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tangents conjugated to each other in this way are in correspondence (4,4). By projecting the pairs of this correspondence out of a line a, the pencil of planes (a) is arranged in a correspondence (12,12). As each coincidency furnishes a line $p = R_0 S_0$ resting on a the *developable* under discussion is of *order* 24; it has ϱ^3 and σ^3 for *fourfold curves*.

Any chord r^* of ϱ^3 meeting σ^3 belongs to the complex, for in the common point of r^* and σ^3 the points P and S coincide. The chords r of ϱ^3 cutting a line l generate a scroll of order four with ϱ^3 as nodal curve; so the locus of the chords r is a scroll of order 12. On the latter surface ϱ^3 is a *sixfold curve*, for through any of its points pass the common edges of the two cones projecting ϱ^3 and σ^3.

So the complex (p) contains *two scrolls of order twelve*, the generatrices of which are chords of one of the curves ϱ^3, σ^3 and secants of the other.

Let p^* be a chord of ϱ^3 not meeting σ^3; then the tangent r_0 in one of the points R_0 common to ϱ^3 and that chord must contain the point P. If P moves along that tangent, S describes a curve $\sigma_0{}^3$; the cone projecting the latter curve out of R_0 has 6 edges in common with that of which ϱ^3 is director curve. So any point of ϱ^3 bears 6 rays p^*. As an arbitrary chord r can be cut by chords p^* in its points common to ϱ^3 only, so all in all by 12, the locus of the chords under discussion is of order 12.

So the complex contains *two scrolls of order twelve*, built up out of chords of one of the curves ϱ^3, σ^3.

Physics. — "*Determinations of the refractive indices of gases under high pressures.*" Second communication. "*On the dispersion of air and of carbon dioxide.*" By Prof. L. H. SIERTSEMA. (Communicated by Prof. H. KAMERLINGH ONNES).

(Communicated in the meeting of November 30, 1912).

4. *The dispersion of air.*

This has already been repeatedly determined both for the visible spectrum and for the ultra-red and ultra-violet rays. The results, however, diverge considerably, and, moreover, the dispersion has never been measured under high pressure.

Through the kindness of Prof. KAMERLINGH ONNES compressed air was placed at my disposal with which dispersion determinations were made in exactly the same way as those for hydrogen described in a former paper.

In the following Table are given the results of three series of observations. For the meaning of the symbols employed reference may be made to the corresponding Table for hydrogen published in the paper just mentioned.

Air.

Pressure in atm.	Temperature.	k_b (mean)	$\lambda_b = 0.546$ $\lambda_a = 0.644$	$\lambda_c = 0.509$	$\lambda_d = 0.472$	$\lambda_e = 0.436$	$\lambda_f = 0.405$
			$\frac{k_a}{k_b}$	$\frac{k_c}{k_b}$	$\frac{k_d}{k_b}$	$\frac{k_e}{k_b}$	$\frac{k_f}{k_b}$
71.4	12.75°C.	823.52	0.84350	1.07697	1.16423	1.26722	1.37179
71.4	12.90	821.23	0.84346	1.07702	1.16418	1.26724	1.37176
70.6	13.19	810.84	0.84352	1.07697	1.16420	1.26718	1.37175
66.4	13.42	759.81	0.84350	1.07697	1.16419	1.26727	1.37183
48.7	13.58	559.27	0.84352	1.07698	1.16421	1.26716	1.37177
31.9	13.67	359.50	0.84339	1.07701	1.16417	1.26725	1.37171
67.7	9.25	790.70	0.84345	1.07697	1.16415	1.26723	1.37187
67.6	10.15	787.92	0.84349	1.07698	1.16419	1.26735	1.37191
66.2	10.35	766.84	0.84353	1.07695	1.16414	1.26724	1.37184
49.2	10.75	567.32	0.84351	1.07698	1.16411	1.26723	1.37178
32.2	10.95	367.70	0.84350	1.07695	1.16414	1.26721	1.37175
101.9	12.75	1170.71	0.84343	1.07698	1.16418	1.26729	1.37203
101.8	12.80	1168.50	0.84340	1.07699	1.16412	1.26732	1.37204
100.0	13.03	1147.43	0.84338	1.07700	1.16419	1.26732	1.37204
82.8	13.39	947.50	0.84337	1.07699	1.16415	1.26729	1.37207
65.5	13.89	745.71	0.84339	1.07704	1.16427	1.26728	1.37206
39.6	13.96	446.50	0.84328	1.07697	1.16415	1.26725	1.37199

The values obtained for the various gas densities are pretty well constant, just as was found to be the case with hydrogen. The deviations are not any more one way than the other, and we can therefore conclude that the dispersion of air is constant up to pressures of about 100 atm.

The mean values are:

$\frac{k_a}{k_b}$	$\frac{k_c}{k_b}$	$\frac{k_d}{k_b}$	$\frac{k_e}{k_b}$	$\frac{k_f}{k_b}$
0,84345	1,07698	1,16417	1,26725	1,37188
± 1,7	± 0,6	± 1,0	± 1,2	± 3,1

Hence we get for the dispersion constants

	λ (vac.)	$c = \frac{n-1}{n_b-1}$
a	0.64403	0.99446 ± 0.000020
b	0.54623	1
c	0.50873	1.00304 ± 0.000006
d	0.47234	1.00669 ± 0.000009
e	0.43597	1.01145 ± 0.000010
f	0.40478	1.01662 ± 0.000023

In order to be able to compare these with the results obtained by other observers, dispersion constants for the wave-lengths I used have been obtained from the results given by MASCART [1]), KAYSER and RUNGE [2]), PERREAU [3]), SCHEEL [4]), HERMANN [5]), RENTSCHLER [6]), LORIA [7]), KOCH [8]), CUTHBERTSON [9]), and GRUSCHKE [10]), either by graphical interpolation or by calculation from dispersion formulae given by them; these results are collected in the following Table.

Correspondence between the present results and those given by PERREAU and by KOCH is quite good both with hydrogen and with air. Only at $\lambda = 0{,}644$ does KOCH find a greater dispersion for both gases. For air, the agreement with HERMANN and with CUTHBERTSON is also very good.

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1) E. MASCART. Ann. de l'éc. norm. (2) **6** p. 60 (1877).
2) H. KAYSER and J. RUNGE. Ann. d. Physik **50** p. 312 (1893).
3) F. PERREAU. Ann. d. Ch. et de Ph (7) **7** p. 325 (1896).
4) K. SCHEEL. Verh. d. D. phys. Ges. **9** p. 27 (1907).
5) K. HERMANN. Verh. d. D. phys. Ges. **10** p. 477 (1908).
6) H. C. RENTSCHLER. Astrophys. J. **28** p. 357 (1908).
7) S. LORIA Ann. d. Physik. (4) **29** p. 619 (1909).
8) J. KOCH. Nova acta reg. soc. Scient. Upsaliensis (4) **2** N°. 5. p. 40 (1909).
9) C. and M. CUTHBERTSON. Proc. R. S. (A) **83** p. 153 (1909/10).
10) G. GRUSCHKE. Ann. d. Physik. (4) **34** p. 807 (1911).

λ	Mascart	Kayser and Runge	Perreau	Scheel	Hermann	Rentschler.	Loria	Koch	Cuthbertson	Gruschke	Siertsema
0.644	0.9949	0.9952	0.9946	0.9948	0.9946		0.9947	0.9955	0.9947	0.9946	0.9945
0.546	1	1	1	1	1	1	1	1	1	1	1
0.509	1.0029	1.0027	1.0030	1.0028		1.0024	1.0023	1.0029	1.0030	1.0034	1.0030
0.472	1.0067	1.0060	1.0064	1.0063		1.0054		1.0070	1.0066	1.0071	1.0067
0.436		1.0103		1.0106	1.0116	1.0089		1.0116			1.0114
0.405		1.0152				1.0143					1.0166

The following interpolation formula was calculated using the method of least squares:

$$c = \frac{n-1}{n_b-1} = 0{,}98086\left(1 + \frac{0{,}0056376}{\lambda^2} + \frac{0{,}00005401}{\lambda^4}\right)$$

in which λ is the wave length in microns.

The degree of accuracy of this formula is evident from the following table·

	$\lambda_{(air)}$	$c_{(cal)}$	$c_{(obs)}$	$\Delta \times 10^5$
a	0.64385 μ	0.99451	0.99446	5
b	0.54608 „	1	1	
c	0.50859 „	1.00303	1.00304	— 1
d	0.47221 „	1.00672	1.00669	3
e	0.43585 „	1.01144	1.01145	— 1
f	0.40467 „	1.01660	1.01662	— 2

5. *The dispersion of carbon dioxide.*

In the following table are given results of two series of measurements made with carbon dioxide. The gas used for the first series was only dried over calcium chloride, and contained about 96 % of carbon dioxide. The gas used for the second series was, in addition, distilled several times, and it contained 98 % of carbon dioxide. The measurements were made in exactly the same fashion as in the case of hydrogen and of air.

Carbon dioxide.

Pressure in atm.	Temperature.	k_b (mean)	$\lambda_b = 0.546$ $\lambda_a = 0.644$ $\frac{k_a}{k_b}$	$\lambda_c = 0.509$ $\frac{k_c}{k_b}$	$\lambda_d = 0.472$ $\frac{k_d}{k_b}$	$\lambda_e = 0.436$ $\frac{k_e}{k_b}$	$\lambda_f = 0.405$ $\frac{k_f}{k_b}$
45.2	12.95°C.	1180.44	0.84286	1.07733	1.16499	1.26864	1.37388
45.5	13.47	1185.20	0.84288	1.07733	1.16503	1.26867	1.37397
43.7	13.99	1084.33	0.84290	1.07735	1.16501	1.26867	1.37392
41.5	14.34	983.51	0.84289	1.07735	1.16505	1.26863	1.37387
38.6	14.58	882.68	0.84288	1.07735	1.16503	1.26864	1.37384
37.9	11.94	883.19	0.84289	1.07735	1.16503	1.26869	1.37380
35.1	12.33	783.79	0.84289	1.07736	1.16507	1.26861	1.37379
31.9	12.39	684.53	0.84284	1.07730	1.16502	1.26865	1.37381
31.6	10.99	685.93	0.84286	1.07733	1.16504	1.26867	1.37390
28.0	11.09	586.93	0.84286	1.07739	1.16503	1.26870	1.37393
24.1	11.28	486.18	0.84284	1.07741	1.16496	1.26869	1.37390
24.1	11.75	487.03	0.84287	1.07737	1.16499	1.26864	1.37385
20.0	11.83	387.11	0.84281	1.07735	1.16496	1.26861	1.37393
46.8	14.00	1281.62	0.84276	1.07735	1.16501	1.26876	1.37395
46.9	14.17	1278.52	0.84275	1.07735	1.16497	1.26875	1.37404
46.5	14.25	1248.50	0.84276	1.07735	1.16498	1.26876	1.37405
41.4	14.75	997.38	0.84278	1.07734	1.16496	1.26872	1.37395
34.2	15.00	748.64	0.84280	1.07732	1.16501	1.26874	1.37397
24.8	15.21	498.73	0.84279	1.07732	1.16497	1.26874	1.37395

Just as with the other gases there is here no definite direction to be recognized in the differences, so that we may again conclude that in this case the dispersion is independent of the gas pressure up to the saturation pressure.

The mean values are:

0,84284	1,07735	1,16501	1,26874	1,37391
± 1,2	± 0,6	± 0,8	± 1,2	± 1,7

from which follow these values for the dispersion constants:

	′(vac.)	$c = \frac{n-1}{n_b-1}$
a	0.64403	0.99374 ± 0.000014
b	0.54623	1
c	0.50873	1.00339 ± 0.000006
d	0.47234	1.00742 ± 0.000007
e	0.43597	1.01259 ± 0.000010
f	0.40478	1.01813 ± 0.000013

In the next table these results are compared with values obtained either by graphical interpolation or by calculation from interpolation formulae from the observations of PERREAU [1]), RENTSCHLER [2]), KOCH [3]), STUCKERT [4]), and GRUSCHKE [5]) MASCART'S [6]) results, which show irregularities which were not confirmed by subsequent observers, are not included.

′	Perreau	Rentschler	Koch	Stuckert	Gruschke	Siertsema
0.644	0.9936		0.9938	0.9917	0.9929	0.9937
0.546	1	1	1	1	1	1
0.509	1.0033	1.0020	1.0031	1.0051	1.0033	1.0034
0.472	1.0072	1.0053	1.0071	1.0110	1.0082	1.0074
0.430		1.0096	1.0127	1.0173		1.0126
0.405		1.0154				1.0181

The agreement with PERREAU and KOCH is good, and with GRUSCHKE not quite so good. RENTSCHLER'S results deviate considerably, just as with air, and so too do STUCKERT'S.

The interpolation formula calculated as before becomes

$$c = \frac{n-1}{n_b-1} = 0,97781\left(1 + \frac{0,0067868}{\lambda^2} - \frac{0,00000614}{\lambda^4}\right)$$

[1]) F. PERREAU Ann. de Ch. et de Ph. (7) **7** p. 345 (1896).
[2]) H. C RENTSCHLER Astrophys J. **28** p. 357 (1908).
[3]) J. KOCH Nova acta reg. soc. scient. Upsaliensis (4) **2** No. 5, p. 46 (1909).
[4]) L. STUCKERT. Zeitschr f. Elektrochemie **16** p. 67 (1910).
[5]) G. GRUSCHKE. Ann. d. Ph. (4) **34** p. 810 (1911).
[6]) E. MASCART. Ann. de l'éc. norm. (2) **6** p. 61 (1877).

in which λ represents the wave-length in air. It gives the following differences between observed and calculated values:

	λ(air)	c(cal)	c(obs)	$10^5\Delta$
a	0.64385 μ	0.99379	0.99374	5
b	0.54608 „	1	1	
c	0.50859 „	1.00338	1.00339	— 1
d	0.47221 „	1.00745	1.00742	3
e	0.43585 „	1.01258	1.01259	— 1
f	0.40467 „	1.01811	1.01813	— 2

In this case, too, the theoretical dispersion formula [1])

$$\frac{n^2-1}{n^2+2}=\Sigma\,\frac{Ne_1{}^2}{3m_1(\nu_1{}^2-\nu^2)}$$

even with only *one* erm in the sum, gives quite good agreement. As with hydrogen we obtain from it:

$$c=\frac{n-1}{n_b-1}=\frac{\frac{1}{\lambda_1{}^2}-\frac{1}{\lambda_b{}^2}}{\frac{1}{\lambda_1{}^2}-\frac{1}{\lambda^2}}$$

and, taking λ as the wave-length in vacuo, we calculate $\lambda = 0{,}07982\,\mu$. The following table gives an idea of the degree of correspondence:

	λ(vac.)	c(cal)	c(obs)	$10^5\,\Delta$
a	0.64403	0.99391	0.99374	+ 17
b	0.54623	1	1	
c	0.50873	1.00334	1.00339	— 5
d	0.47234	1.00741	1.00742	— 1
e	0.43597	1.01258	1.01259	— 1
f	0.40478	1.01823	1.01813	+ 10

A subsequent paper will deal with the absolute values of the refractive indices of air and of carbon dioxide.

[1]) These proceedings 1911—12 p 602.