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**Physics.** — *"The diffraction of Electromagnetic waves by a crystal."* By Dr. L. S. ORNSTEIN. (Communicated by Prof. H. A. LORENTZ).

(Communicated in the meeting of Febr. 22, 1913).

In the "Sitzungsberichte der Konigl. Bayerischen Akademie der Wissenschaften" [1]) M. LAUE has published a theory — and together with Messrs. FRIEDERICH and KNIPPING experiments also — about this highly remarkable phenomenon. W. L BRAGG, in a paper entitled "The diffraction of short electromagnetic waves by a crystal" [2]) doubts of the explication of this experiments given by LAUE being satisfactory. He proposes an elementary theory, in which he points out that we can describe the phenomenon of LAUE by regarding all as if the RÖNTGEN rays were reflected on the sets of planes that can be brought through the molecules of the crystal. In the following lines I will develop the theory proposed by BRAGG, and at the same time I will give a provisory discussion of some experiments made in the Physical Laboratory of the University of Groningen which Prof. HAGA has been so kind as to put at my disposal, for which I may cordially thank him here.

I will confine myself to a regular crystal, the extension to crystals with other BRAVAIS or SOHNKE point-systems being possible without any difficulty.

1. Let us suppose a plane beam of RÖNTGEN rays (direction of ray: $x$-axis) to strike a regular crystal, of which one of the cubical axes of the point system is set parallel to the incident beam. The origin of coordinates is chosen in a molecule lying within the crystal in the middle of the part through which the rays are propagated. The $y$ and $z$-axes are oriented parallel to the other cubic axis. Be the length of the side of the cubes $a$ The coordinates of a molecule of the crystal then are

$$x = k_1 a \qquad y = k_2 a \qquad z = k_3 a \quad . \quad . \quad . \quad . \quad (1)$$

in which $k_1$, $k_2$ and $k_3$ are positive or negative whole numbers.

We shall examine the influence of the rays in a point with coordinates $\xi$, $\eta$, $\zeta$, at a distance $r$ from the origin.

Now whatever may be the constitution of primary RÖNTGEN rays, we can always imagine the disturbance of equilibrium being dissolved, according to the theorem of FOURIER, into periodical movements. In

1) Loc.cit. June 8 & July 6 1912. Interferenzerscheinungen bei Röntgenstrahlen.

2) Proc. Cambridge Phil. Soc. Vol. XVII, Part 1. The diffraction of short electromagnetic w by a crystal.

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the same way, the movement and radiation of molecules can be described. Thus knowing the effect of the radiation from the molecules when a periodical radiation strikes them, we can from this calculate for each case the influence of a crystal on RÖNTGEN rays. I will therefore consider the problem of a radiation of the wavelength $\lambda$ striking the crystal. Under the influence of this radiation the molecules will emit spherical waves. I will indicate the vector of radiation for the radiation emitted by a molecule situated at the origin, by

$$\frac{A}{r} \cos 2\pi \left(\frac{t}{T} - \frac{r}{\lambda}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

this formula representing the vector of radiation in the point $\xi \eta \zeta$, while $A$ depends on the direction. The radiation of a point (1) in the point $\xi \eta \zeta$ is now represented by

$$\frac{A}{\varrho} \cos 2\pi \left(\frac{t}{T} - \frac{\varrho}{\lambda} - \frac{k_1 a}{\lambda}\right),$$

where $\varrho$ denotes the distance of $\xi \eta \zeta$ from (1). This distance is given by

$$\varrho = r - \frac{a}{r}(\xi k_1 + \eta k_2 + \zeta k_3) + \frac{a^2}{2r}(k_1^2 + k_2^2 + k_3^2) + \frac{a^2}{2r}\left(\frac{\xi}{r} k_1 + \frac{\eta}{r} k_2 + \frac{\zeta}{r} k_3\right)^2 \ldots$$

Substituting in the amplitudo $\varrho$ by $r$ (which is allowed since $k_x a$ is small compared with $r$ etc.) then we get for the vector of light considered

$$\frac{A}{r} \cos 2\pi \left(\frac{t}{T} - \frac{r}{\lambda} - \frac{a}{\lambda}\left\{\left(1 - \frac{\xi}{r}\right) k_1 - \frac{\eta}{r} k_2 - \frac{\zeta}{r} k_3\right\} - \right.$$
$$\left. - \frac{a^2}{2r\lambda}\left\{(k_1^2 + k_2^2 + k_3^2) + \left(\frac{\xi}{r} k_1 + \frac{\eta}{r} k_2 + \frac{\zeta}{r} k_3\right)^2\right\} \right) \quad . \quad . \quad . \quad (3)$$

And in order to find the total vector of radiation we have to sum up the expression (3) over all molecules struck (or rather put into vibration) by the primary radiation. In doing so we obtain the formula given by LAUE and with that, his cones of maximal intensity.

However, we can show that there are other maxima still, besides the cones of LAUE. I will suppose $r$ to be so great that we can neglect the fourth term.

The maxima that do not appear in LAUE's theory can be made to appear by first taking into account the interference of the points for which

$$k_1 \left(1 - \frac{\xi}{r}\right) - \frac{\eta}{r} k_2 - \frac{\zeta}{r} k_3 = 0$$

Further I wil substitute $\xi/_r$ by $\alpha$, $\eta/_r$ by $\beta$, $\zeta/_r$ by $\gamma$, then $\alpha^2+\beta^2+\gamma^2=1$, thus in this notation we have to fix our attention upon the interference of the radiation from those points for which the numbers $k$ satisfy the equation

$$k_1(1-\alpha)-\beta k_2-\gamma k_3=0.$$

Now if this equation determines a great number of points, the pulses originating from the molecules will interfere without difference of phase.

This will be the case when the plane

$$x(1-\alpha)-y\beta-z\gamma=0$$

passes through the molecules of the crystal. Now, a plane through molecules may in general be represented by

$$\mathfrak{a}x+\mathfrak{b}y+\mathfrak{c}z=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where $\mathfrak{a}\,\mathfrak{b}\,\mathfrak{c}$ are whole numbers, that we constantly suppose to be reduced to their smallest values possible. The values of $\alpha\beta\gamma$, where maximal intensity is thus to be found on account of the cooperation of the points of a plane, we can find by putting

$$\frac{1-\alpha}{\mathfrak{a}}=-\frac{\beta}{\mathfrak{b}}=-\frac{\gamma}{\mathfrak{c}}$$

while $\alpha^2+\beta^2+\gamma^2$ must be 1. From this we find $\beta=0$, $\gamma=0$, $\alpha=1$ (i. e. the light transmitted directly, a point of interference that is not observable) and

$$\left.\begin{aligned}\alpha&=\frac{\mathfrak{b}^2+\mathfrak{c}^2-\mathfrak{a}^2}{\mathfrak{a}^2+\mathfrak{b}^2+\mathfrak{c}^2}\\ \beta&=\frac{-2\,\mathfrak{a}\,\mathfrak{b}}{\mathfrak{a}^2+\mathfrak{b}^2+\mathfrak{c}^2}\\ \gamma&=\frac{-2\,\mathfrak{a}\,\mathfrak{c}}{\mathfrak{a}^2+\mathfrak{b}^2+\mathfrak{c}^2}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Now we can easily show the direction thus found to agree with the direction in which the RÖNTGEN-beam would be reflected if the chosen plane rich in molecules should be a mirror. For the angle of the normal of (4) forms with the $x$-axis an angle of which the cosine is $\dfrac{\mathfrak{a}}{\sqrt{\mathfrak{a}^2+\mathfrak{b}^2+\mathfrak{c}^2}}$, the plane of incidence has for equation: $\mathfrak{c}y-\mathfrak{b}z=0$, the direction cosines of the reflected ray are $\alpha'\beta'\gamma'$. Thus we have

$$\left.\begin{aligned}(\alpha'+1)\,\mathfrak{a}+\beta'\,\mathfrak{b}+\gamma'\,\mathfrak{c}&=0\\ \beta'\,\mathfrak{c}-\gamma'\,\mathfrak{b}&=0\\ \alpha'^2+\beta'^2+\gamma'^2&=1\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The set of values (5) satisfies (6).

In this way we have shown the maximum to lie really in the direction of reflection. We can see this without calculation, and I principally gave the above calculation to show the connection between LAUE's considerations and mine.

For if $P$ the origin of rays, and $L$ the point of observation, both are situated at a distance from the molecules of a plane which is infinite with respect to the dimensions of the plane of which $A$ and $B$ are arbitrary molecules, then the way $PAL = PBL$, and there is interference of the light emitted by the molecules, if the angles of $PA$ and $AL$ with the normal of the plane are equal. Thus there is interference in $L$, if the point lies in the direction of the ray reflected in the plane. For the rest the disturbance of equilibrium, if $N$ is the number of particles of the plane, will be $N$ times as great as the disturbance caused by one particle, and therefore the intensity will be $N^2$ times as great.

The intensity of the maximum is of the order of the number of molecules in a plane, i. e therefore, of the order of the "two-cone" maxima of LAUE. As we may now presume, all pulses will interfere in the same direction which originate from planes in the crystal parallel to the one considered. The equation of similar planes is

$$\mathfrak{a}x + \mathfrak{b}y + \mathfrak{c}z = \pm sa$$

where I must be a whole number, $xyz$ being whole multiples of the side $a$, the coefficients $\mathfrak{a}$, $\mathfrak{b}$, and $\mathfrak{c}$ also being whole numbers.

Expressed in $\alpha\beta\gamma$ the equation takes the form

$$x(1-\alpha) - y\beta - 2\gamma = d.$$

We therefore have

$$\frac{\mathfrak{a}}{1-\alpha} = -\frac{\mathfrak{b}}{\beta} = -\frac{\mathfrak{c}}{\gamma} = \frac{sa}{d} = \lambda$$

which gives for $\alpha\beta\gamma$ the same values as in the preceding formula, whereas we have

$$d = \frac{2\mathfrak{a}}{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2} sa$$

or

$$a\{k_1(1-\alpha) - k_2\beta - k_3\gamma\} = \frac{2s\mathfrak{a}a.}{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2}.$$

It is easy to introduce into this formula the smallest distance of the planes under consideration. It amounts to $\frac{a}{\sqrt{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2}}$. For if

$\mathfrak{a}x + \mathfrak{b}y + \mathfrak{c}z = d$ is a plane, we pass to another plane of the same kind by putting:

$$\mathfrak{a}x + \mathfrak{b}y + \mathfrak{c}z = d + (\alpha_1\mathfrak{a} + \beta_1\mathfrak{b} + \gamma_1\mathfrak{c})a$$

where $\alpha_1\,\beta_1\,\gamma_1$ are whole numbers. Now the distance of the two planes considered is

$$\frac{a}{\sqrt{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2}}(\alpha_1\mathfrak{a} + \beta_1\mathfrak{b} + \gamma_1\mathfrak{c})$$

which, $\mathfrak{a}\,\mathfrak{b}\,\mathfrak{c}$ being given, must be a minimum. This minimum is reached if $\alpha_1, \beta_1, \gamma_1$, are such that

$$\alpha_1\mathfrak{a} + \beta_1\mathfrak{b} + \gamma_1\mathfrak{c} = 1.$$

$\mathfrak{a}$, $\mathfrak{b}$ and $\mathfrak{c}$ being given, this equation can always be satisfied in $\infty^2$ ways. The minimum distance of the planes I will represent by $l_m$. We may still observe that in applying the above results we have the means of easily comparing the number of molecules lying in the different planes. The number of molecules that each plane contains will be greater, the greater the distance of the planes of a given kind is. If the number of molecules pro unit of volume is $\mathfrak{r}$, then a plane with parameters $a\,b\,c$, contains $\frac{v}{\sqrt{a^2 + b^2 + c^2}}$ molecules pro unit of surface.

The plane of the kind considered, denoted by the parameter $s$, contains $N_s$ molecules. The contribution to the vector of radiation, originating from this plane, thus amounts to

$$\frac{NsA}{r}\cos 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} - \frac{2s\,\mathfrak{l}\,l_m}{\lambda\sqrt{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2}}\right)$$

Taking the sum with respect to $s$ over all possible values, then we obtain the total vector of radiation originating from the emission of molecules. Generally, however, the contributions to the vector of radiation here considered and originating from parallel planes, are incoherent, unless, which may exceptionally occur, $\lambda$ and $\frac{\mathfrak{a}\,l_m}{\sqrt{\mathfrak{a}^2 + \mathfrak{b}^2 + \mathfrak{c}^2}}$ are mutually measurable. If we have to do with several wave-lengths, this will certainly cause incoherence.

Now, the intensity of the maxima observed can easily be found if for a moment we imagine an equal number of points getting into vibration in all planes considered. Then, if $n$ is the number of planes considered, the intensity is

$$nN^2,$$

where $nN^2$ is therefore substituted for

$$\Sigma\, N_s^2$$

Taking into consideration that $nN$ represents the total number of the molecules struck by radiation $\Re$, then we see that the intensity of the maxima is proportional to

$$\Re N$$

so that the spots are the more intense according as they are caused by planes in which the number of molecules pro unit of plane is greater.[1]) We can even to some degree extend what was observed above, so as to come to a conclusion which perhaps can be controlled by experiments. Take an $x$-axis in the direction of the normal of the planes, then $x$ will pass through the values $\pm l_m \pm 2l_m \pm kl_m$ etc., in which the same positive and negative value ought to be taken for $\gamma$, when the origin is chosen in the centre of the plate. For each value of $x$ the part cut off from the plane by the incident beam can be calculated. Be this part $S_\gamma$, the number of molecules pro unit of plane is $\nu l_m$, the contribution to the intensity of the plane $S_\gamma$, therefore

$$\nu^2\, l_m{}^2\, S_\gamma{}^2$$

and the total intensity is therefore $\nu^2\, l_m{}^2\, \Sigma S_x{}^2$, for which we may approximately write

$$\nu^2\, l_m \int S^2\, dx.$$

By applying this formula in different cases, we may come to a further trial of the theory, however, we do not yet possess the necessary photometrical experimental measurements. The intensity of the maxima now under consideration is greater than that of the "two-cone" maxima of LAUE (of the order $10^7$ times as great), it is, however, of the order $10^7$ times as small as that of the 3 cone maxima of LAUE. However, the experiment forces us to such a degree to accept the explication by reflection, that probably in no other way than in the one described above the photograms may be explained, as I will show below.

We may still observe, that in the consideration as given above, the molecules are assumed to contain only one electron. We can, however, easily get rid of this supposition by multiplying $N$ and $\nu$ by $s$, where $s$ is the number of electrons pro molecule. Perhaps, by taking this into account, we may derive an estimation of the proportion of the numbers of electrons pro molecule in different crystallised matter.

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[1]) We may here observe, that by this we have the means of comparing the numbers $N$, in matter with given density, for planes that are struck by equal radiation under similar circumstances.

We may also observe, that in the direction of the propagation of the primary radiation too an interference can be noticed between the secondary pulses emitted and the primary radiation. At this interference a difference of phase shows itself, which to such a degree diminishes the primary radiation as is necessary to deliver the energy of secondary pulses emitted in the directions of reflection.

We can still somewhat nearer consider the influence of a single plane. Be the reflecting plane chosen as $yz$-plane, be the $xy$-plane the plane of incidence, and $a$ the angle of incidence Let us now consider the vector of radiation in a point

$$x = r \cos a, \qquad y = r \sin a + \eta, \qquad z = \zeta.$$

The vector of radiation is given by

$$\frac{A}{r} \sum_{k_1} \sum_{k_2} \cos 2\pi \left( \frac{t}{T} - \frac{r}{\lambda} + \frac{k_1 a}{\lambda r} \eta + \frac{k_2 a}{\lambda r} \zeta \right).$$

which, when summed up with respect to $k_1$ and $k_2$, will give

$$\frac{a_1 A}{r} \cos a\pi \left( \frac{t}{T} - \frac{r}{\lambda} \right) \cos N \frac{a\eta}{2\lambda} \pi \cos N \frac{a\zeta}{2\lambda} \pi \sin(N+1) \frac{a\eta}{2\lambda} \pi \sin(N+1) \frac{a\zeta\pi}{2\lambda} : \sin \frac{a\eta\pi}{2\lambda} \sin \frac{a\zeta\pi}{2\lambda}$$

For $\eta = 0$ $\zeta = 0$ we obtain the maximum found above (diffraction maximum of the order zero) with the intensity there given.

A second maximum (first maximum of diffraction) could appear if $\frac{\eta a}{2\lambda} = 1$, or $\frac{a\zeta}{2\lambda} = 1$, or thus if $\eta = \frac{2\lambda}{a}$ or $\zeta = \frac{2\lambda}{a}$. Now $r$ is about 4 in the experiments, and $a$ is of the order $10^{-8}$; should $\lambda$ be much smaller than $a$, then this second maximum would be observable. In the photograms we do not find diffraction-rings of this kind. Thus if the wavelength is very small with respect to $10^{-8}$ then such images do not occur, but if $\lambda$ is of the order of $a$ or not much smaller, then we can neither observe such images, the latest estimation giving for $\lambda$ a quantity of the order $10^{-9}$. This might well thought to be consistent with the result that circular fringes do not appear on the plates.

Bragg has explained the form of the spots, — ellipses whose long axis has the direction of the line perpendicular to the plane of incidence which belongs to the plane observed — by observing that the different layers are struck by waves not wholly parallel. However, he does not take into account that in each point the radiation of molecules of all the planes interferes. The form might rather be explained by observing that the intensity in the said direction approaches less rapidly to zero than that in the direction perpendicular to it, whereas we have also to take into account that the distance between the source of radiation and the point of observation is not infinitely great with respect to the dimensions of the plane struck by radiation. Trying to explain the form of the spots by assuming a rectilinear propagation we do not come to the right result. E. g., if we have to do with a reflecting plane lying oblique to the beam, then the photographic plate would cut the reflected cylindre just in an ellipse, whose longest axis is perpendicular to the direction in the plane already considered, whereas on the photograms we observe just the contrary.

In the pencil the beams are not wholly parallel. What is the influence of this on the diffraction image? If the beams forming a small angle will have to give the same reflected beam then the reflecting planes must form a small angle too, and otherwise. Now if $ax + by + cz = 0$ is the plane rich in molecules, then a plane very little differing from it as to its direction will be

$$\left(a + \frac{1}{p}\right) x + \left(b + \frac{1}{q}\right) y + \left(c + \frac{1}{r}\right) z = 0 \, ,$$

where $p$, $q$, $r$ are large whole numbers; or,

$$qr\,(pa + 1) + (bq + 1)\,pz + (cq + 1)\,pq = 0 \, .$$

This plane however will be very poor since $l_m$ here becomes $\frac{1}{\sqrt{q^2r^2(pa+1)^2+\ldots}}$, which is very small. The forming of the patterns is thus exclusively ruled by the planes very rich in molecules. Of course, each of the pencils in the incident beam gives a reflected pencil to a plane rich in molecules, but since the incident beams differ but a little, the reflected ones will not do so either. Always, when among the planes considered one is rich in molecules the spot will be formed by the influence of one of the pencils.

When we want to consider directly very thin pulses, we come to a problem which agrees in some way with the one treated by Prof. LORENTZ [1]). However, we can now directly consider the pulses reflected by the molecules, which were dealt with in this treatise, to be combined to pulses formed by the planes rich in molecules, since in this case each of such planes gives only one pulse. This fact hinders the coinciding of the pulses considered in the publication mentioned. Take e. g. pulses originating from a definite set of planes, be the dimension in the direction of the normal $l$, then we have $\frac{l}{l_m}$ pulses,

[1]) Verslagen Kon. Akad. v. Wet. XXI 1912/13 p. 911. „Over den aard der Röntgenstralen".

of pulse thickness $\Delta$, together having a thickness $l' = \frac{l}{l_m}\,\Delta$ or $\frac{l'}{l} = \frac{\Delta}{l_m}$, which is a small quantity so long as $\Delta$ is small with respect to $l_m$, as is generally the case. When the pulses do coincide, which again will be the case when we take into account the primary disturbances of equilibrium emitted successively by the anticathode, then the considerations developed by Prof. LORENTZ must be applied. Thus also when operating with the hypothesis that the RÖNTGEN rays exist in pulses, the incoherence of the pulses originating from the different parallel planes is a matter of fact, and therefore also on this assumption the intensity of the spots in the photogram will be proportional to the number of molecules pro unity of surface of the corresponding plane. We may suppose that in this direction also the solution is to be found of the question why the effect of the motion of heat which causes the molecules to vibrate around the corners of the net, is so small.

Now we may still with a single word discuss the photograms which were at our disposal.

The way in which they were taken agrees in many points with that of LAUE, only it has been somewhat less complicated. In order to shorten the time of exposition, a fluorescent screen was used. The spots occurring on the plates may be arranged very conveniently into ellipses, hyperbolas, straight lines and sometimes parabolas; as BRAGG has already explained, points of such a conical section originate from the reflection on planes rich in molecules, which have a line rich in molecules in common. The conic section then will be the inter-section of the photographic plate and a cone, produced by letting the incident beam turn about the said line rich in molecules.

The photograms at my disposal were:

1. Rock-salt. The direction of incidence was lying along a cubical axis. The diagram produced agrees with the one for zinc-blende. The distance of the crystal from the photographic plate was 4 cm., while 3.56 in LAUE's experiment. By magnifying LAUE's pattern in the corresponding proportion I got one *perfectly* congruent with that of Prof. HAGA. Only a few ellipses were missing or were represented less intense, which may be attributed to the fact that with NaCl the net is centric cubical, whereas $ZnS$ shows cubes with centric cube faces. This agrees with the crystallographically deducted cleavability, which lies in the direction of the plane richest in molecules. The fact that the patterns for matters of totally different kinds are identical, is a strong proof for the above developed theory.

2. $CaF^2$ transmitting the radiation along a triangular axis, gave a pattern identical with $ZnS$.

3. Topaz, transmitting radiation in the direction of the bisectrix of the acute angle of the optical axes, gave a pattern which can be explained by assuming the net of the molecules to be built up from parallellograms with equal sides in the plane perpendicular to the bisectrix, and by points perpendicularly placed above the net points obtained in this way.

From the photogram I calculated the angle of the *pg.* It amounts to 66°10'. A trying of this angle with the angles of the planes of the prism, known from crystallographic data, gives a suitable agreement. I hope to have an opportunity to calculate the proportion of sides etc. for more types of BRAVAIS nets. We may suppose that in this way we shall obtain the possibility of deciding between the different structure theories, and of coming to a rational description of crystals.

4. The experiment of reflecting RÖNTGEN rays on the cleavage plane published by BRAGG in "Nature" of 23 of Dec., was repeated with mica. Because of the plate being longer exposed this time, there appeared on the plate, besides the reflected spot upon the planes parallel to the cleavage plane already found by BRAGG, also a number of other points of which by far the greater part were lying upon an ellipse rather changed into a circle. For plane of incidence the principal cross-section had been chosen, the photographic plate was placed perpendicular to the plane of incidence. The circle was lying asymmetrically, although the plane of incidence had been chosen in a principal cross-section.

Supposing the monoclinic net for mica to exist in a rectangle (in the cleavage-plane) and a side inclining with respect to this rectangle, lying in a plane perpendicular to the cleavage plane, then in order to explain the patterns we must take for the proportion of the sides of the rectangle and the inclining side 8 : 13 : 100, and besides we must suppose the angle of the cleavage plane and the inclining side to amount to 85°. The pattern obtained can still better be explained by using the second net of the monoclinic system. The basis then is a *pg* with very long and almost equal sides, and an angle of about 85° between the short diagonal and one of the sides. The third side is perpendicular to the *pg* considered, the rectangle through the short diagonal of the basis is centric. The cleavage plane then is // to this rectangle. This structure shows for mica an approach to the hexagonal type.

The same results were shown by the pattern obtained when mica was crossed by a radiation in a direction perpendicular to the cleavage plane. The photogram so obtained was much weaker, although the time of exposition was taken equally long, and although the intensity of the primary radiation was the same. This may be explained by observing that in the reflection the cleavage plane rich in molecules gives a spot, which does not appear with the transmitted radiation. But the other images are to be taken with respect to corresponding planes. The explication therefore must run otherwise. In both cases a cylindrical pencil with cross-section of about 1 mm. strikes the plate. Consequently the part struck by radiation of the plane richest in molecules, the reflection taking place under an angle $\alpha$ near 90°, is a good deal greater, namely in the proportion $\frac{1}{\cos\alpha}$, the number of working layers being the same. In the most unfavourable case of the vector of radiation lying in the plane of incidence, the working vector of radiation, if $\alpha = 90 - \beta$ where $\beta$ is a small angle, is $-S \sin 2\beta$. The intensity of the image reflected thus will be proportional to $\frac{I^2 \sin^2 2\beta (o\omega)^2}{\sin^2 \beta}$ (where $o$ is the diameter of the pencil, $\omega$ the number of particles pro unit of surface). For the case of the vector of radiation lying in the plane of incidence, $\sin 2\beta$ in the numerator is to be substituted by the unity; then the intensity will be great. As the incident pencil is not polarised, we have to expect a stronger effect with the reflection than with the light being directly transmitted.

5. The reflection on rock-salt (perpendicular to a cubical axis) again gave a set of spots very clearly observable, situated on conical sections through the central spot. The spots were lying close together on the plate; as may be supposed they are partly to be assigned to different not wholly parallel layers in the crystal.

**Anatomy.** — *"Nerve-regeneration after the joining of a motor nerve to a receptive nerve."* By Prof. J. BOEKE.

(Communicated in the meeting of February 22, 1913).

After the primary discoveries of FONTANA, MONRO, CRUIKSHANK, at the end of the 18[th] century, no phenomenon of life has been more closely studied than the process of nerve-regeneration. Attention was drawn to the primary degeneration of the peripheral portion of a cut nerve deprived of its trophic centre, the ganglion cells (WALLER), and the manner after which a new nervous union was established