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This causes that many of the properties already mentioned may be deduced and expressed in a much more simple manner. I will refer to this later when discussing the vapour pressures and boiling points of aqueous solutions saturated with salts and double salts, which in some cases have been determined experimentally.

(To be continued).

Chemistry. — "*Equilibria in ternary systems.*" III. By Prof. SCHRÖNEMAKERS.

(Communicated in the meeting of Dec. 28, 1912).

In the previous communications ¹⁾ we have assumed that in the system liquid-vapour occurs neither a maximum or minimum, nor a stationary point; we have also limited ourselves to the appearance of two three-phase triangles.

We will now discuss first the case that in the ternary system occurs a point with a minimum vapour pressure.

Let us imagine that in fig. 1 (1) the liquidum line de and the vapour line d_1e_1 of the heterogeneous region LG surround the saturation line of F , so that we get a diagram as in fig. 1. The saturation line of F is here surrounded by the liquidum region L , this by the heterogeneous region LG and this in turn by the vapour region. All liquids saturated with F therefore occur at the stated P and T in a stable condition.

On reduction of pressure, the liquidum region contracts so as to disappear simultaneously with the heterogeneous region LG in a point. This point represents for the stated temperature, the liquid and the vapour which, at the minimum pressure of the system liquid

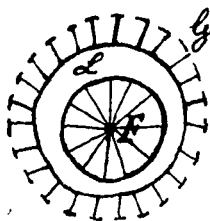


Fig. 1.

+ gas can be in equilibrium with each other. This point may occur without as well as within the saturation line of F . As at lower temperatures the region FL is generally large, but small at temperatures in the vicinity of the melting point of F , the said point will appear, at high temperatures, usually without, and at lower temperatures as well within as without the saturation line of F .

We now first consider the case where the point with a minimum vapour pressure falls outside the saturation line of F , or in other words that the liquidum and the heterogeneous region disappear in a point outside the saturation line of F .

¹⁾ These Proc. p. 700 and 853.

If starting from fig. 1 we now reduce the pressure, the liquidum line of the heterogeneous region approaches the saturation curve of F and meets this at a definite temperature. The diagram now formed may be deduced from fig. 2 (1) if we suppose the saturation curve of F therein to be surrounded by the curves de and d_1e_1 . The diagrams appearing on further reduction of the pressure can be represented by figs. 3 (1), 4 (1), 5 (1), 6 (1), or 3 (1), 8 (1), 9 (1) and 10 (1). In each of these figures, however, the curves de and d_1e_1 must be imagined to be bent in such a manner that they entirely surround the liquidum region, they finally disappear in the point with the minimum pressure.

From this it now follows that the liquid as well as the vapour of the three-phase equilibrium $F + L + G$ proceeds along a closed curve like in fig. 7 (1) or 11 (1), the saturation line under its own pressure is, therefore, again circumphased and the correlated vapour line circumphased or exp phased.

If we consider temperatures very close to the melting point of F , we find as in the first communication, that the saturation line under its own vapour pressure becomes exp phased and that we obtain diagrams such as in figs. 12 (1) and 13 (1).

We now consider the case where the point with minimum vapour pressure falls within the saturation line of F , or in other words, that the liquidum and the heterogeneous region disappear in a point within the saturation surface of F .

We again start from fig. 1 and reduce the pressure first of all until the liquidum and saturation curve come into contact, then until both curves intersect. We now obtain a diagram as in fig. 3 (1) in which, however, the saturation curve of F is supposed to be surrounded by the heterogeneous region $L G$.

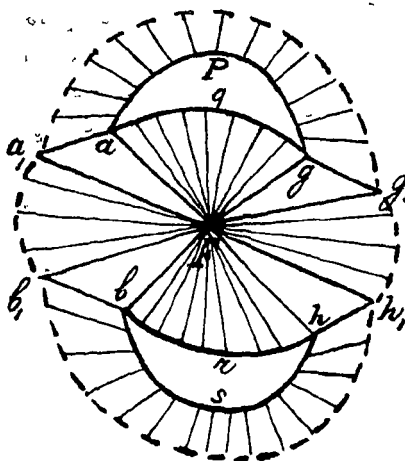


Fig. 2.

On further reduction of pressure, the liquidum line of the heterogeneous region and the saturation line of F may once more come into contact, so that on further reduction of pressure two new three-phase triangles are formed; we then obtain a diagram such as fig. 2 with four three-phase triangles. The liquidum region now consists of the two isolated pieces $apyq$ and $brhs$, the heterogeneous region likewise of two isolated parts, namely of a_1g_1gpa

and b^1h^1hsb , whereas the vapour region forms a coherent whole.

In fig. 2 we find the following equilibria:

Curve a^1g^1 represents vapours in equilibrium with liquids of the curve apg ;

Curve b^1h^1 represents vapours in equilibrium with liquids of the curve bsh ;

Curve a^1b^1 and g^1h^1 represents vapours in equilibrium with the solid substance F ;

Curve apg represents liquids in equilibrium with the vapours of the curve a^1g^1 ;

Curve bsh represents liquids in equilibrium with vapours of the curve b^1h^1 ;

Curve apg (and brh) represents liquids saturated with the solid substance F .

If, at the temperature and the pressure applying to fig. 2, we join the components, then, according to the situation of the figuring point, there is formed within:

the gasregion an unsaturated vapour:

the liquidum region an unsaturated solution;

$apgg_1a_1$ a vapour of a_1g_1 + a liquid of apg ;

$bshh_1b_1$ a vapour of b_1h_1 + a liquid of bsh ,

a_1b_1F a vapour of a_1b_1 + solid F ;

g_1h_1F a vapour of g_1h_1 + solid F ;

$aggF$ a liquid of agg + solid F ;

$brhF$ a liquid of brh + solid F ;

aa_1F vapour a_1 + liquid a + solid F ;

bb_1F „ b_1 + „ b + „ F ;

gg_1F „ g_1 + „ g + „ F ;

hh_1F „ h_1 + „ h + „ F .

On further reduction of pressure, the liquidum line apg and hsb which surrounds the liquidum region contracts still more so that on the one side the points a and g coincide at a pressure P , this will be likewise the case with their conjugated points a_1 and g_1 ; the two triangles Fa_1a and Fg_1g then coincide along a straight line and the pressure P for the system $F + L + G$ is a minimum pressure. The same applies when the two triangles Fbb_1 and Fhh_1 coincide.

After the four three-phase triangles have disappeared from fig. 2 owing to reduction of pressure, the vapour saturation line of F composed in Fig. 2 of the two branches a_1b_1 and g_1h_1 forms a closed curve which surrounds the heterogeneous region LG as well

as the saturation line of F . Hence, at these pressures only unsaturated vapours and those saturated with solid F can occur in the stable condition.

From a consideration of the equilibrium $F + L + G$ it appears that the saturation curve of F under its own vapour pressure is a curve surrounding the point F , on which however, now occur two points with a maximum vapour pressure. The same applies to the correlated vapour curve surrounding the former curve. Each maximum or minimum point of the one curve lies with the correlated maximum or minimum point of the other curve and the point F on a straight line.

We have assumed above that when the liquidum and the heterogeneous region disappear in a point within the saturation line of F two three-phase triangles, as in fig. 2. appear. We may, however, also imagine that the liquidum line of the heterogeneous region LG in fig. 1 contracts in such a manner that it intersects the saturation line of F in two points only; only two three-phase triangles are then formed.

The saturation line of F under its own vapour pressure and the correlated liquidum line are then both circumphased and exhibit one point with a maximum and one with a minimum vapour pressure. When the liquidum region disappears at one temperature within and at another temperature without the saturation point of F , it will, at a definite temperature disappear in a point of the saturation line. Among all solutions saturated at this temperature with F and in equilibrium with vapour there will be one which is in equilibrium with a vapour of the same composition. The saturation line of F under its own vapour pressure and the correlating vapour line then meet in the point with the minimum vapour pressure.

We have noticed above that there exist saturation lines of F under their own vapour pressure which exhibit two vapour pressure maxima and two minima. Such curves must, of course, be capable of conversion into curves with one maximum and one minimum; this takes place by the coincidence of a maximum and a minimum of the first curve causing the part of the curve situated between these two points to disappear. The two other parts then again merge in each other.

We have deduced above the saturation line under its own vapour pressure with two maxima and two minima in the assumption that the liquidum region disappears somewhere within the saturation line of F . We may also however, imagine similar cases if this disappearance takes place in a point outside the saturation line of F . We have only to suppose that in fig. 1 the liquidum line of the hetero-

geneous region LG contracts so as to disappear in a point outside the saturation line of F .

After the contact of the liquidum and saturation lines two points of intersection appear; if now no further contact takes place, these points finally coincide in a point of contact so that the saturation line under its own vapour pressure exhibits but one maximum or minimum.

If, however, after the appearance of the first two points of intersection a second point of contact occurs we obtain four points of intersection of which, at first two, and afterwards the other two coincide in a point of contact, so that in all four of these points are formed. The saturation line under its own vapour pressure then exhibits two maxima and two minima.

By way of a transition case it might happen that the second point of contact, which appears after the formation of the two first points of intersection, coincided with one of these points so that a point of the second order was formed. On further change of pressure two points of intersection then again occurred, which finally coincided in a new point of contact. The saturation line under its own vapour pressure then represents the transition form between that with one maximum and one minimum and that with two maxima and two minima.

After what has been stated it will surely be unnecessary to consider the case where, in the system liquid-vapour, a vapour pressure maximum or a stationary point occurs; we will refer to this and to a few peculiar boiling point lines perhaps later.

We will now just consider what happens if we take the compound F only and apply heat. If we imagine F placed in a vacuum at a low temperature a portion of this compound F will evaporate and there is formed the equilibrium: solid F + vapour F . On increase of temperature the vapour pressure of F is raised; in a $P.T$ -diagram we thus obtain a curve such as aK of fig. 3, namely the sublimation curve of the substance F . At a definite temperature T_k and a pressure P_k an infinitely small quantity of liquid is now formed; this, of course, has not the composition F but another composition K . As only an infinitely small amount of liquid has formed as yet, the vapour still has the composition F . The point K is, therefore the terminal point of the sublimation line, called by VAN DER WAALS in his binary systems the upper sublimation point of the compound.

If we increase the temperature, say, to T'' , more of the compound melts; there is then formed the three-phase equilibrium $F + L + G$ in which neither L nor G have the composition F . L and G have

such a composition that we can form from both the solid substance F ; the three figuring points are, therefore, situated on a straight

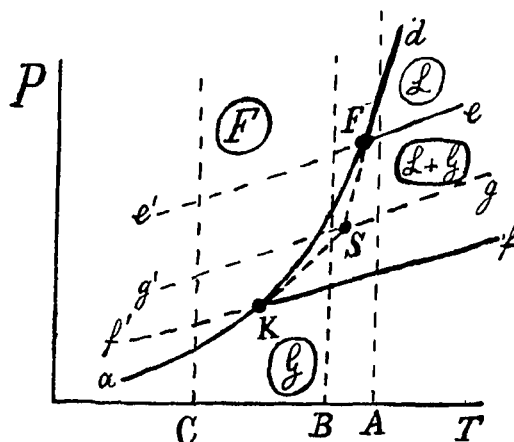


Fig. 3.

line. Besides, L and G are always present in quantities equivalent to the reaction $L + G \rightarrow F$; L and G are, consequently, present in such amounts that from both we can form F without any L or G remaining.

As a rule, the three-phase equilibrium $F + L + G$ can exist, at the temperature T' with a whole series of pressures, namely, with the pressures occurring on the saturation line under its own vapour pressure of the solid substance F at the temperature T' . As in this particular case a phase reaction is possible between the three phases or in other words, as the points F , L , and G lie on a straight line, the three-phase equilibrium exists here only at a definite pressure, namely, the maximum or minimum pressure which occur at the temperature T' on the saturation line of F under its own vapour pressure. In this particular case it is the minimum pressure, as will appear later.

At a further increase of temperature more of the substance F keeps on melting and L and G alter their composition; we will regulate the volume in such a manner that there is but an infinitely small amount of vapour which, of course, does not affect the pressure. If we represent the pressure and temperature graphically, a curve is formed such as curve KF of fig. 3.

Finally we now arrive at a temperature and correlated pressure at which all solid F has fused; as particularly at the last moments, we have taken care that but infinitely little vapour is present, the liquid now has the composition F ; the vapour has quite a different composition D .

As the solid substance F and the liquid now have the same composition we have attained the melting point of F . If now we regulate the temperature and pressure in such a manner that the solid matter F remains in equilibrium with its melt the system proceeds along the melting point line Fd of fig. 3. Here, it has been assumed that the volume v of the solid substance is much smaller than the volume V of its melt. If this is not the case, the melting point line Fd starts from F towards lower temperatures. In binary systems, VAN DER WAALS has called the initial point F of the melting point line, the minimum melting point of the solid substance F .

Hence, we have forced the substance F to proceed along:

- a. the sublimation line aK
- b. the three phase line KF
- c. the melting point line Fd

we can, however, consider still other lines.

In the upper sublimation point K we have solid F + vapour F + infinitely little liquid. We now increase the volume until the solid substance F has been converted totally into vapour, or else we remove the solid substance. We then have the system: vapour F + infinitely little liquid or we may also say, a vapour F which can be in equilibrium with a liquid. If the temperature is increased the vapour F will continue to exist; it is then, however, no longer in equilibrium with liquid. In order, to again form an infinitely small quantity of liquid, or in other words to again bring the vapour in equilibrium with a liquid, it will generally be necessary to increase the pressure.

Hence, at an increase in temperature, one can always regulate the pressure in such a manner that a vapour of the composition F is in equilibrium with an infinitely small quantity of liquid which, of course, changes its composition with the temperature. If pressure and temperature are represented in fig. 2, the curve Kf of this figure is formed.

In the minimum melting point F we can start from the system solid F + liquid F + infinitely little vapour after we have first eliminated the solid substance F thereof. If now, we elevate the temperature, the pressure may be always regulated in such a manner that this liquid of the composition F is in equilibrium with an infinitely small quantity of vapour which, of course, changes its composition with the temperature. The corresponding PT -line is represented in fig. 3 by the curve Fv .

As, on the line eF , a liquid of the composition F is in equilibrium with vapour we will call this line the evaporation line of F . On

the line Kf a vapour of the composition F is in equilibrium with liquid; we will, therefore, call Kf the condensation line of F . The metastable prolongations of Fe and Kf are represented in the figure by Fe' and Kf' . Hence, in point F three curves coincide namely, the melting point line (Fd), the evaporation line (Fe) and the three-phase line (FK); in point K three curves also meet, namely, the sublimation line (aK), the three-phase line (KF) and the condensation line (Kf).

The metastable prolongations of the sublimation line aK and the melting point line dF intersect in a point S ; at this temperature T_S and pressure P_S now occurs, in a metastable condition, the equilibrium: solid F + liquid F + vapour F . If now the substance F behaved as a simple substance which can only yield a liquid and a vapour of the same composition, S would represent the triple point of the substance F ; owing to the occurrence of the three-phase equilibrium $F + L + G$ this triple point is, however, metastable here. Through this metastable triple point S now also passes, besides the sublimation and the melting point curve of F , the evaporation line $g'Sg$ of F . This represents the equilibrium liquid F + vapour F occurring in the metastable condition; on this curve $g'Sg$ liquid and vapour, therefore, have the composition F and not, as on $f'Kf$, only the vapour, and as in $e'Fe$ only the liquid. We will call the curve $g'Sg$ the theoretical evaporation line.

In order to find what conditions of the substance F are represented by the points of the different regions we take this substance in a condition answering to a point of the sublimation line aK . We then have solid F + vapour F . From a consideration of what takes place on supply or withdrawal of heat, or on increase or decrease in volume we now deduce: to the right and below the line aK occurs the vapour region, to the left and above the line aK is found the solid region of F .

Acting in a similar manner with the points of the other lines, we find that four regions may be distinguished, namely, a gas region indicated in the figure by an encircled G , a solid region indicated by an encircled F , a liquidum region indicated by an encircled L and a liquidum-gas region indicated by an encircled $L + G$. Hence if the substance F is brought to a temperature and under a pressure corresponding with a point of the solid region, the substance F is solid; if brought to a temperature and under a pressure corresponding with a point of the liquidum-gas region, F is resolved into liquid and gas etc.

We will also consider fig. 3 just once more in connection with the

previously mentioned saturation lines of F and the liquidum and vapour lines of the heterogeneous region $L + G$. For this, we first choose a temperature T_A corresponding with point A of fig. 3 and a very high pressure so that we find ourselves in the solid region.

On the pressure being reduced we arrive from the solid region into the liquidum region, then into the liquidum-gas region and finally into the gas region. If we choose a temperature T_B corresponding with point B of fig. 3, the substance F on reduction of pressure first traverses the solid region, then the liquidum-gas region and finally the gas region. Reduction of pressure at the temperature T_C transfers the substance from the solid region to the gas region.

We now start from the temperature T_A and a very high pressure: the corresponding diagram then consists of fig. 1 (1) wherein, however, is still wanted the gas region and the heterogeneous region $L + G$ of this figure. It is now evident that the compound F can only exist in the solid condition; it can, of course, be in equilibrium with a liquid, but this liquid cannot form unless to the compound is added a little of at least one of its components. The pure compound F which we have still under consideration can only occur in the solid condition.

On reduction of pressure, the saturation line of F contracts so as to coincide finally with point F of fig. 1 (1). At this pressure occurs, therefore, the equilibrium solid F + liquid F , so that in fig. 3 we proceed from the solid region to a point of the melting point line Fd . The heterogeneous region $L + G$ of fig. 1 (1) may, or may not, have appeared at this pressure, in any case, however, it has not yet extended to the point F of this figure.

As, on further reduction of pressure, the saturation line of F disappears from fig. 1 (1) (in order to keep in with fig. 3 we take $V > v$) F is now situated in the liquidum region of fig. 1 (1). Hence, in fig. 3 we must also arrive in the liquidum region. As on further reduction of pressure the gas region of fig. 1 (1) is further extended, the liquidum line ed of the heterogeneous region passes, at a definite pressure, through the point F . This means that the liquid F can be in equilibrium with vapour. This is in agreement with fig. 3; therein we proceed from the liquidum region to the line Fe .

On further reduction of pressure, the heterogeneous region $L + G$ shifts over the point F ; the compound F is now resolved into a liquid of the liquidum line and into a vapour of the vapour line which on further decrease in pressure always change their composition. Hence the compound F traverses the liquidum gas region

which is in agreement with fig. 3. This will continue until on further reduction of pressure the vapour line of the heterogeneous region passes through point F . This means that a vapour F can be in equilibrium with a liquid; this again is in harmony with fig. 3; therein we proceed from the liquidum gas region to the curve Kf . On still further reduction of pressure the gas region of fig. 1 (1) moves over the point F so that, in harmony with fig. 3 the compound F can occur only in the state of vapour.

Between the liquidum line de and the vapour line d_1e_1 of the heterogeneous region $L + G$ of fig. 1 (1) is situated the projection of the line of intersection of the liquidum and the vapour side of the ζ -surface. This line indicates a series of solutions which each can be in equilibrium with a vapour of the same composition; all these liquids and vapours, however, are metastable and break up into a liquid of the liquidum line and a vapour of the vapour line of the heterogeneous region $L + G$. We will call this line of intersection the theoretical liquidum-vapour line.

As this theoretical line passes, at a definite pressure, through the point F , there exists at this pressure the equilibrium: liquid $F +$ vapour F in a metastable condition; hence, we have a point of the theoretical evaporation line Sg of fig. 3 and it is, moreover, evident that this must be situated in the liquidum-gas region of fig. 3.

We now choose a temperature T_B lower than T_A ; this will cause the saturation line of F to disappear at T_B at a lower pressure than at T_A . We now choose T_B so low that, on lowering the pressure the saturation line of F has not yet disappeared when the liquidum line of the heterogeneous region passes through the point F ; T_B is, therefore lower than the minimum melting point of F . If we now choose a very high pressure, the corresponding diagram will then consist of fig. 1 (I) wherein, however, the gas region and the heterogeneous region $L + G$ are still wanting. On reducing the pressure fig. 1 (I) is formed first, then fig. 2 (I) and further fig. 3 (I); at these pressures the compound F still occurs in the solid condition so that it finds itself in the solid region of fig. 3. At a definite pressure the metastable part of the liquidum line $dabe$ situated between the points a and b in fig. 3 (I) will pass through the point F ; this means that a liquid of the composition F may be in equilibrium with vapour, this is only possible in the metastable condition for in the stable condition F only occurs as a solid. Hence, in fig. 3 we find ourselves in the solid region on a point of the metastable curve eF .

On further reduction of pressure there is now formed from fig.

3 (I) the figure 4 (I) or 8 (I), we first choose T_B in such a manner that on lowering the pressure, the vapour saturation line has not yet disappeared when the vapour line of the heterogeneous region passes through the point F . So as to be in harmony with fig. 3, T_B has been chosen lower than the minimum melting point and higher than the upper sublimation point of the compound F . In consequence of this, fig. 3 (I) is converted into fig. 4 (I) on reduction of pressure, and afterwards at a definite pressure into fig. 5 (I). At this pressure the as yet solid compound F melts with formation of the vapour m_1 and the liquid m ; hence in fig. 3 we proceed from the solid region to a point of the three-phase line KF .

On further decrease of pressure F is resolved into liquid and gas; in fig. 3 we, therefore, proceed from the line KF to the liquidum gas region. On further reduction of pressure the vapour curve $e_1 d_1$ of fig. 5 (I) passes, at a definite pressure through the point F ; this means that a vapour of the composition F can be in equilibrium with a liquid. The compound F then passes, in fig. 3, from the liquidum-gas region to the line Kf . On further decrease of pressure is now formed fig. 6 (I), the point F lies now in the vapour region so that the compound F can only still occur in the state of vapour.

In fig. 3 we, therefore, proceed from the line Kf to the gas region.

Between fig. 3 (I), in which we assume the metastable part ab of the liquidum line $dabe$ to pass through the point F , and fig. 5 (I), in which we assume the vapour line $d_1 e_1$ to pass through F , there must, of course, lie another one where the theoretical liquidum vapour line passes through point F . This means that, in fig. 3, we must find, at the temperature T_B , between the curves $e'F$ and Kf a point of the curve $g'Sg$. If this theoretical vapour curve already passes through the point F before fig. 5 (I) is formed through reduction of pressure, the point of intersection of $g'Sg$ with the vertical line then lies in the point B of fig. 3 above the three-phase line; if, however, this theoretical line passes through the point F when, through reduction of pressure, fig. 5 has formed, the above point of intersection in fig. 3 lies below the three-phase line. These results, as follows from fig. 3, are in harmony with this figure.

The situation of the metastable sublimation line KS and of the metastable melting point line FS may be found in this manner. Here, we will just determine the situation of the triple point S . In this point there exists an equilibrium between solid $F +$ liquid $F +$ vapour F .

The equilibrium liquid $F +$ vapour F requires that the theore-

tical liquidum vapour line passes through point F ; if this equilibrium occurs in the stable condition, the liquidum and the vapour line of the heterogeneous region must then also pass through the point F ; this is the case when, incidentally, a ternary maximum, minimum or stationary point occurs in F . If, however, this equilibrium appears in the metastable condition, the liquidum and vapour line of the heterogeneous region do not pass through F which is then situated between these two. As, from the equilibria solid F + liquid F + vapour F and solid F + vapour F , it follows that the saturation and the vapour saturation line of F coincide to one point in F , the metastable triple point S must be situated in the liquidum gas region of fig. 3.

We now choose a temperature T_c (fig. 3) lower than the upper sublimation point T_k of fig. 3; the vapour saturation line of F has, therefore, not yet disappeared when the vapour line of the heterogeneous $L + G$ passes through the point F . Starting from high pressures and then reducing the same there is first formed fig. 1 (I) wherein, at first, the gas and heterogeneous regions are still wanting, then figs. 1 (I), 2 (I) and 3 (I) which is now converted into 8 (I); then are formed figs. 9 (I) and 10 (I) and finally a figure which we will call 10a and which is formed from fig. 10 when the vapour saturation line of F coincides with the point F .

During this lowering of the pressure, as shown from the figures, the substance F only occurs solid in the stable condition; the substance F , therefore, traverses the solid region of fig. 3. Not until the pressure has been so reduced as to form fig. 10a can solid F be in equilibrium with vapour F . We then proceed in fig. 3 from the solid region to a point of the sublimation line aK .

On continued reduction of pressure the vapour saturation line of F disappears from fig. 10a, so that F lies within the gas region; hence, F can occur only in the form of vapour, so that in fig. 3 we proceed to the vapour region.

In the conversion of fig. 3 (1) into fig. 8 (1) the substance F passes through different metastable conditions. On reduction of pressure the metastable piece $a b$ of the liquidum line passes through the point F first, then the theoretical liquidum-vapour line and then the metastable piece $a_1 b_1$ of the vapour line of the heterogeneous region $L + G$. This also agrees with fig. 3; on lowering the pressure at the temperature T_c we meet in the solid region, successively, the metastable curves $e' F$, $g' S$, and $f'' K$.

When in a system liquid-gas a liquid and a vapour of the same composition are in equilibrium, we will call this a singular point of

the system $L + G$. The appearance of such a point has no influence on fig. 3 unless this accidentally coincides with the point F of one of the previously examined figures. Such a singular point, that at each T occurs only at a definite P , proceeds in the component triangle along a curve which may happen to pass through F . If this should take place, and if this point is a stationary point, then, in the case of the correlated P and T , the vapour and liquidum line of the heterogeneous region $L + G$ and the theoretical liquidum vapour line pass through F ; if this point is a maximum or minimum one these three lines coincide in F . From this it follows that in fig. 3 the singular point must always lie simultaneously on the lines $g'Sg$, $e'Ee$ and $f'Kf$. The coincidence of a singular point with the point F therefore causes the above three curves of fig. 2 to have one point in common; from other considerations it follows that they get into contact with each other.

This point of contact may lie in the solid as well as in the liquidum-gas region; in the first case, the system liquid F + vapour F is metastable, in the second case it is stable.

This point of contact may — but this is not very likely — also coincide with point S of fig. 3. The system solid F + liquid F + vapour F would then occur in the stable condition and the sublimation and melting point curves would then continue up to the point S . (To be continued).

Mathematics. — “On complexes which can be built up of linear congruences”. By Prof. JAN DE VRIES.

(Communicated in the Meeting of December 28, 1912).

§ 1. We will suppose that the generatrices a of a scroll of order m are in (1,1)-correspondence with the generatrices b of a scroll of order n , and consider the complex containing all the linear congruences admitting any pair of corresponding generatrices a, b as director lines. The two scrolls admit the same genus p ; as the edges of a complex cone are in (1,1)-correspondence with the generatrices a, b on which they rest, p is also the genus of all the complex cones¹⁾. The rays of a pencil are arranged in a correspondence (m, n) by the generatrices of the scrolls $(a), (b)$; so in general the complex is of order $m + n$.

¹⁾ For $m = n = 1$ (two pencils) we get the *tetrahedral complex*. In a paper “On a group of complexes with rational cones of the complex” (Proceedings of Amsterdam, Vol. VII, p. 577) we already considered the case of a pencil in (1,1) correspondence with the tangents of a rational plane curve.