0(2\beta\sqrt{x})\,\beta\,d\beta = f(x) \quad . \quad . \quad (12)$$

Thus we have established the result, that every function $f(x)$ which satisfies the conditions of DIRICHLET for all values between 0 and ∞ may be expanded in a series of the form

$$f(x) = a_0 + a_1 \varphi_1(x) + a_2 \varphi_2(x) + \dots \qquad 0 \leqq x < \infty \quad . \quad (1)$$

where

$$a_n = \int_0^\infty e^{-\alpha} f(\alpha) \varphi_n(\alpha) \, d\alpha$$

It is to be remarked that the values $f(c+0)$ and $f(c-0)$ being different, the second member reduces to $\frac{1}{2}[f(c+0) + f(c+0)]$.

4. We now proceed to give two interesting examples of this expansion and to show the value of this expansion for the problem of the momenta.

As a first example suppose it is required to express $f(x) = \frac{1}{1+x}$ in a series of ABEL's functions $\varphi_n(x)$.

Evidently this function satisfies the conditions of DIRICHLET from $x = 0$ to $x = \infty$, thus

$$\frac{1}{1+x} = a_0 + a_1 \varphi_1(x) + a_2 \varphi_2(x) + \dots$$

where

$$a_n = \int_0^\infty \frac{e^{-\alpha} \varphi_n(\alpha) \, d\alpha}{1+\alpha}.$$

Now the following relation holds between successive functions φ:

$$(n+1) \varphi_{n+1}(\alpha) = (2n+1-\alpha) \varphi_n(\alpha) - n \varphi_{n-1}(\alpha) \quad . \quad . \quad (6)$$

Multiplying this by $\frac{e^{-\alpha}}{1+\alpha} d\alpha$, and integrating between 0 and ∞ we obtain

$$(n+1) a_{n+1} = (2n+1) a_n - n a_{n-1} - \int_0^\infty \frac{\alpha e^{-\alpha}}{1+\alpha} \varphi_n(\alpha) \, d\alpha \cdot$$

But, as

$$\frac{\alpha}{1+\alpha} = 1 - \frac{1}{1+\alpha}$$

we have

$$\int_0^\infty \frac{\alpha e^{-\alpha}}{1+\alpha} \varphi_n(\alpha) \, d\alpha = \int_0^\infty e^{-\alpha} \varphi_n(\alpha) \, d\alpha - a_n$$

where the latter integral, which may be written

$$\int_0^\infty e^{-\alpha} \varphi_0(\alpha) \varphi_n(\alpha) \, d\alpha$$

vanishes according to (2) if $n > 0$.

Therefore three successive coefficients of this expansion are related in the following way

$$(n+1) a_{n+1} = 2(n+1) a_n - n a_{n-1} \quad (n > 0)$$

so that all the coefficients may be expressed in a_0 and a_1.

Now

$$\varphi_1(\alpha) = 1 - \alpha$$

hence

$$a_1 = \int_0^\infty \frac{e^{-\alpha}(1-\alpha)}{1+\alpha} d\alpha = \int_0^\infty \frac{e^{-\alpha}[2-(1+\alpha)]}{1+\alpha} d\alpha = 2a_0 - 1$$

which proves that all the coefficients are dependent on the first

$$a_0 = \int_0^\infty \frac{e^{-\alpha} d\alpha}{1+\alpha} = - e l i \left(\frac{1}{e}\right) = 0{,}596347 \ldots$$

These coefficients may also be obtained in another way.

From Abel's expansion

$$\frac{1}{1-v} e^{-\frac{1}{1-v}} = \sum_0^\infty \varphi_n(x) v^n$$

which holds where

$$mod\ v < 1$$

we see, by putting

$$t = \frac{v}{1-v}$$

that

$$e^{-xt} = \frac{1}{1+t} + \frac{t}{(1+t)^2} \varphi_1(x) + \frac{t^2}{(1+t)^3} \varphi_2(x) + \ldots$$

if

$$mod \frac{t}{1+t} < 1 .$$

Multiplying this equation by $e^{-t} dt$ and integrating between the limits 0 and ∞, we obtain

$$\frac{1}{1+x} = a_0 + a_1 \varphi_1(x) + a_2 \varphi_2(x) + \ldots$$

where

$$a_n = \int_0^\infty e^{-t} \frac{t^n}{(1+t)^{n+1}} dt \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Comparing this result, with the former, we obtain the interesting formula

$$\int_0^\infty e^{-t} \frac{t^n}{(1+t)^{n+1}} dt = \int_0^\infty \frac{e^{-t} \varphi_n(t)}{1+t} dt \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which is evident if we put $n = 0$.

From (13) we see also that

$$\sum_0^\infty a_n = \int_0^\infty e^{-t} \frac{1}{1+t} \sum_0^\infty \left(\frac{1}{1+t}\right)^n dt = \int_0^\infty e^{-t} dt = 1 ,$$

which shows, that the expansion

$$\frac{1}{1+x} = \sum_0^\infty a_n \varphi_n(x)$$

holds for $x = 0$.

5. As a second example we will expand a discontinuous function.

Supposing $f(x) = 1$ from $x = 0$ to $x = 1$ and $f(x) = 0$ for $x > 1$ we have

$$f(x) = a_0 + a_1 \varphi_1(x) + a_2 \varphi_2(x) + \ldots$$

where

$$a_n = \int_0^1 e^{-x} \varphi_n(\alpha) d\alpha .$$

This coefficient may be determined in the following way. From the differential equation

$$\frac{d}{dx}[x e^{-x} \varphi_n'(x)] + n e^{-x} \varphi_n(x) = 0 \quad . \quad . \quad . \quad . \quad (3)$$

it appears that

$$x e^{-x} \varphi_n'(x) + n \int_0^x e^{-x} \varphi_n(x) dx = 0$$

therefore, putting $x = 1$, we have

$$a_n = -\frac{1}{ne} \varphi_n'(1) \qquad (n > 0)$$

or, according to (5)

$$a_n = \frac{1}{e} [\varphi_{n-1}(1) - \varphi_n(1)] \qquad (n > 0)$$

The two first coefficients may be obtained directly, for

$$a_0 = \int_0^1 e^{-\alpha} d\alpha = 1 - \frac{1}{e}$$

and

$$a_1 = \frac{1}{e} [\varphi_0 (1) - \varphi_1 (1)] = \frac{1}{e} .$$

The remaining coefficients are dependent on these. For putting $x = 1$ in the recurrent relation

$$(n + 1) \varphi_{n+1} (x) - (2n + 1 - x) \varphi_n (x) + n \varphi_{n+1} (x) = 0 \quad . \quad (6)$$

we get

$$(n + 1) \varphi_{n+1} (1) - 2n \varphi_n (1) + n \varphi_{n-1} (1) = 0$$

and, changing n into $n + 1$

$$(n + 2) \varphi_{n+1} (1) - 2 (n + 1) \varphi_{n+1} (1) + (n + 1) \varphi_n (1) = 0 .$$

thus, subtracting the former from the latter equation

$$(n + 2) a_{n+2} - (2n + 1) a_{n+1} + n a_n = 0 .$$

6. The expansion holding for the value $x = 0$, we must have

$$\sum_0^\infty a_n = 1$$

and remarking that $x = 1$ is a point of discontinuity

$$\sum_0^\infty a_n \varphi_n (1) = \tfrac{1}{2} .$$

To prove these equations directly we may remark that

$$\sum_1^n a_p = \frac{1}{e} \sum_1^n [\varphi_{p-1} (1) - \varphi_p (1)] = \frac{1}{e} [1 - \varphi_n (1)]$$

so

$$\sum_1^\infty a_p = \frac{1}{e} - \frac{1}{e} \lim_{n=\infty} \varphi_n (1) .$$

Now, the number n being very large, we have

$$\varphi_n (x) \equiv 1 - nx + \frac{n^2 x^2}{(2!)^2} - \frac{n^3 x^3}{(3!)^2} + \ldots = J_0 (\sqrt{nx})$$

and

$$\lim_{n=a} \varphi_n (x) = \lim_{n=\infty} J_0 (\sqrt{nv}) = \lim_{n=\infty} \sqrt{\frac{2}{\pi \sqrt{nx}}} \cos \left(\sqrt{nx} - \frac{\pi}{4} \right) = 0$$

therefore

$$\sum_1^\infty a_p = \frac{1}{e}$$

and finally

$$a_0 + \sum_1^\infty a_p = 1 - \frac{1}{e} + \frac{1}{e} = 1\,.$$

The second equation may be obtained as follows.

From the differential equation

$$\frac{d}{dx}\left[xe^{-x}\varphi_p'(x)\right] + pe^{-x}\varphi_p(x) = 0 \qquad \ldots\ldots \quad (3)$$

we may conclude

$$\int_0^1 e^{-x}\varphi_p^2(x)\,dx = -\frac{1}{p}\int_0^1 \varphi_p(x)\,d\left[xe^{-x}\varphi_p'(x)\right] =$$

$$= -\frac{1}{p}\left[xe^{-x}\varphi_p(x)\varphi_p'(x)\right]_0^1 + \frac{1}{p}\int_0^1 xe^{-x}\varphi_p'^2(x)\,dx$$

so

$$\int_0^1 e^{-x}\left[\varphi_p^2(x) - \frac{x}{p}\varphi_p'^2(x)\right]dx = -\frac{1}{pe}\varphi_p(1)\varphi_p'(1) = a_p\,\varphi_p(1).$$

Now, the equations (4) and (5) give

$$\varphi_p^2(x) = \varphi_p(x)\left[\varphi_p'(x) - \varphi_{p+1}'(x)\right]$$

$$\frac{x}{p}\varphi_p'^2(x) = \varphi_p'(x)\left[\varphi_p(x) - \varphi_{p-1}(x)\right]$$

hence

$$\varphi_p^2(x) - \frac{x}{p}\varphi_p'^2(x) = \varphi_{p-1}(x)\varphi_p'(x) - \varphi_p(x)\varphi_{p+1}'(x)$$

and

$$\sum_1^n\left[\varphi_p^2(x) - \frac{x}{p}\varphi_p'^2(x)\right] = \varphi_0(x)\varphi_1'(x) - \varphi_n(x)\varphi_{n+1}'(x).$$

This shows that

$$\int_0^1 e^{-x}\left[\varphi_0(x)\varphi_1'(x) - \varphi_n(x)\varphi_{r+1}'(x)\right]dx = \sum_1^n a_p\varphi_p(1)$$

where

$$\int_0^1 e^{-x}\varphi_0(x)\varphi_1'(x)\,dx = -\int_0^1 e^{-x}\,dx = -1 + \frac{1}{e}.$$

To obtain the second integral, the value of n being very large, we observe that according to equation

$$\varphi_n(x) = \varphi_n'(x) - \varphi_{n+1}'(x). \qquad \ldots\ldots\ldots \quad (4)$$

the functions

$$\varphi_n'(x) \text{ and } \varphi'_{n+1}(x)$$

tend to the same limit.

If, therefore n is very large, the second integral, tends to

$$\int_0^1 e^{-x}\varphi_n(x)\varphi'_{n+1}(x)dx = \int_0^1 e^{-x}\varphi_n(x)\varphi_n'(x) =$$

$$= \left[\frac{e^{-x}\varphi_n^2(x)}{2}\right]_0^1 + \int_0^1 e^{-x}\varphi_n^2(x)\,dx = -\tfrac{1}{2}$$

and we obtain

$$\sum_1^\infty a_p\,\varphi_p(1) = -1 + \frac{1}{e} + \tfrac{1}{2}.$$

Thus, adding to this equation

$$a_0\,\varphi_0(1) = 1 - \frac{1}{e}$$

we get finally the required relation

$$\sum_0^1 a_p\,\varphi_p(1) = \tfrac{1}{2}.$$

7 In this article we wish to give a second verification of the former expansion because this leads to a very interesting integral containing BESSEL's functions. This verification is obtained by direct summation of

$$a_0 + a_1\varphi_1(x) + a_2\varphi_2(x) + \ldots$$

where

$$a_0 = 1 - \frac{1}{e} \quad \text{and} \quad a_n = \tfrac{1}{2}\left[\varphi_{n-1}(1) - \varphi_n(1)\right].$$

It appears from the equation (10) that

$$\varphi_{n-1}(1) = \frac{ne}{n!}\int_0^\infty e^{-\alpha}\,\alpha^{n-1}\,J_0(2\sqrt{\alpha})\,d\alpha$$

$$\varphi_n(1) = \frac{e}{n!}\int_0^\infty e^{-\alpha}\,\alpha^n\,J_0(2\sqrt{\alpha})\,d\alpha$$

therefore

$$\varphi_{n-1}(1) - \varphi_n(1) = \frac{e}{n!}\int_0^\infty J_0(2\sqrt{\alpha})\,d\left(e^{-\alpha}\alpha^n\right)$$

or, after partial integration

$$\varphi_{n-1}(1) - \varphi_n(1) = \frac{e}{n!}\int_0^\infty e^{-\alpha}\,\alpha^n J_1(2\sqrt{\alpha})\frac{d\alpha}{\sqrt{\alpha}}.$$

If $n = 0$, the first member has no meaning, as $\varphi_{-1}(1)$ has not been determined. The second member however reduces to

$$e\int_0^\infty e^{-\alpha} J_1(2\sqrt{\alpha})\frac{d\alpha}{\sqrt{\alpha}} = 2e\int_0^\infty e^{-\alpha^2} J_1(2\alpha)\,d\alpha =$$

$$= e\sqrt{\pi}\,J_{\frac{1}{2}}\left(\frac{i}{2}\right)e^{-\frac{1}{2}-\frac{\pi i}{4}} = e - 1 = a_0 e$$

[NIELSEN, Handbuch der Theorie der Cylinderfunctionen p. 185 (7)]. By applying again the equation (10), we have

$$[\varphi_{n-1}(1) - \varphi_n(1)]\varphi_n(x) = \frac{e^{x+1}}{(n!)^2}\int_0^\infty e^{-\alpha}\alpha^n J_1(2\sqrt{\alpha})\frac{d\alpha}{\sqrt{\alpha}}\int_0^\infty e^{-\beta}\beta^n J_0(2\sqrt{\beta x})d\beta$$

and by summation from $n = 0$ to $n = \infty$, as

$$\sum_0^\infty \frac{\alpha^n\beta^n}{(n!)^2} = J_0(2i\sqrt{\alpha\beta})$$

$$e\sum_0^\infty a_n\varphi_n(x) = e^{x+1}\int_0^\infty e^{-\alpha}J_1(2\sqrt{\alpha})\frac{d\alpha}{\sqrt{\alpha}}\int_0^\infty e^{-\beta}J_0(2i\sqrt{\alpha\beta})\,J_0(2\sqrt{\beta x})\,d\beta.$$

Putting β^2 instead of β in the latter integral, this reduces to

$$2\int_0^\infty e^{-\beta^2} J_0(2i\beta\sqrt{\alpha})\,J_0(2\beta\sqrt{x})\,\beta d\beta = e^{-x+\alpha} J_0(2\sqrt{\alpha x})$$

(NIELSEN p. 184); thus

$$\sum_0^\infty a_n\varphi_n(x) = \int_0^\infty J_0(2\sqrt{\alpha x})\,J_1(2\sqrt{\alpha})\frac{d\alpha}{\sqrt{\alpha}}$$

or, changing α into $\frac{\alpha^2}{4}$

$$\sum_0^\infty a_n\varphi_n(x) = \int_0^\infty J_0(\alpha\sqrt{x})\,J_1(\alpha)\,d\alpha.$$

The second member of this equation has different values according to the value of x, for

$$\int_0^\infty J_0 (a\sqrt{x})\, J_1 (a\, da) = \begin{cases} 1 & 0 < x < 1 \\ \frac{1}{2} & x = 1 \\ 0 & x > 0 \end{cases}$$

(NIELSEN p. 200), and for $x = 0$

$$\int_0^\infty J_0 (a\sqrt{x})\, J_1 (a)\, da = \int_0^\infty J_1 (a)\, da = 1.$$

8. Now we will apply our expansion to the problem of the momenta. In this problem the question is to determine the function $f(y)$ from the integral equation

$$a_n = \int_0^\infty f(y)\, y^n\, dy.$$

where a_n is a function which is given for all positive integral values of n.

Putting

$$f(y) = e^{-y}\, \theta(y)$$

we obtain

$$a_n = \int_0^\infty e^{-y}\, y^n\, \theta(y)\, dy.$$

Supposing $\theta(y)$ to be a function which satisfies the conditions of DIRICHLET, we have

$$\theta(y) = b_0 + b_1\, \varphi_1 (y) + b_2\, \varphi_2 (y) + \dots$$

so

$$a_n = \sum_0^\infty b_p \int_0^\infty e^{-y}\, y^n\, \varphi_p (y)\, dy.$$

Now, this integral has the value zero, when $p > n$, therefore

$$a_n = \sum_0^n b_p \int_0^\infty e^{-y}\, y^n\, \varphi_p (y)\, dy$$

Moreover, according to the equation (7)

$$a_n = n!\, \sum_0^n (-1)^p\, b_p\, C_p^n$$

so, with (10)

$$f(y) = e^{-y} \sum_0^\infty b_p\, \varphi_p (y) = \sum_0^\infty \frac{b_p}{p!} \int_0^\infty e^{-x}\, x^p\, I_0 (2\sqrt{xy})\, dx.$$

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If now we expand the function

$$g(x) = e^{-x} \sum_0^\infty b_p \frac{x^p}{p!} = e^{-x} X$$

in a power series, we have, differentiating n times, and putting $D = \frac{d}{dx}$

$$g^{(n)}(x) = D^n (e^{-x} X) = e^{-x} (D + 1)^{(n)} X$$
$$= e^{-x} \sum_0^n (-1)^p C_p^n \; D^{(n-p)} X$$

where

$$D^{(s)} X = \sum_0^\infty b_{s+p} \frac{x^p}{p!}$$

which, for the value $x = 0$, gives

$$D_0^{(s)} X = b_s.$$

Introducing this value, we obtain

$$g^{(n)}(0) = \sum_0^n (-1)^p b_{n-p} C_p^n = (-1)^n \sum_0^n (-1)^p b_p C_p^n = (-1)^n \frac{\alpha_n}{n!}$$

$$g(x) = \sum_0^\infty (-1)^n \frac{\alpha_n}{n!} x^n$$

and finally

$$f(y) = \int_0^\infty I_0(2\sqrt{xy}) \sum_0^\infty (-1)^n \frac{\alpha^n}{(n!)^2} x^n dx$$

This solution agrees with that of LE ROY. In his memoir the discussion of this formula for different values of α_n may be found.

Mathematics. — *"Some remarks on the coherence type η."* By Prof. L. E. J. BROUWER.

In order to introduce the notion of a "coherence type" we shall say that a set M is *normally connected*, if to some sequences f of elements of M are adjoined certain elements of M as their "limiting elements", the following conditions being satisfied:

1st. each limiting element of f is at the same time a limiting element of each end segment of f.

2nd. for each limiting element of f a partial sequence of f can be found of which it is the *only* limiting element.

3rd. each limiting element of a partial sequence of f is at the same time a limiting element of f.

4th. if m is the only limiting element of the sequence $\{m_\nu\}$ and