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KOPSCH with regard to the deviation from SUTHERLAND's formula of the observations on hydrogen at liquid air temperatures.

At hydrogen temperatures SUTHERLAND's formula is shown to be entirely unsuitable for expressing our results. It gives a value two or three times too small. We have tried to represent the series of measurements by another formula, and in column 4 under $\eta_{calc} \cdot 10^7$ we have given the values which we have calculated by the following formula.

$$\frac{\eta}{\eta_0} = \left(\frac{T}{273.1}\right)^{0.647}$$

As can be seen, this empirical formula agrees remarkably well with the values found over the whole extensive field of temperatures. In a following paper we shall discuss the values for

$$\frac{\eta_0}{\eta_t} \frac{\sqrt{T}}{\sqrt{T_0}}$$

which follow from our experiments, and further the viscosities at various temperatures for different substances in connection with the law of the corresponding states.

Physics. — *"Investigation of the viscosity of gases at low tempe-ratures. III. Comparison of the results obtained with the law of corresponding states"*. By H. KAMERLINGH ONNES and SOPHUS WEBER. Communication N°. 134c from the Physical Laboratory at Leiden by Prof. H. KAMERLINGH ONNES.

§ 1. *Dependence of the viscosity upon temperature.* We have already discussed this in our previous papers. It was shown that SUTHERLAND's formula in no way corresponded to the observations at low temperature, either for hydrogen or for helium [1]. REINGANUM's formula although founded upon acceptable hypotheses about the constitution and mutual action of the molecules, is even worse so long as we regard C in it as constant. This can be easily understood when we consider that SUTHERLAND's formula can be taken as a first approximation to REINGANUM's, and that the terms left out must lead to a further divergence from the observations. Neither is it possible to come to an even approximate agreement at low tem-

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[1] Shortly after our paper an important article by EUCKEN appeared in the Phys. Zeitschrift (April 15th 1913) in which observations concerning the visco-sity of helium and hydrogen taken from an as yet unpublished paper by VOGEL were communicated. Within the limits of accuracy, which in VOGEL's observations are given at 5% at hydrogen temperature, these confirm our measurements, of which the accuracy at hydrogen temperatures is to be put at about 1%.

91*

.perature with the empirical correction of Reinganum's C which Rappenecker has suggested.

We might for instance with helium take $C = C' T \log T$, to come to agreement with the observations. But then Reinganum's formula would simply be converted into our interpolation formula.

Keesom, in Suppl. N°. 25 and 26 of these communications has shown, that the second virial coefficient in the equation of state for hydrogen, at temperatures at which this gas may be regarded as di-atomical, can be very statisfactorily explained by the supposition that hydrogen molecules are hard spheres with electric doublets in the centia. His formula for the virial of the collisional forces under these circumstances gives a change of the radius of the molecule with the temperature, which for higher temperatures agrees fairly well with that deduced from the viscosity by $\dfrac{\eta_t \sqrt{T_0}}{\eta_0 \sqrt{T}}$. At lower temperatures at which hydrogen behaves like a monatomic gas, the formula for hard spheres with a central force according to the law r^{-q} becomes applicable, and Keesom finds this again confirmed by the change of the viscosity with the temperature.

But when we go down to $-193°$ C. deviations appear, in accordance with what we said above about Sutherland's formula, and at lower temperatures the value of the viscosity becomes much too small.

None of the formulas deduced from theoretical suppositions can represent the observations for helium ; for the present we can only use our empirical representation for this substance, which for hydrogen also holds good for lower temperatures than the theoretical formulas, viz. as far as the temperature of reduced oxygen. As regards the formula for helium, it is not impossible, that the straight line in the logarithmic diagram must be replaced by one that at low temperatures, and perhaps at higher ones also, curves somewhat towards greater values of the viscosity.

§ 2. *Application of the principle of mechanical similarity upon the comparison of the viscosities in corresponding conditions.*

If two substances may be taken as mechanically similar systems of molecules, it follows [1] that the viscosities for both in corresponding conditions must be in a constant ratio which may be calculated from the ratios of the units of length, time, and mass in both systems. On the other hand from the values of

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[1] H. Kamerlingh Onnes Verh. Kon. Akad. Amsterdam 21, p. 22, 1881 Beibl. 5. p. 718. 1881.

$$- \frac{1}{2} log \frac{\eta}{\sqrt{T}\sqrt{M}} = log\, \sigma - log\, c$$

where M is the molecular weight, σ the mean radius of a molecule, as it is effective in viscosity, and c a constant, the same for all substances, we may, when the curves which express the connection between η and $\frac{T}{T_k}$ are the same, infer the ratio between the units of length which have to be ascribed to the two mechanically similar [1]) systems. With the help of the viscosity we can, therefore, make a comparison of the just defined mean molecular radii and we may inquire how far the ratio found corresponds to that of the mean molecular radii, determined in the way that is necessary in the deduction of the equation of state. If this correspondence were complete, then, when the expression of σ given above, is expressed in reduced quantities, the curves which express the logarithm of the reduced σ as a function of the logarithm of the reduced temperature for the various substances, would coincide. The accompanying diagram shows in how far this is the case. In the construction $M^{-1/2} T_k^{-1/6} p_k^{2/3}$ has been used as the ratio by which the viscosities [2]) are deduced to the same imaginary system. In this we have taken p_k and T_k which hold for the critical state [3]), as determining quantities, and postponed the consideration of deviation functions still later [4]).

The first thing that strikes one is the great deviation of helium. In § 1 we remarked that the character of the viscosity of helium can be expressed by replacing the constant c, which may be understood as a measure for the attraction between the molecules, in REINGANUM's formula (differing by a constant factor from v in KEESOM's formula) by $c'\, T log\, T$. Perhaps this points to an increase with the temperature of the quantity which determines the attraction of helium.

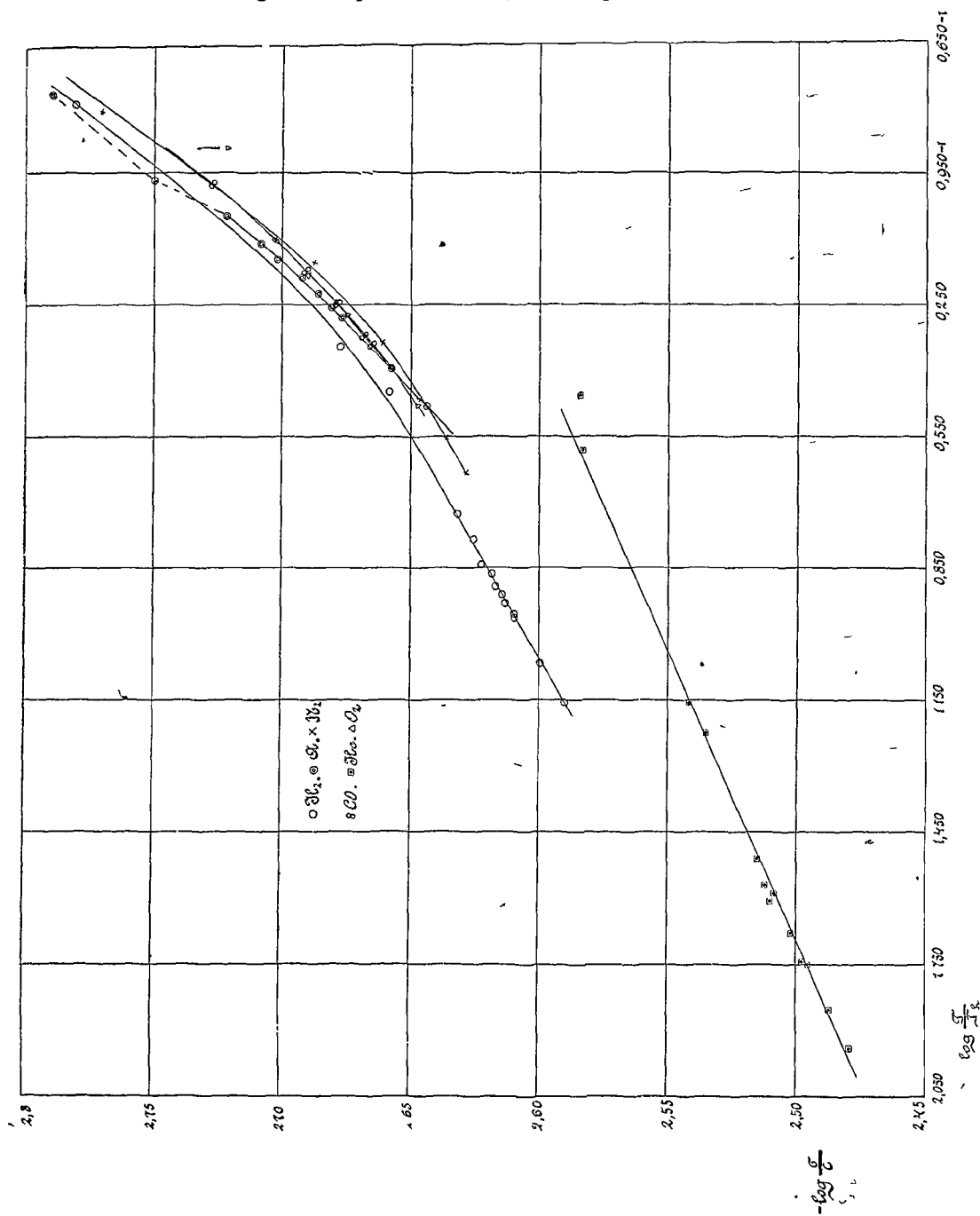[1]) More correctly: mechanically and statistically similar.

[2]) H. KAMERLINGH ONNES Leiden Comm. No. 12, p. 9.

[3]) The cristical dates we have used are the following.

	p_k (Atm.)	T_k
H_2	15.0	32°.0 K.
He	2.26	5 .25
O_2	50.0	155 .0
N_2	33.0	127 .1
Ar	48.0	150 .7
CO	35.9	132 0 [Note added in the translation]

[4]) KAMERLINGH ONNES and KEESOM. Suppl No. 23, § 38. The ratios found by KEESOM in Suppl. No. 25. p. 12, note 3, give 6 °/₀ deviation for hydrogen and argon, those used here 9 °/₀.

KAMERLINGH ONNES was also led to assume a similar increase of the attraction with the temperature, to explain peculiarities in liquid helium, and KEESOM in discussing the second virial coefficient of helium at higher temperatures, found that peculiarities of this coeffi-

cient might be ascribed to the same cause, in this case the receding of the attraction sphere (or the greater receding than in other substances) might be the cause of a slighter decrease of the viscosity at the reduction to lower temperatures. There might, however, also be an expansion of the molecule (in this case the atom) with temperature, and finally both phenomena might be dependent upon one cause, and go together. The possible small curvature for helium of the line in the logarithmic diagram [that we mentioned in § 1] in the opposite sense to that of the other lines which expresses the difference between helium and other substances, could be ascribed to this change in the attraction.

That which might explain the deviation for helium of the slope of the line from that which holds for a large range of temperatures for other substances, may also possibly help to explain the deviation from REINGANUM's formula at low temperatures, by the quantity which determines the attraction becoming smaller.

With hydrogen at the temperature of liquid air there is a distinct change in the slope of the curve. It is remarkable that the same is found with nitrogen, and perhaps also with oxygen and carbon monoxide, and that the point at which it occurs seems to lie at the same reduced temperature for hydrogen and nitrogen and perhaps also for oxygen and carbon monoxide. If this is the case, then the change which in the hydrogen molecules may according to KEESOM be taken as a change from hard spheres with electric doublets into hard spheres with a central force $r-q$ as far as the vis cosity, is concerned would be a similar process for all these different substances, determined by the same units of length, time, and mass as hold for the critical quantities, while this point only coincides with the point of transition in the specific heat of diatomic substances in the special case of hydrogen.

We must further notice the systematic differences between the different substances which appear from the non coincidence of the curves. It is remarkable that most of them (except a part of argon) can be removed by shifting the curves. The mean value of the molecular radius which comes into consideration for the viscosity seems thus to differ from the mean value which comes into consideration for the equation of state at the critical temperature, but both are in a fixed relation for the various materials over the whole field of temperature. This might be ascribed for instance, to a more elongated shape of the molecules in substances which give the smallest viscosity.