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mined by phosphate- and acetate-mixtures, we found at $p_H = 6.00$ an optimal reaction to the action of the enzyme. On either side the action decreases, first slowly, afterwards rapidly. Even at $p_H = 4.5$ and 7.5 it is stopped almost completely. At these p_H's injury to the enzyme is out of the question during the whole time of the test. The place of the optimal p_H does not change even when the digestion-time is five times the ordinary duration. The influence of citrate-mixtures is much more inhibitory than that of phosphate- and acetate-mixtures. The inhibition is energetic especially on the side of the minor p_H's. This accounts for the fact that in citrate-mixtures the optimal reaction has shifted towards the neutral point.

Astronomy. — *"On absorption of gravitation and the moon's longitude."* By Prof. Dr. W. DE SITTER. Part I.

(Communicated in the meeting of November 30, 1912).

By absorption of gravitation we mean the hypothesis that the mutual gravitational attraction of two bodies is diminished when a third body is traversed by the line joining the first two. If this absorption exists, it will manifest itself by diminishing the attraction of the sun upon the moon during a lunar eclipse. Therefore, in order to test the reality of our hypothesis, we must compute the perturbations in the longitude of the moon which are a consequence of this decrease of attraction, and compare these computed perturbations with the well known deviations of the observed longitude from that derived in accordance with the rigorous law of NEWTON. NEWCOMB, in the last paper from his hand (M. N. Jan. 1909) has put before the scientific world the great problem of these deviations or "fluctuations" in the moon's longitude. They can be represented by a term of long period, for which NEWCOMB finds an amplitude of 12''.95 and a period of 275 years (great fluctuation), upon which are superposed irregular deviations (minor fluctuations), which amount to not more than ± 4'' in NEWCOMB's representation. Mr. F. E. Ross, NEWCOMB's assistant, has afterwards represented these minor fluctuations by two empirical terms having periods of 57 and 23 years and amplitudes of 2''.9 and 0''.8 respectively (M. N. Nov. 1911). The outstanding residuals are very small: after 1850 they seldom reach 1''. In the years before 1850 the minor fluctuations are not so well marked, probably because (owing to the smaller number and greater uncertainty of the available observations) too many years have been combined in each mean result.

The idea of explaining these fluctuations by an absorption of the gravitational attraction of the sun upon the moon by the earth during lunar eclipses, has for the first time been publicly worked out by Mr. BOTTLINGER[1]), the investigation having been proposed as the subject of a prize essay by the philosophical faculty of the University of Munich. I had also towards the end of 1909 commenced a similar investigation, which was however of a preliminary character and, as it did not lead to positive results, was discontinued and not published. The publication of Mr. BOTTLINGER's dissertation led me to resume the investigation.

The decrease of the attraction of the sun upon the moon can be taken into account by adding to the forces considered in the ordinary lunar theory a perturbing force acting in the direction of the line joining the sun and the moon, in the direction away from the sun. If the sun and moon are treated as material points, this force is

$$H = \varkappa \frac{m'}{\Delta^2} = \frac{n_0{}^2 m'^2 a'^3}{r'^2}(1-2\alpha)\,\varkappa \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The meaning of the letters is:
m' = mass of the sun,
n', a' = mean motion and mean distance of the earth.
n, a = the same elements of the moon (osculating values),
n_0, a_0 = the mean values of these elements,
Δ, r' = distance of sun from moon and earth,
$\alpha = a_0/a' \qquad m = n'/n_0$.

The effect on the elements of the moon's orbit can be computed by the ordinary formulas. The perturbing forces are:

radial	force	$H \cos\beta \cos(\zeta-\zeta')$,
transversal	,,	$H \cos\beta \sin(\zeta-\zeta')$,
orthogonal	,,	$-H \sin\beta$,

where ζ and ζ' are the selenocentric longitudes of the earth and sun, and β is the selenocentric latitude of the sun, the moon's orbital plane being taken as fundamental plane. For the instant of central eclipse we have $\zeta-\zeta' = 0$. The transversal force therefore changes its sign during the eclipse, and its total effect is very nearly zero. The effect of the orthogonal force is entirely negligible. In the expression of the radial force, we can put $\cos(\zeta-\zeta') = 1$. We have further with sufficient accuracy

$$\beta = s, \quad \zeta = w + 180^\circ, \quad \zeta' = w',$$

[1]) K. F. BOTTLINGER. Die Gravitationstheorie und die Bewegung des Mondes. Inaugural-Dissertation (München). 1912.
See also "The Observatory" November 1912.

where

$s =$ the moon's latitude,

$w, w' =$ true longitudes of moon and sun.

The radial force thus becomes $H \cos s$. It is easily verified that the mean motion (whose perturbation must be *twice* integrated to give the perturbation in longitude) is practically the only element which need be considered. We find

$$\frac{dn}{dt} = -\frac{3\, e \sin v}{a\sqrt{1-e^2}} H \cos s = -\frac{3n_0^2 m^2}{a}(1-2\alpha)\frac{a_0}{a}\left(\frac{a'}{r'}\right)^2 \varkappa \frac{e \sin v}{\sqrt{1-e^2}} \cos s, \quad (2)$$

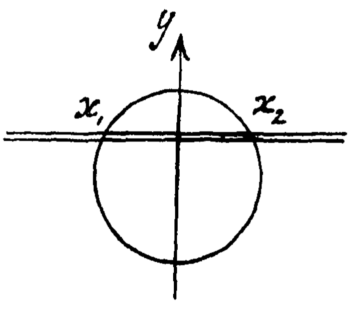
where v is the moon's mean anomaly. For the excentricity e we must use the osculating value. The mean value will be denoted by e_0, as for the other elements.

During the eclipse we can for the coordinates and elements of the moon use their values for the epoch of central eclipse. We then find for the addition to n as the effect of one eclipse:

$$\delta n = \int_{-T}^{+T} \frac{dn}{dt}\, dt = -3n_0^2 m^2 \frac{1-2\alpha}{a}\left(\frac{a'}{r'}\right)^2 \frac{a_0}{a} \frac{e \sin v}{\sqrt{1-e^2}} \cos s \int_{-T}^{+T} \varkappa\, dt, \quad (3)$$

where the time is counted from the middle of the eclipse, and T is the half duration.

Now assume the absorption of gravitation to be proportional to the mass of the absorbing body. We have then $\varkappa = \mu . \gamma$, where γ is the coefficient of absorption and μ the mass of that part of the earth that is traversed by the "ray of gravitation". This ray of gravitation, i. e. the infinitely thin cone enveloping the sun and moon, which are considered as points, by its motion during the eclipse cuts an infinitely thin disc out of the body of the earth. In the plane of this disc take two coordinate axes, of which the axis of x is parallel to the line joining sun and moon at the instant of centrality. If then ρ is the density and x_1 and x_2 are the points where the "ray" enters and leaves the earth, we have



$$\mu = \int_{x_1}^{x_2} \rho\, dx.$$

Further we have

$$dy = \frac{r'}{\Delta} \cdot r \frac{dw}{dt} \cdot dt$$

or

$$dt = \frac{(1+\alpha)\, dy}{r \dfrac{dw}{dt}}.$$

Consequently:

$$\int_{-T}^{+T} \varkappa dt = \frac{(1+\alpha)\,\gamma}{r \dfrac{dw}{dt}} \iint \varrho \; dx \; dy.$$

The double integral must be taken over the entire surface of the above considered section of the earth, and represents the mass of the infinitely thin disc. Its value therefore depends on the distribution of mass within the body of the earth. Like BOTTLINGER I take the distribution according to WIECHERT, i. e. a central core of density $\sigma_2 = 8.25$ surrounded by a mantle of density $\sigma_1 = 3{\cdot}30$. The radius of the core is $R_1 = 0{\cdot}77\ R$. If we call D the radius of the above considered disc, we can take $D = R \cdot \frac{T_0}{112}$, where T_0 is the half-duration of the eclipse computed with the *mean* elements of the moon's orbit, i. e. the value which is given in OPPOLZER's Canon der Finsternisse, expressed in minutes of time. The number 112 is the maximum of this half-duration.

We then find easily, in the case when the section is entirely in the outer mantle

$$\iint \varrho \; dx \; dy = \pi\ R^2 \sigma_1 \left(\frac{T_0}{112}\right)^2,$$

and when it also traverses the inner core (i. e. for $T_0 > 71.5$):

$$\iint \varrho \; dx \, dy = \pi\ R^2 \sigma_1 \left\{ 2.5 \left(\frac{T_0}{112}\right)^2 - 0.62 \right\}.$$

Now put, in the first case

$$J_0 = 100 \left(\frac{T_0}{112}\right)^2$$

and in the second case

$$J_0 = 100 \left\{ 2.5 \left(\frac{T_0}{112}\right)^2 - 0.62 \right\}.$$

The function J_0 which is thus defined, is tabulated in Dr. BOTTLINGER's dissertation, with the argument T_0. We have now

$$\int_{-T}^{+T} \varkappa\, dt = \frac{(1+\alpha)\,\pi\, R^2 \delta_1 \gamma}{100\, r \dfrac{dw}{dt}}\, J_0 \quad , \quad . \; . \; . \; . \; . \; . \qquad (4)$$

and this value must be subtituted in the formula (3).. In doing this, we can either express the coordinates and velocities in the osculating elements, or the latter in the former, by the well known formulas

$$\pi = \frac{1}{r} = \frac{1 + e\cos v}{a\,(1-e^2)} \qquad \frac{dr}{dt} = \frac{ane \sin v}{\sqrt{1-e^2}}$$

$$r^2 \frac{dw}{dt} = a^2 n \sqrt{1-e^2}$$

We then find

$$\delta n = -q J_0 \left(\frac{a'}{r'}\right)^2 \cos s\, \frac{a_0^{\,2} n_0}{a^2 n}\, \frac{e \sin v}{(1 + e\cos v)}, \quad . \; . \; . \; . \qquad (5)$$

or

$$\delta n = q J_0 \left(\frac{a'}{r'}\right)^2 \cos s \sqrt{1-e^2}\, \frac{a_0^{\,2} n_0\, \pi \dfrac{d\pi}{dt}}{\left(\dfrac{dw}{dt}\right)^2} . \; . \; . \; . \; . \qquad (6)\ ^{1)}$$

where we have put

$$q = \frac{3 n_0 m^2 (1-\alpha)\, \pi R^2 \delta_1}{100 a_0 \alpha}\, \gamma .$$

We can with sufficient accuracy [2]) take in the formula (5) $a_0^{\,2} n_0 = a^2 n$, and in the formula (6) $\sqrt{1-e^2} = \sqrt{1-e_0^{\,2}}$. The formulas can, however, not be used for the computations, unless they are so developed as to contain only such quantities as can be easily derived from existing tabl es.

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[1]) The formula (6) is derived by BOTTLINGER from the *vis viva* integral. In this derivation he introduces a couple of approximations, which are unnecessary, and which are the reason why the factor $\sqrt{1-e^2}$ does not appear in his formula. On his page 12 he takes *tan i* for *sin i*. If we retain *sin i* and replace it by its value $\frac{1}{V}\frac{dr}{dt}$, the square root drops out of the formula, and consequently the approximation introduced on page 13 in the development of this same root is also unnecessary. We then find $\Delta n = -\frac{3}{a^2 n} J \frac{dr}{dt}$. Now we have $\sqrt{1-e^2} = r^2 \frac{d\varphi}{dt}$ and $r = \frac{c}{\pi}$. BOTTLINGER's formula (I) on page 13 thus becomes $\Delta n = \frac{3\sqrt{1-e^2}}{c} \frac{d\pi}{d\varphi} J$, and his formula (I) on page (18) then becomes identical to our formula (6).

[2]) See however the footnote on p. 815.

The coordinates of the moon are developed in the lunar theory in series depending on the four arguments l, l', F and D, where l and l' are the mean anomalies of the moon and sun, F the mean argument of the moon's latitude, and D the difference of the mean longitudes of the moon and sun. For the *mean* opposition we have $D = 0$. The other three arguments are contained, under the names of I, II and III, in OPPOLZER's "Tafeln zur Berechnung der Mondfinsternisse". We have

$$l_1 = \frac{9}{10} II, \qquad l_1' = \frac{9}{10} I, \qquad 2F_1 = \frac{9}{10}(III - 37.66)$$

Denoting the mean longitudes by λ and λ', and the true longitudes by w, w', we have

$$w = \lambda + \delta l + \Delta\lambda, \qquad w' = \lambda' + \delta l',$$

where

$$\delta l = 2e \sin l + \frac{5}{4} e^2 \sin 2l - \gamma^2 \sin 2F$$

represents the elliptic term $\left(\gamma^2 = \sin^2 \frac{1}{2} i\right)$, and $\Delta\lambda$ the sum of all perturbations in longitude. The perturbations in the motion of the earth can be neglected. Then, denoting the values for mean opposition by the suffix 1, we have

$$\lambda_1 - \lambda_1' = 180°, \qquad w_1 - w_1' = 180° + \delta l_1 + \Delta\lambda_1 - \delta l_1';$$

for the instant of central eclipse on the other hand we have

$$w - w' = 180° - \gamma^2 \sin 2F.$$

We now put

$$\Delta = (w_1 - w_1') - (w - w') = \delta l_1 + \Delta\lambda_1 - \delta l_1' + \gamma^2 \sin 2F,$$

Then, $n(1-c)$ and $n(1-g)$ being the mean motions of the perigee and the node, we have, neglecting perturbations [1]):

$$\mu = \frac{dw}{dt} = n\,(1 + 2ce \cos l + \frac{5}{2} ce^2 \cos 2l - 2g\gamma^2 \cos 2F),$$

$$\mu' = \frac{dw'}{dt} = nm\,(1 + 2e' \cos l' + \ldots),$$

The time elapsed between the epochs of mean opposition and central eclipse is then

$$\Delta t = -\frac{\Delta}{\mu - \mu'}$$

At the instant of central elipse we have thus

$$l = l_1 + nc\Delta t, \qquad v = l + \delta l + \Delta l,$$

[1]) See however the next footnote.

where $\Delta l = \Delta\lambda - \Delta\omega$, $\Delta\omega$ being the perturbation in the longitude of the perigee. Further we have, to the order of accuracy here required, $\delta l = \delta l_1 - 2e\Delta \cos l$. Therefore, neglecting the difference between the perturbations Δl and Δl_1 at the two epochs, and putting $c = (1-m)\,c'$, we find

$$v = l_1 + \delta l_1' - \gamma^2 \sin 2F_1 - \Delta\omega - (c'-1)\Delta. \quad . \quad . \quad . \quad (7)$$

Now we have approximately $l' = l_1' - m\Delta$, and also $c'-1$ differs not much from m, therefore, if $\Delta\omega$ is neglected, we find from (7)

$$v - v' = l_1 - l_1' - \gamma^2 \sin 2F, \quad \text{or} \quad w - w' = \lambda_1 - \lambda_1' - \gamma^2 \sin 2F.$$

The term $\gamma^2 \sin 2F$ is the reduction from true opposition to central eclipse. Consequently the meaning of these formulas is: The difference of the true longitudes of moon and sun at true opposition is equal to the difference of the mean longitudes at mean opposition.

In the expression for Δ, which only occurs multiplied by the small factor $c' - 1$, we can neglect all perturbations except the evection. This latter is very easily applied by replacing e_0 in δl by $\frac{6}{7} e_0$ (see e. g. TISSERAND III p. 134). We have thus

$$\Delta = \frac{12}{7} e \sin l_1 - 2e' \sin l_1'.$$

We must now develop the quantity

$$K = \left(\frac{a'}{r'}\right)^2 \cos s \frac{e \sin v}{1 + e \cos v}.$$

where for v we must introduce the value (7). We can take with sufficient accuracy

$$\left(\frac{a'}{r'}\right)^2 = 1 + 2e' \cos l'.$$

Further we can take $\cos s = 1$, and we put

$$\Delta e = \Sigma \varkappa \cos x, \qquad e_0 \Delta\omega = \Sigma \varkappa \sin x,$$

It appears, in fact, on investigation that all perturbations which need be considered, are of this form. We then find easily

$$K = e_0 \sin l_1 - \left[\frac{1}{12} + \frac{6}{7}(c'-1)\right] e_0^2 \sin 2l_1 + (c'+1)\, ee' \sin (l_1 + l_1') + \Sigma \varkappa \sin (l_1 - x).$$

The perturbations Δe and $\Delta\omega$ are not as such contained in the existing lunar theories. I have therefore derived them, neglecting all perturbations that do not exceed $0.01\, e_0$. The only remaining term is again the evection. Those terms in the perturbing function, which in longitude give rise to the variation, produce a large perturbation in e and ω, but its argument is $x = l \pm 2D$, and consequently the corresponding term in K is zero, since $2D_1 = 0$ [1]). The evection-term has the argument $x = 2l - 2D$. The resulting term in K therefore has the same argument as the principal term. Finally I found in this way

$$K = e_0 \{0.858 \sin l_1 - 0.031 \sin 2l_1 + 0.033 \sin (l_1 + l_1')\}$$
$$= 0.0471 \{\sin l_1 (1 - 0.072 \cos l_1) + 0.039 \sin (l_1 + l_1')\} \quad . \quad . \quad (8)$$

In order to verify this result, I have also computed the formula (6). The values of π and $w - \lambda$ expressed in the arguments l, l', D and F were taken from BROWN's lunar theory. From these we easily derive $\frac{d\pi}{dt}$ and $\frac{dw}{dt}$.

We must then substitute for the arguments their values

$$l = l_1 + cn\,\Delta t \qquad D = 180^\circ + (1-m)\, n\Delta t$$
$$l' = l_1' + mn\,\Delta t \qquad 2F = 2F_1 + 2g\, n\Delta t$$

The value of Δt is given in OPPOLZER's "Syzygien-Tafeln fur den Mond", page 4. The value there given is the interval of time between mean and true opposition. To get the value for the epoch of central eclipse it is sufficiently accurate to omit the term $+0.0104 \sin (2g' + 2\omega')$. The interval thus computed must then be reduced to our unit of time (see below). The developments, which are rather long, finally led to the following formula, where nothing is neglected that can affect the third decimal place:

$$a_0^2 n_0 \frac{\pi \frac{d\pi}{dt}}{\left(\frac{dw}{dt}\right)^2} \cos s = 0.05404 \{0.8075 \sin l_1 - 0.0300 \sin 2l_1$$
$$+ 0.0300 \sin (l_1 + l_1') - 0.0020 \sin (2l_1 + l_1')$$
$$- 0.0033 \sin l'_1 - 0.0050 \sin (l_1 - l'_1)$$
$$+ 0.0016 \sin 2F_1 - 0.0055 \sin 2F_1 \cos l_1$$
$$+ 0.0114 \cos 2F_1 \sin l_1\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Eclipses occur near the node. Consequently $\sin 2F < \frac{1}{2}$. Thus, if we neglect all but the first three and the last term, none of the neglected terms exceeds $\frac{1}{200}$. Further $\cos 2F$ is always included between the limits 1 and 0.866. Therefore if we take $\cos 2F_1 = 0.96$ throughout, we cannot make a larger error than about $\frac{1}{10}$ of the last term. This latter then becomes $0.0110 \sin l_1$ and can be added to the principal term. We thus finally get the formula

[1]) The influence of the variation on the osculating values of a and n, is considerable, but it is the same in all oppositions, so that $a^2 n$ is a constant. The same thing is true of the error which is produced by our taking in μ, in the computation of Δt, the mean instead of the osculating value of n.

$$\delta n = -q_1 J_0 \{\sin l_1 (1-0.074 \cos l_1) + 0.037 \sin (l_1 + l_1')\}, \quad (10)$$

where

$$q_1 = 0.8185 \times 0.05404 \times q \sqrt{1-e_0^2} = 0.04473 q.$$

The agreement with (8) is very satisfactory [1].

We adopt as unit of time the mean interval between two successive eclipses, i. e. 6 synodic months or 177.18 days. Then taking as units of length and of density the earth's radius and the density δ_1 of the outer mantle, we find

$$q_1 = 1262''.\gamma$$

Calling λ the coefficient of absorption in the *C.G.S.* system of units, we have $\gamma = R\delta_1 \lambda$, and therefore

$$q_1 = 2656.''10^9.\lambda.$$

The formula (10) has been used to compute the value of δn for all eclipses occurring in OPPOLZER's Canon between 1703 and 1919. The coefficient q_1 was omitted, the results are therefore expressed in q_1 as unit.

Eclipses occur in groups of six. The interval of time between two successive eclipses of a group is 6 synodic months. In some groups there are only five or four eclipses: we can then still treat the group as consisting of 6 eclipses, if for the missing eclipses we assume $\delta n = 0$ [2].

Between each group and the next one or two eclipses are missed out, the interval of time between the last eclipse of one group and the first of the next group being in those cases 11 or 17 synodic months instead of 12 or 18.

Five groups make a Saros of 223 synodic months = 6585.2 days = 18.03 years.

The interval of 6 synodic months being the unit of time, the perturbation in n is derived by simply adding up the individual values of δn, i. e. forming the first series of sums. Then to get the perturbations in longitude we must again form the successive sums of these values of n, after having filled in so many times the final value of n of each group as there are empty places corresponding to the eclipses dropped out between that group and the next, remembering however that for *one* of these missing eclipses we must only take $^5/_6$ of this final value.

[1]) The difference in the multiplier outside the brackets is produced by the neglect of the influence of the variation in (8) (see preceding footnote).

[2]) In the course of time eclipses drop out at the beginning of the groups and new eclipses appear at the end. The limits of the groups are thus displaced within the Saros. During the interval of two centuries treated in this paper, it is not necessary to take account of this displacement.

In each of the two series of sums we can start with an arbitrary constant.

When the computations were carried out it appeared that always the values of ∂n summed up over a complete Saros gave a very small total, while the perturbation in longitude showed a very marked periodicity, with the Saros as period.

Accordingly I have divided the total perturbation into two parts: the periodic Saros and the remaining non-periodic part. I call Δn_p and $\Delta \lambda_p$ the increase of the mean motion and the longitude during the pth Saros, if the initial constants for both series of sums are taken zero. The purely periodic part of the perturbation during that Saros is then derived by taking for the initial constant of the first series of sums — i. e. the initial value of the perturbation in n — a value n_0 determined from the condition $37\frac{1}{6}\, n_0 + \Delta\lambda = 0$ $\left(37\frac{1}{6}\right.$ is the length of the Saros in our units of $\left.\text{time}\right)$. The perturbation in longitude at the end of the p^{th} Saros is then:

$$\lambda_p = \Delta\lambda_0 + \sum_{k=1}^{p} \Delta\lambda_k + 37\frac{1}{6}\left\{p\,\Delta n_0 + \sum_{k=1}^{p}(p-k)\,\Delta n_k\right\},$$

where Δn_0 and $\Delta\lambda_0$ are the initial constants of the two series of sums, i. e. the values of n and λ at the beginning of the first Saros. Putting now

$$\Delta\lambda_k = \Delta_0\lambda + (\Delta_1\lambda)_k\,, \qquad 37\frac{1}{6}\,\Delta n_k = \Delta_0\nu + (\Delta_1\nu)_k\,,$$

$$37\frac{1}{6}\,\Delta n_0 = -\,\Delta_0\lambda + \frac{1}{2}\,\Delta_0\nu + \nu_1\,,$$

we have:

$$\lambda_p = \Delta\lambda_0 + p\nu_1 + \frac{1}{2}\,p^2\Delta_0\nu + \sum_{k=1}^{p}(\Delta_1\lambda)_k + \sum_{k=1}^{p}(p-k)\,(\Delta_1\nu)_k\,, \quad (11)$$

which formula still contains two arbitrary contants $\Delta\lambda_0$ and ν_1. If for $\Delta_0\lambda$ and $\Delta_0\nu$ we choose the mean values of $\Delta\lambda_k$ and $37\frac{1}{6}\Delta n_k$, the terms under the signs Σ are small and of varying sign. The term containing p^2 is of the nature of a secular acceleration. If we denote the time expressed in centuries by τ, then p is equivalent to 5.55 τ, or $\frac{1}{2}p^2$ to 15.4 τ^2.

The individual values of ∂n will be given in the second part of this paper. Table 1 contains the values of Δn, $\Delta\lambda$, $\Delta_1\nu$ and $\Delta_1\lambda$ for each Saros.

TABLE I.

Year	Saros	Δn	$\Delta \nu$	$\Delta_1 \nu$	$\Delta_1 \lambda$	λ_1	λ_2	$\frac{\lambda_2}{160}$	Newc.
1703.0						+1091	— 628	- 3.9	— 5″4
	I	— 7.5	+1839	+103	—756				
1721.0						0	0	0	— 0.4
	II	— 2.3	+2180	+297	—415				
1739.1						— 647	+ 690	+ 4.3	+ 4.6
	III	— 2.2	+2299	+300	—296				
1757.1						— 878	+1414	+ 8.8	+ 9.1
	IV	—16.9	+2197	—246	—398				
1775.1						— 911	+1954	+12.2	+11:5
	V	—11.8	+2415	— 57	—180				
1793.1						— 972	+2084	+13.0	+12.9
	VI	— 8.3	+2565	+ 74	— 30				
1811.2						— 940	+1925	+12.0	+11.7
	VII	— 6.2	+2537	+152	— 58				
1829.2						— 862	+1430	+ 8.9	+ 8.7
	VIII	—11.1	+2627	— 30	+ 32				
1847.2						— 542	+ 795	+ 5.0	+ 4.1
	IX	—15.7	+2874	—202	+281				
1865.3						— 3	— 3	0	— 1.1
	X	—14.8	+3200	—168	+605				
1883.3						+ 658	—1061	— 6.6	— 6.1
	XI	—21.4	+3135	—413	+540				
1901.3						+1086	- 2734	—17.1	—10.2
	XII	— 5.3	+3269	+185	+674				
1919.4						+1237	—5066	—31.7	

We have $\Delta_0 \nu = - 382$, $\Delta_0 \lambda = + 2595$. If we neglect the term in p^2, and choose the values of $\Delta \lambda_0$ and ν_1 so as to make $\lambda_p = 0$ for 1721 and 1865, the perturbation in longitude given under the heading λ_1 results. If we add the term $\frac{1}{2} p^2 \Delta_0 \nu$, at the same time altering the initial constants so that the perturbation remains zero at the same two epochs, we get the values λ_2 [1]).

The reliability of these results of course depends on the reliability of the individual values of δn. The values of l_1 in two successive eclipses differ by 155°, consequently the values of δn have opposite signs and nearly destroy each other. Therefore, to arrive at a tolerable accuracy in the final perturbation in longitude, it is necessary to compute the individual δn to a much higher accuracy. The sum of the neglected terms in the series (9) will generally not exceed $^1/_{200}$, or in some cases perhaps $^1/_{100}$, of the whole. The maximum value of δn is about 190, we may thus expect on this account an error of one, or in extreme cases, 2 units.

The chief source of uncertainty is the function J_0. This function contains the hypothesis regarding the distribution of mass in the

1) In the original Dutch there was a mistake in the values of λ_1 and λ_2, which has here been corrected. The conclusions remain the same.

body of the earth. If a distrubution differing from WIECHERT's is adopted, the function J_0 is considerably altered. What is the effect of this on the final result can only be decided by actually carrying out the computation with a different hypothesis. This has been done, as will be related in the second part of this paper. Here it must suffice to state that, although there are some differences, the general character of the results is remarkably similar to those of the first computation. It may be mentioned that also my preliminary investigation of 1909, though based on a totally different and only roughly approximate formula, gave results of the same character.

The hypothesis that the sun and moon can be treated as points, is also, of course, only approximate, and it is very difficult to say in how far it affects the reliability of the results. It seemed however better, at the present state of the question, to rest content with this approximation.

The function J_0 however gives rise to errors in still another way. It is tabulated with the half-duration T'_0 as argument. This is taken from the Canon, where it is given in minutes of time, and can thus be a half, or in some cases perhaps even a whole minute in error. The resulting error in δn may occasionally amount to 4 units. Thus, neglecting the uncertainty introduced by the hypothesis regarding the distribution of density, the purely numerical error in δn may reach an amount which can be taken to correspond to a mean error of say $\pm$ 3 units. The mean error of the perturbation in n after p eclipses is then $\pm 3\sqrt{p}$. For a Saros (30 eclipses) this gives $\pm$ 16. Also the m. e. of the second sum (i. e. the perturbation in longitude, if we neglect the fact that sometimes the interval between successive eclipses differs from the normal value) is found to be $\pm \frac{1}{2}\sqrt{6\,p\,(p+1)\,(2\,p+1)}$. For the Saros this becomes $\pm$ 292.

It thus appears that all the values which have been found for Δn might very well be due to accidental accumulation of the inaccuracies of the computations. On the other hand the circumstance that they have the same sign throughout might lead us to consider them as at least partly real; by which I mean as necessary consequences of the adopted hypotheses. The values of $\Delta_1\lambda$ also are not so large that their reality can be considered as certain, but here also the systematic change with the time may be an indication of their being not entirely due to accidental errors of computation. The only thing that can be asserted with confidence is that the values of $\Delta_1 \nu$ and $\Delta\lambda_1$ are small, and consequently that the non-periodic part of the perturbations in longitude has a smooth-running course: no other irregularities with short periods can exist in the longitude than those which are contained in the periodic part.

This periodic part is very nearly the same in all Saros-periods. It will be given in detail in the second part of this paper. To show its general character I give here in Table II the mean for the last five periods VIII—XII (1829—1919), which are the most important for the comparison with the observations. The first column contains the time t counted in synodic months from the beginning of the

TABLE II.

t	λ_s	Form.	t	λ_s	Form.	t	λ_s	Form.	t	λ_s	Form.	t	λ_s	Form.
0	0	0	41	−521	−523	88	−573	−546	129	−34	−36	176	+312	+360
6	−75	−84	47	−560	−572	94	−557	−491	135	+64	+45	182	+311	+356
12	−143	−167	53	−578	−608	100	−545	−426	141	+133	+120	188	+321	+337
18	−309	−359	59	−654	−631	106	−362	−354	147	+237	+208	194	+305	+306
24	−321	−331	65	−538	−640	112	−319	−273	153	+239	+247	200	+240	+262
30	−441	−406	71	−603	−635	118	−216	−190	159	+262	+295	206	+272	+206
36	−487	−474	77	−582	−616	124	−117	−106	165	+316	+330	212	+174	+140
			83	−577	−583				171	+314	+352	218	+76	+66

Saros. This periodic perturbation can be represented with unexpected accuracy by the formula:

$$\lambda_s = -140 - 500 \sin\left[\frac{2\pi t}{223} - 16°26\right] \quad . \; . \; . \quad (12)$$

The values computed by this formula are given in the table under the heading "Form". The constant term, of course, is unimportant, and could be added to the arbitrary constant of integration $\Delta\lambda_0$. It would almost entirely disappear, if the Saros was begun at the end of the third group, say at about $t = 121$. If the time is expressed in years, the formula becomes

$$\lambda_s = -140 + 500 \sin\left[19°.967\,(t - 1900) + 137.°1\right] \quad . \quad (13)$$

The course of the perturbation in longitude is remarkably similar in the different periods, the irregularities, i.e. the deviations from the sine-formula, recurring in each period at the same values of t. The coefficient of the sine on the other hand varies from one period to another. For the first eight periods it oscillates between about 350 and 400, in the later periods it increases up to about 600 for the Saros XII (1901—1919).

Comparison with the observations. The excesses of the observed longitude of the moon over the longitude as computed by pure gravitational theory, which have been given by NEWCOMB, must still be corrected by the differences between the new lunar theory of BROWN and HANSEN's theory which has been used by NEWCOMB. The corrections necessary on this account have been collected by BATTERMANN[1]). Out of the 43 terms given by him we need only consider the terms of long periods (14)—(22) and (43). For the discussion of the non-periodic part of the perturbation in longitude we must take account of the terms (16) to (19), which have periods between 128 and 1921 years[2]). I have, however, not applied these terms, the reality of the non-periodic part being too uncertain to warrant much labour to be bestowed on it. For the discussion of the periodic part, we have to consider the terms (14), (15), (20), (21), (22) and (43), which can be written as follows:

(14)	$+0''.48 \sin 40°.67\ (t-1894.3)$	period	8.84	years
(22)	$+0\ .13 \sin 30\ .35\ (t-1894.6)$	„	11.87	„
(20)	$+0\ .24 \sin 20\ .66\ (t-1890.7)$	„	17.41	„
(43)	$+0\ .56 \sin 19\ .35\ (t-1892.2)$	„	18.60	„
(15)	$+0\ .13 \sin 10\ .34\ (t-1870.4)$	„	34.76	„
(21)	$+0\ .28 \sin 9\ .69\ (t-1877.6)$	„	37.14	„

The therm (43) contains the correction given by BATTERMAN in his "Zusatz". It is very similar to the term which was already applied by ROSS, viz: $-0''.50 \sin \Omega = +0''.50 \sin 19°.35\ (t-1894.8)$. These corrections must be added to the tabular longitudes, or subtracted from the residuals.

Considering now first the non-periodic part, it is very remarkable that the values of λ_2 as given in Table I are between the years 1703 and about 1894 almost identical to NEWCOMB's great fluctuation, if 160 of our units are taken equal to 1''. This is at once apparent from the last two columns of table I, of which the last contains the great fluctuation according to NEWCOMB. Therefore, if we assumed the absorption of gravitation to be the true explanation of the great fluctuation, we should have

$$160 \times 1262''.\gamma = 1'' \qquad \gamma = 5.10^{-6} \qquad \lambda = 25.10^{-16}.$$

However, after 1894 the similarity ceases. The agreement before that date depends on the assumption of the reality of the values

[1]) Beobachtungs Ergebnisse der K. Sternwarte zu Berlin, N⁰. 13, 1910.

[2]) The most important of these is a correction of 0.''85 to the coefficient of the well known Venus-term of 273 years period.

which have been found for Δn and $\Delta\lambda$, especially the negative value of the mean $\Delta_0\nu$. This latter is equivalent to a secular acceleration of which the coefficient would, with the above value of q, become — 37″. This, of course, is entirely inadmissible and consequently it is not possible to consider the value of $\Delta_0\nu$ as real unless we take for q such a small value that the whole effect becomes entirely negligible [1]. The partial agreement of λ_2 with the empirical terms of long period can therefore not be considered as a proof for the existence of an absorption of gravitation.

We now come to the comparison with the observations of the periodic part of our computed perturbation. This comparison was only carried out for the time after 1829. From 1847 to 1912 I had the advantage of being able to make use of a new and careful reduction of the Greenwich meridian observations which Prof. E. F. van de Sande Bakhuyzen most kindly placed at my disposal. Prof. Bakhuyzen applied to the meridian observations the correction for the difference of right ascension of the moon between the epochs of true and of tabular meridian passage, for those years in which this correction had not yet been applied at Greenwich. Then the systematic corrections, which in his former reduction (These Proceedings, Jan. 1912), were taken constant over the whole interval from 1847 to 1910, were derived anew. The following are the systematic corrections finally adopted by Prof. Bakhuyzen for the observations of the limb:

1847—48	49—57	58—68	69—78	79—98	1899—1911
0″.00	—1″.61	—0″.83	—0″.93	—0″.62	+0″.39

For the observations of the crater Mösting A the correction was derived in two different ways, which gave — 0″.22 and + 0″.34 respectively. The adopted correction is 0″.00. Prof. Bakhuyzen then formed the means of the meridian observations of the limb, of the crater and the occultations, the latter being taken from Newcomb's paper, but corrected by + 0″.18, for reasons explained in his paper of Dec. 1911. The corrected results of the meridian observations and the means thus derived are given in Table VII in the second part of this paper. From these means I then subtracted the theoretical corrections given by Battermann and quoted above. The resulting corrected means which are thus the excesses of the longitude of the moon over the pure gravitational value, diminished by Newcomb's great fluctuation, were plotted and a smooth curve was drawn through

[1]) In my former investigation I was led to a similar conclusion (see "The observatory" Nov. 1912 page 892).

them. From this curve were read off the values given below in Table III under the heading "Obs." If these are compared with the computed perturbation, of which the periodic part is also given in the table under the heading λ_s, there appears at first sight to be

TABLE III.

Year	Obs.	λ_s	Obs. $-\frac{\lambda_s}{500}$	Year	Obs.	λ_s	Obs. $-\frac{\lambda_s}{500}$	Year	Obs.	λ_s	Obs. $-\frac{\lambda_s}{500}$
1829	−0″3	+ 20	−0 3	1865	+3″8	+ 60	+3′7	1892	−2″8	−340	−2′1
35	−0.8	−550	+0.3	68	+2.4	−500	3.4	95	−3.1	+300	−3.7
41	−0.5	+60	−0.6	71	0.0	−630	+1.3	98	−2.0	+380	−2.8
47	+1.3	+ 10	+1.3	74	−1.8	−350	1.1	1901	+0.5	+ 40	+0.4
50	+1.1	−440	+2.0	77	−2.5	+230	−3.0	04	+1.4	−560	+2.5
53	+1.1	−550	+2.3	80	−1.4	+330	−2.1	07	+2.7	−640	+4.0
56	+2.0	−330	+2.7	83	−1.4	+ 50	−1.5	10	+4.4	−350	+5.1
59	+3.0	+170	+2.7	86	−2.2	−580	−1.0	12	5.1	+ 70	+5.0
62	+3.8	+270	+3.3	89	−3.0	−630	−1.7				

a certain similarity in the course of the two curves. Mr. BOTTLINGER, whose results on the whole agree with mine, has been led by this similarity to consider the existence of an absorption of gravitation as being established "mit guter Wahrscheinlichkeit". In fact, from about 1840 to 1868 the observed deviations can be very satisfactorily represented by about $\frac{1}{500}\lambda_s +$ a smooth curve, which latter then must either be ascribed to the non-periodic part, or remain unexplained. After 1868, however, the agreement is lost. We have again a partial parallelism between 1886 and 1891, and also the increase after 1908 coincides with an increase of λ_s, but it is impossible so to represent the observed values over the whole interval 1829 to 1912 by λ_s multiplied by a constant coefficient, that the remaining differences form a smooth curve. Still I think we cannot consider the probability of the existence of an absorption of gravitation as established unless the residuals remaining after applying the perturbation produced by this absorption (and which then remain unexplained), are small and form a smooth curve, or at least are less irregular than the original fluctuations. The values of Obs. — $k\lambda_s$ however, whatever value we adopt for k, always are considerably more irregular than the observed values themselves. The sudden fall between 1868 and 1874 coincides

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with a horizontal stretch (minimum) of λ_s, the quick rise from 1897 to 1906 corresponds to a decrease of λ_s. The effect of absorption cannot have another period than 18.03 years, while in the observed fluctuations periods of different length are certainly present.

It appears to me, therefore, that so far we have no reason to consider the existence of a sensible absorption of gravitation as proved, or even as probable.

(To be continued).

Astronomy. — *"On Absorption of Gravitation and the moon's longitude"*. By Prof. Dr. W. DE SITTER. Part II.

(Communicated in the meeting of December 28, 1912).

The conclusions derived in the first part of this paper are entirely confirmed by the second computation, which was already referred to in that part, and which was based on a different hypothesis regarding the distribution of mass in the body of the earth. I now assumed a core of density $\delta'_2 = 20$ and radius $R'_1 = 0.55\ R$, surrounded by a mantle of density $\delta'_1 = 2.8$ [1]). In the same way as before, I put, for $T_0 < 93.5$

$$J_0' = 84.7 \left(\frac{T_0}{112}\right)^2,$$

and for $T_0 > 93.5$

$$J_0' = 84.7 \left\{ 7.1 \left(\frac{T_0}{112}\right)^2 - 4.27 \right\}.$$

The multiplier 100 has been replaced by $84.7 = 100\ \delta_1'/\delta_1$ in order to get the same value of q for both computations. The result of the introduction of this new distribution of mass instead of the formerly assumed one is to increase the amount of absorption for long eclipses and to diminish it for short eclipses. The ratio J_0'/J_0 varies from 0.51 to 1.25. It is smallest for those eclipses in which with WIECHERT's hypothesis the core also contributes to the absorption, while in the new hypothesis the ray of gravitation is situated entirely in the mantle. For the purpose of computation this ratio J_0'/J_0 was tabulated with the argument T_0. We have then

$$\delta n' = \frac{J_0'}{J_0}\ \delta n.$$

[1]) This hypothesis has been suggested by recent investigations by Mr. GUTENBERG, which were kindly communicated to me by Dr. BOTTLINGER. Mr. GUTENBERG finds that the real distribution of mass is included between the limits given by $\delta'_2 = 20$, $\delta'_1 = 2.8$ and $\delta_2' = 8$, $\delta'_1 = 4.4$. It being my intention to investigate the effect of a change in the function J_0, I purposely took the upper limit, which differs most from WIECHERT's assumption.