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were entirely due to the influence upon the polonium of the condensation of gases still present in the apparatus.

Experiments made in Leiden in liquid hydrogen with a provisional apparatus have convinced us that one might get rid of the condensations completely, even with liquid hydrogen, by using a ionisation chamber filled with pure gaseous hydrogen and a side tube with charcoal, immersed in liquid hydrogen.

Conclusions.

All these experiments which unfortunately are not so complete as we could have wished, confirm the independence of the radiation from the temperature, over a larger range of temperatures than had heretofore been done. Moreover these experiments have brought to light sources of error which must be taken into account, if one wants to make very accurate measurements at low temperatures.

Astronomy. — *"The periodic change in the sea level at Helder, in connection with the periodic change in the latitude"*. By Prof. H. G. v. d. Sande Bakhuyzen.

At the meeting of the Academy in February 1894 I read a paper about the variation of the latitude, deduced from astronomical observations, and added to this a determination of the change in the mean water level in consequence of the variation of the latitude.

Roughly speaking, one may regard the variation of latitude, as consisting of two parts, a periodic variation which takes place in one year, probably due to meteorological influences, and a periodic variation which takes place in about 431 days, which depends amongst other things upon the coefficient of elasticity of the earth, its resistance to change of shape. As a consequence of these changes of position of the axis of the earth oscillations of the same periods must take place in the mean sea level and if we eliminate the annual oscillation, the periodic variation of 431 days remains.

For the determination of the latter variation, I had made use of the mean sea level during the different months of the years 1855—1892, taken by the tide gauge at Helder. The results attained then for the amplitude and the phase of the periodic variation confirmed the opinion that such variations actually existed in the water, but as the changes in question are very small, it was desirable to extend the investigation in order to increase the accuracy of the results. I resolved therefore to submit to the calculations all the tidal observations made at Helder in the years 1855 -1912, and as the results of the years 1893—1912 were not at my disposal, Mr. GOCKINGA, Chief engineer Director of the "Waterstaat", was so good as to let me have the monthly averages of these years.

2. Before I give an account of how the monthly averages were used by me, it is desirable to explain the exact significance of the observation material. The tide curve of Helder, with its double maximum, has an asymmetrical form, which differs considerably from a sinecurve, so that to deduce the exact mean sea level during a day from the observations, one must either determine the area of the surface enclosed by the tide curve with a planimeter, or, as will also be sufficiently accurate, determine the average value of the 24 hourly heights. From the daily means one can then deduce the monthly means.

It will be clear that the work which is necessary to calculate all the observations in this way for the more than 21,000 days from 1855 to 1912 is very great; fortunately for our purpose we can use an easier way, as we do not need to know the actual mean heights, but only their mutual differences. If the tidal curve were symmetrical with respect to the mean sea level, the half of the sum of high and low water would correspond to the mean sea level of that day; but the form is not symmetrical, and even changes periodically, so that there is not only a difference between the half of the sum of high and low water, and the mean sea level, but this difference changes from day to day. If, however, we determine the average form of the tide curve during the period of a month, then we get a fairly constant shape, and for such a period one may assume, that the difference between the half sum of all the high and low waters and the mean sea level is almost constant. This assumption will differ even less from the truth, if we take the average of a great number of monthly means from different years, which is the case with my calculations.

On these grounds I have taken as the monthly means of the sea level the half of the sum of the high and low waters during these months, deduced from the registered tidal curves in the years 1855—1912.

These monthly means show rather marked deviations from the annual mean, due partly to the yearly and half-yearly sun tide, and partly to the regularly changing meteorological conditions. From 58 years, I found for Helder the following mean values for yearly means—monthly means in millimetres.

January,	February,	March,	April,	May,	June,	July,	August,
— 17.8	+ 28.5	+ 60.9	+ 102.4	+ 92.9	+ 48.0	— 1.6	— 38.4

September,	October,	November,	December.
— 42.6	— 85.2	— 75.1	— 72.5.

By the introduction of these corrections I have eliminated the influence of the yearly periodic variations in the water level.

In order to increase the accuracy of the values from which the results must be deduced and to remove entirely or partially the error that might arise, if the number of low waters in a month should be one less or more than the number of high waters, I have always taken the averages of two consecutive months: Jan. and Febr., Febr. and March, etc. The further calculations are based upon these two-monthly means.

Corrections for known tides are not introduced into these values. The influence of tides of short period is very slight upon the two-monthly means, and if, as is the case in my calculations, the average is taken of nearly 50 such means, it may be altogether neglected.

Of the tides of longer period we must mention, besides the yearly and half-yearly sun tide, the influence of which has been taken into account, the tide Mm, with a period of over 27 days. It appears from the calculations that the influence of this tide upon the two-monthly means can rise to about ± 6 mm. but as the amplitude and phase constant of this tide are very little known, we cannot calculate the exact value of the correction. We may, however, assume that in an average of about 50 of these values, for dates that correspond to very various phases of this tide, its influence may be neglected.

3. The length of the period of the latitude variation of about 431 days (CHANDLER's period) was deduced from long series of astronomical observations, by E. F. v. D. SANDE BAKHUYZEN, Dr. ZWIERS and me; the results obtained by us differ very little, but I take as the most accurate that deduced by Dr. ZWIERS in a paper in These Proceedings of June 24th, 1911, Vol. XIV, p. 111, that is 431,24 days.

In order to determine whether a variation in the sea level takes place in that period, I have, starting from the first bi-monthly mean for 31 Jan. 1855, determined the dates of the days, which fall 431,24 days later, or a multiple of that interval and then selected the bi-monthly means which are nearest to these dates, sometimes a little earlier and sometimes a little later, with a difference at most of 15 days. From all these mean sea levels, 49 in number, corresponding to the same phase of the latitude variation, an average is then formed. In a similar way the averages are taken from the series of sea levels which correspond to the phases of the latitude variation 1, 2, 3, ... 13 months later than 31 January 1855. These 14 months contain over 426 days, almost the entire CHANDLER period therefore.

I found for the deviations of these 14 values from their general mean:

− 10.1 mm.
− 9.6
− 5.8
+ 1.7
+ 11.0
− 4.2
− 13.4
+ 2.9
+ 1.6
+ 1.2
+ 9.0
+ 7.4
+ 4.1
+ 3.3

These numbers with the exception of the 4th and 5th seem to show, a periodic variation, and the assumption is permissible that the sea level at Helder undergoes a periodic change in the course of 431.24 days, and that the height, t days after the end of January 1855 is represented by

$$h = a \sin\left(\frac{t}{431.24}\times 360^\circ + \alpha_0\right) = a \sin(\varphi + \alpha_0) = a \cos\alpha_0 \sin\varphi + a \sin\alpha_0 \cos\varphi$$

$$= p \sin\varphi + q \cos\varphi.$$

The heights given in the above column are got by taking the average of the bi-monthly means; if at the beginning of the period $\varphi = \varphi_0$, and at the end $\varphi = \varphi_1$, then that average is

$$H = a\frac{\cos\varphi_0 - \cos\varphi_1}{\varphi_1 - \varphi_0}\cos\alpha_0 + a\frac{\sin\varphi_1 - \sin\varphi_0}{\varphi_1 - \varphi_0}\sin\alpha_0$$

or

$$H = p\frac{\cos\varphi_0 - \cos\varphi_1}{\varphi_1 - \varphi_0} + q\frac{\sin\varphi_1 - \sin\varphi_0}{\varphi_1 - \varphi_0}.$$

After the substitution of $\frac{\cos\varphi_0 - \cos\varphi_1}{\varphi_1 - \varphi_0}$ and $\frac{\sin\varphi_1 - \sin\varphi_0}{\varphi_1 - \varphi_0}$ in which $\varphi_1 - \varphi_0 = \frac{1}{1{,}1275}$, we get the following equations

$$
\begin{aligned}
+0.415\,p + 0.874\,q &= -10.1\\
+0.750\,p + 0.611\,q &= -\ \ 9.6\\
+0.940\,p + 0.230\,q &= -\ \ 5.8\\
+0.948\,p - 0.195\,q &= +\ \ 1.7\\
+0.772\,p - 0.529\,q &= +11.0\\
+0.447\,p - 0.858\,q &= -\ \ 4.2\\
+0.036\,p - 0.967\,q &= -13.4\\
-0.382\,p - 0.889\,q &= +\ \ 2.9\\
-0.727\,p - 0.639\,q &= +\ \ 1.6\\
-0.930\,p - 0.265\,q &= +\ \ 1.2\\
-0.954\,p + 0.160\,q &= +\ \ 9.0\\
-0.793\,p + 0.554\,q &= +\ \ 7.4\\
-0.479\,p + 0.841\,q &= +\ \ 4.1\\
-0.072\,p + 0.965\,q &= +\ \ 3.3
\end{aligned}
$$

Solving these by the method of least squares, we get

$$p = -4.40, \qquad q = +0.42,$$

therefore

$$h = 4.42 \sin(\varphi + 174°33').$$

The mean error of the unit of weight (mean of two consecutive months) is ± 51.5 mm., the mean errors of p and q are ± 2.86 and ± 2.89, and the probable errors ± 1.93 and ± 1.95 millimetres.

4. So far, we may deduce from this that the periodicity of the sea level in a period of 431.24 days is presumably real, although considering the small amount of this variation and the comparatively large value of the mean errors, a more detailed investigation as to the probability of the results is desirable.

For this purpose I have in the first place calculated the mean error of the unit of weight in another way, namely by taking the yearly means, and in the assumption of a small change in the sea level, proportional to the time, determining the mean error of a yearly mean and therefrom the mean error of the unit of weight; I found for the latter value ± 93.3 mm., much greater than the first value given. This shows that there are fairly large systematic errors in the sea levels, probably to a large extent caused by the circumstance, that the causes of deviations in the normal sea level are of lengthy duration, and thus can cause abnormally high or low sea levels during a long time.

In order to investigate this, I have taken the means of a series of 12 months in a different way, by combining the height in Jan. of the year a, with that in Feb. of the year $a+1$, in March of the year $a+2$ etc. From this follows for the mean error of the unit of weight ± 60.2 mm. which agrees much better with the value we found ± 51.5. The real mean errors of p and q therefore probably do not differ greatly from the values calculated.

5. A second way of judging of the reliability of the results obtained is the calculation of the same quantities from another combination of observations. For this purpose I chose the observations of 1855—1892, which I had calculated in 1894, but had now reduced to the yearly means with better values for the deviations of the monthly means and further the observations of 1893—1912. I found from both series of observations:

$$h = 4.50 \sin(\varphi + 168°.59) \quad . \; . \; . \quad (1855-1892)$$

and

$$h = 3{,}74 \sin(\varphi + 176°13') \quad . \; . \; . \quad (1893-1912).$$

By the change in the reduction numbers and a more accurate calculation, the formula for the sea level during the period 1855—1892 differs somewhat from the formula found in 1894. The striking correspondence between the three formulas now found for the periods 1855—1892, 1893—1912 and 1855—1912 is certainly largely due to accident, but it confirms the view that the variation in the sea level is real.

6. In order to test the efficiency of the method that I had followed, I applied it to two cases in which one could not a priori expect a periodic variation, and to another case in which the existence of such a variation was certain.

First I arranged the bi-monthly means in a period of 13 months or 395.75 days which is not a multiple of any period of a sun or moon tide, and in which therefore we could not expect any periodic variation of level. For this purpose I used the observations of 1855—1892, and got the following deviations of the sea level from their general averages.

	4.0 m.m.
—	14.8
—	4.1
+	8.4
+	5.1
+	2.0
+	9.3
—	2.8
+	0.8
+	2.9
—	9.8
—	4.6
+	11.5

A periodicity looks less likely here than in the first case. Further I arranged the bi-monthly averages according to the period of 438.096 days, which, according to a paper by SCHUMANN from Vienna, should represent the length of the CHANDLER's period. This value differs very greatly from the results obtained in Leiden, and is a priori improbable as it is only theoretically deduced from the elements of the moon's orbit, without taking into account the elasticity of the earth, which certainly has a great influence upon this value. From all the observations from 1855 to 1912, arranged according to the phases of a periodic variation in 438.096 days, in distances of a month, I got the following figures for the sea level.

—	18.0 m.m.
—	4.8
+	13.9
+	8.6
—	4.4
—	0.7
—	5.3
—	2.9
+	21.1
+	17.1
—	5.6
—	0.9
—	1.6
—	16.7

94*

In this series there is again little trace of a periodicity in a period of 438 days.

Finally I arranged the mean sea levels according to the phases of a period of 440.872 days, which is 16-times the period of the monthly moon tide Mm. the length of which is 27.5545 days. It is plain that the influence of this tide will only be felt to a very small degree in the bi-monthly means, as these are the means of two complete periods or 55.11 days and 5.7 days. The periodic variation in the bi-monthly means will be about $^1/_{16}$ of that which is due to the actual tide Mm.

After arranging and combining the bi-monthly means I got for the sea level at 14 different epochs with intervals of one month

— 2,6 mm.
— 7,2
— 7,3
+ 3,4
+ 7,2
+ 9,4
+ 7,1
+ 3,7
+ 12,2
+ 8,0
— 13,2
— 20,8
— 4,3
+ 4,3

The periodic character is here undeniable, and if we determine the amplitude of the Mm.-tide itself from these figures, we get for the amplitude 118.0 mm., whereas from the observations in 1892 I formerly got for the amplitude 83.4 mm. (Versl. Kon. Akad. v. Wet. Vol III, p. 197). The correspondence is satisfactory, if we consider that the error in the observations made in the above series appears in the amplitude multiplied by about 16

These different considerations give me reason to take the value found above for the periodic change of the mean water levels in the time of 431.24 days as correct within the limits of the probable errors.

The probable error of the amplitude 4.42 is ± 1.93, the probability that the amplitude lies between 0 and 8.84 mm. may therefore be put at 7.

7. We have next to discuss the question what the connection is

between this variation in the sea level and the change in the position of the pole. If the sea level always corresponded to the position of the pole, the lowest sea level at a given place would always correspond to the maximum of the latitude at that place.

In the formula for the periodic variation in the water level $\varphi=0$ for 1 Jan. 1855 = 2398585 Julian date, and as the change of φ per day is 0°. 83478, we may represent the formula for the height of the sea level on a day for which the Julian date is t by

$$h = 4{,}42 \; Sin \; \{(t - 2398585 + 209{,}1) \; 0°{,}83478\}$$

$$=$$

$$h = 4{,}42 \; Sin \; \{(t - 2398375{,}9) \; 0° \; 83478\}.$$

The height of the sea level is a maximum when the expression under the sine is 90°; thus we find

Maximum height of sea level for $t = 2398483{,}7$,
Minimum „ „ „ „ „ $t = 2398699{,}3$

and if we add to this $23 \times 431{,}25 = 9918.7$ we find

Minimum height of sea level for $t = 2408618{,}0$.

According to Zwiers (These Proceedings XIV p. 211) the Julian date for the maximum latitude for Greenwich is 2408580, and if we reduce this for the difference of longitude between Greenwich and Helder, the date for the maximum latitude at Helder is 2408585,7 which gives a difference with the date of the minimum height of the sea level of only 32,3 days.

If the latitude variation is really the cause of the variation in the sea level, some time will elapse between the maximum latitude and the moment of the lowest sea level; how much this will be, cannot be theoretically determined: it depends upon the configuration of the continents, but the small difference which has been found is an argument in favour of the hypothesis that there is a connection between the two phenomena.

We will now investigate the relation between the amplitude of the 431-days tide and the magnitude of the latitude variation. The distance from a point of the ellipsoid of the earth to the centre of the earth is approximately expressed by

$$log \; \varrho = C + {}^1/_2 \, M \, a \, Cos \, 2 \, \varphi$$

if a is the ellipticity of the earth, and the radius of the equator is taken equal to 1. If the pole moves through an angle $\Delta\varphi$ in the direction of the meridian of this point, so that the latitude becomes $\varphi + \Delta\varphi$, and the liquid and solid parts of the earth could immediately change so as to both acquire in relation to the new axis the same shape as they had to the original axis, then the distance from that point to the centre of the earth would vary by the amount $\Delta \, \varrho$, given by

$$\Delta \varrho = -\varrho\, a\, Sin\, 2\, \varphi\, \Delta\, \varphi$$

If we take for ϱ a mean value of 6367000 meters and for $a\ \frac{1}{297}$ then expressing $\Delta\, \varphi$ in seconds:

$$\Delta \varrho = 104\, Sin\, 2\, \varphi\, \Delta\, \varphi \text{ mm.}$$

The amplitude $\Delta\, \varphi$ of the latitude variation seems to be variable, as shown by the investigations of Dr. ZWIERS; as mean value I take $\Delta\, \varphi = 0'',16$, then for Helder with a latitude of about 53°,

$$\Delta \varrho = 16 \text{ mm.}$$

The displacement of water that is necessary for this change in the surface of the sea will be lessened by the attraction of the earth; NEWCOMB in his paper (M. N. R. S. vol 52 p. 336) estimates that the displacement is only half as great· $\Delta\varrho$ would be in this case about 8 mm.

The sea level is measured with reference to the solid earth, so that, in order to determine the relative variation of the sea level, one must also know the variation in shape of the solid earth, which of course depends upon its rigidity. In my former publication of 1894 I had deduced from a very approximate theory and very rough estimates, that the amplitude of the water movement would be about 4.5 mm. I do not venture to give such a theoretical deduction any more, especially as so little is known about the rigidity of the earth; whereas SCHWEYDAR found by observations with a horizontal pendulum at Potsdam,

$$17.6 \times 10^{11},$$

for the coefficient of elasticity of the earth, HAID of Karlsruhe deduced a much smaller value in exactly the same way from observations with horizontal pendulums in Freiberg and Durlach, namely

$$3.2 \times 10^{11} \text{ and } 3.0 \times 10^{11}.$$

So long as this great uncertainty about the elasticity of the earth exists, estimations are of little value, and we can only state that the theoretical value of the amplitude of the variation of the sea level is of the same order as that which is deduced from the observations.

These various considerations confirm the opinion, that the periodic variation of the sea level in 431,24 days, as it is deduced from the observations, is real within the limits of the probable errors and that it is a consequence of the latitude variation.

I think it is of importance to apply similar calculations to other long series of sea levels, as they might contribute towards the determination of the coefficient of elasticity of the earth.