Physics. — "*Considerations concerning light radiation under the simultaneous influence of electric and magnetic forces and some experiments thereby suggested.* (First Part) by Prof. P. ZEEMAN.

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I. Theoretical considerations.

1. After the discovery of the influence of magnetic forces on radiation frequency in 1896, many physicists certainly have put the question to themselves whether electric fields also influence the emission of light. We may imagine an atom or molecule containing one single electron, which is drawn back to its position of equilibrium by quasi-elastic forces $-kx, -ky, -kz$, where x, y, z, are the components of the displacement of the electron. This is just as in LORENTZ's elementary theory of magnetic separation. Let our molecule be placed in a uniform *electric* field parallel to the axis of X. If the force on the electron be denoted by X, then the displacement x_0 of the electron is given by

$$X = + kx_0 .$$

In the new position there is *equilibrium*. If the electron performs vibrations about the new position, then the coordinates may be represented by

$$x_0 + \xi, \eta, \zeta$$

ξ, η, ζ being supposed to be infinitely small. The components X', Y', Z' of the quasi-elastic force become

$$-k(x_0+\xi), \ -k\eta, \ -k\zeta$$

and therefore the components of the total force ($X+X'$, etc.)

$$-k\xi, \ -k\eta, \ -k\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In the new position the electron is subjected to infinitely small forces, which are independent of the direction of the displacement. The frequency of the vibrations of the electron, being determined by k, has the same value as before the application of the electric field.

2. VOIGT developed the consequences of the hypothesis, which presents itself, if the simple law followed by the quasi-elastic force of § 1 no longer holds.

The potential energy of a displacement (x, y, z) is represented in the supposition of § 1 by

$$\varphi = \frac{1}{2} k(x^2+y^2+z^2) = \frac{1}{2} kr^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

k being a constant.

If the displacement of the electron can no longer be regarded as infinitely small, the value of the potential energy may be expanded according to ascending powers of x, y, z. In a perfectly isotropic molecule we may write therefore, only retaining the first correction term

$$\varphi = \frac{1}{2} kr^2 + \frac{1}{4} k'r^4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

k' being a second constant.

If the electron now performs vibrations (ξ, η, ζ) about the new position of equilibrium, which it shall take under the action of the force X in an electric field parallel to the axis of X, then one finds easily with VOIGT[1]) for the components of the total force ($X + X'$, etc.)

$$-(k+3k'x_0^2)\xi, \; -(k+k'x_0^2)\eta, \; -(k+k'x_0^2)\zeta. \quad . \quad . \quad . \quad (4)$$

The factor by which the displacement is to be multiplied in order to find the force, has now another value with a displacement parallel to the lines of force than with a displacement at right angles to the field. The frequencies of vibrations in these principal directions are therefore different.

Applying these considerations to all the electrons contained in the atoms of a substance such as luminous sodium vapour, VOIGT comes to the following remarkable theoretical result.

If by means of a spectroscope we examine the light that is radiated along the lines of force of the electric field, we shall observe a displacement of the unpolarized spectral line from its original position. At right angles to the field we may expect a transverse electric effect, the original line being separated into two polarized components. The component vibrating parallel to the field undergoes a displacement three times as large as that of the component vibrating perpendicularly to the field.

It is easily seen that the electric change of frequency must be proportional to the square of the electric force.

It is remarked by VOIGT that, if the negatively charged electrons are embedded in a positively charged sphere with a density *decreasing* from the centre outward, the expected displacement of the spectral lines must be towards the *red.*

In what follows I shall, in order to fix the ideas, suppose that the spectral lines are shifted in the sense indicated, but this is not essential.

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[1]) VOIGT. Zur Theorie der Einwirkung eines elektrostatischen Feldes auf die optischen Eigenschaften der Körper. Ann. d. Phys. **69** S. 297. 1899, Ueber das electrische Analogon des ZEEMAN-effektes. LORENTZ-bundel. Archiv. Néerl. 1900, Magneto- und Elektrooptik. Kapitel IX u. X. 1908.

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3. If the expression (3) for the potential energy of an electron is replaced by

$$\varphi = \frac{1}{2} k (x^2+y^2+z^2) + \frac{1}{4} k'' x^4$$

the isotropy has disappeared.

One of the components of the electric doublet now coincides with the original line.

We will do well therefore, not to attach too much importance to the simple ratio of the displacements of the components of the doublet, which follows from the considerations in § 2. In the following discussion we consider a doublet, which can have yet very different positions relatively to the original line.

4. Ten years have passed already since the appearance of VOIGT's first paper concerning an electric analogue of the magnetic spectral effect, but till now physicists have not succeeded in verifying its existence. Two reasons can at once be given for this negative result.

Some idea of the probable order of magnitude of the electric effect can be inferred from observations concerning the influence of an electric field on the refractive index. This estimate gives extremely small values for the electric change of frequency.

According to VOIGT's estimate the change of frequency in a field of 30.000 Volts per cm. would hardly amount to the $^1/_{1000}$th part of the distance of the sodium lines. A field of 3000 Volts per cm. would again diminish it 100 times.

Even if a source of light giving very narrow spectral lines could be placed in the mentioned intense electric fields, the observation of the electric effect would not be without difficulties.

A greater difficulty than the smallness of the effect is, however, due to the impossibility of subjecting metallic vapours to intense electric fields. A sodium flame almost immediately equalizes a large potential difference between the plates of a condenser.

Circumstances are perhaps somewhat more favourable with rapid electric oscillations. During part of the period of discharge of the spark of a condensor the luminous vapour between the electrodes may be subjected to intense electric forces.

Of course a mere displacement of the spark lines relatively to the flame or arc lines is not to be explained by the influence of electric forces, now under consideration. Yet a displacement of spectral lines is the first thing one may expect to observe. It will depend upon circumstances whether a polarization at the borders of the displaced line shall be visible. Finally this also involves the establishing of an extremely small displacement under rather unfavourable circumstances.

The failure of all attempts hitherto made to observe an electric spectral effect [1]), induced me to try a new way for attacking the problem. I have imagined a method which would reveal an action of the electric field by an asymmetrical change of a magnetic triplet, or by a remarkable variation of a magnetic doublet.

I shall prove that the mentioned asymmetry must change its sign if the direction of the electric field is rotated through an angle of 90°. In some of my experiments the electric field existing between metallic electrodes during the passage of the spark is used.

The spark passes across an air space, in a longitudinal or in a transverse magnetic field as the case may be. [In more recent experiments the absorption lines of a xenotime crystal were studied].

Besides the mentioned asymmetries different, delicate particularities of triplets originating under the simultaneous influence of magnetic and electric forces can be predicted.

The observation of the whole of these particularities would give almost as strong evidence for the existence of the electric effect as a direct observation of the effect in an experiment made with electric forces alone.

There is one particularity, which, if it could be observed, would prove by itself most strongly the existence of an electric spectral effect. I shall show (see § 10) that the components of the magnetic doublet, observed along the horizontal magnetic lines of force cease to be *completely* circularly polarized, but must become slightly elliptically polarized, if the vibrating electrons are at the same time under the influence of a vertical electric field. The ellipticity hinted at must be much more easily observable than a change of frequency under the sole action of electric forces.

For some time I privately held the opinion that the asymmetry of some magnetic triplets, first studied in detail by myself and afterwards by GMELIN, DUFOUR, NAGAOKA, and others, could be explained by a cooperation of electric and magnetic fields.

I shall show, however, experimentally that such cannot be the case.

A description of the experiments hitherto made seems to be of some interest, although at the present moment the question of the existence of a specific action of electric fields on the emission of light cannot yet be answered affirmatively.

The experiments certainly are of some value for our understanding of the asymmetry of triplets, and may show the way to better

[1]) Cf. HULL. Proc. R. S. p. 80. Vol. 78. 1907.

methods for investigating the electric effect. A description of the experiments follows in the second part of this paper; in the present communication I shall explain their theoretical foundations. A criterion for a dissymmetry, governed by electric actions, can be established, by means of which it shall be possible to fix a limit for the magnitude of the electric effect.

5. The first problem that I will consider relates to the vibrations of an electron under the simultaneous influence of *parallel* electric and magnetic fields.

Let a system of three rectangular axes be chosen and let the magnetic force be parallel to the axis OZ.

Let ξ, η, ζ be the components of the displacement of an electron, then the equations of motion are

$$\ddot{\xi} = -b^2\xi + r\dot{\eta} \qquad \ddot{\eta} = -b^2\eta - r\dot{\xi} \quad . \; . \; . \quad (6)$$

$$\ddot{\zeta} = -a^2\zeta \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

The difference of a and b determines the electric effect, the magnetic one is determined by r. Suppose $b > a$ and put $b - a = s$, hence s positive.

(7) gives the frequency a; the vibrations corresponding to this equation are always parallel to the axis OZ.

In (6) we assume

$$\xi = e^{int} \qquad \eta = qe^{int},$$

q being in general a complex quantity. The real motion of the electron is obtained by taking the real parts of the expressions for ξ and η.

Making the substitution, we get:

$$-n^2 = -b^2 + inrq \quad , \quad -n^2q = -b^2q - irn \quad . \quad (8)$$

$$(n^2 - b^2)^2 = +n^2r^2$$

hence

$$n = b \pm \frac{r}{2} = a + s \pm \frac{r}{2} \quad . \; . \; . \; . \; . \; . \; . \quad (9)$$

From (8) we obtain two values for q, viz:

$$q = \pm i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

6. We shall now consider 3 special cases.

Case I. Electric field $= 0$, hence $s = 0$. Upper signs in (9) and (10). We then have for the motion in the plane of XY.

$$n = a + \frac{1}{2}r \quad , \quad q = +i.$$

The lower signs in (9) and (10) give

$$n = a - \frac{1}{2} r \quad , \quad q = -i.$$

The two solutions represent circular vibrations in the plane of XY, right-handed with the frequency $a + \frac{1}{2} r$, left-handed with the frequency $a - \frac{1}{2} r$. The vibrations parallel to the axis OZ have the frequency a. In short we have to do with LORENTZ's elementary theory.

All this is independent of the sign of r, *i. e.* of the direction of the magnetic field. If r be negative, the right-handed circular vibrations belong to a frequency smaller than a.

Case II. Magnetic field $= 0$, hence $r = 0$.

Vibrations of arbitrary form with frequency b are now performed in the plane of X, Y. Parallel to the axis OZ we still have vibrations with frequency a.

Case III. Simultaneous electric and magnetic fields. Let r be positive. According to (9) and (10) and taking the upper signs we find

$$n = b + \frac{r}{2} = a + s + \frac{1}{2} r \qquad q = + i$$

representing right handed circular vibrations in the plane of $X\,Y$, with the frequency $a + s + \frac{1}{2} r$.

The lower signs give:

$$n = b - \frac{1}{2} r = a + s - \frac{1}{2} r \qquad q = - i$$

being left-handed circular vibrations in the plane of X, Y with the frequency $a + s - \frac{1}{2} r$.

If r be negative, the circular motions are described in the opposite direction.

Vibrations parallel to the axis OZ always have the frequency a.

We therefore obtain when observing at right angles to the fields a dissymmetrical triplet, the relative position of its components being determined by the following rule.

I shall suppose that the violet, consequently the higher frequencies, is on the right.

Let A and B be the lines with the frequencies a and b, if there is solely an electric field.

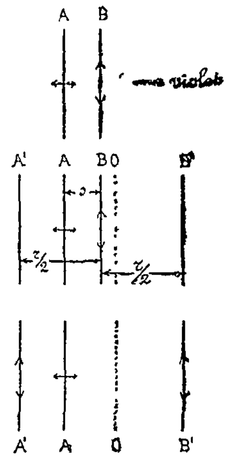

Fig. 1.

Let the electric field be horizontal.

If now the magnetic field is superposed, then A remains.

Two components, however, originate out of B; they have displacements equal to $\frac{r}{2}$, and in oppisite directions. The result is the triplet $A'AB'$.

The electric doublet AB may still have different positions relatively to the original line O. In the supposition of § 3 the line B coincides with the line O. In the supposition of § 1 the original line is at a distance $\frac{1}{2} s$ at the right of B.

7. We shall now suppose that the electric field is an oscillating one. Let B coincide with the original line. If the electric force oscillates according to the formula $a \cos nt$, then s may be represented by $\beta^2 \cos^2 nt = \frac{\beta^2}{2}(1 + \cos 2nt)$.

The lines A' and B' retain their places. The middle line of the dissymmetrical triplet must always be less narrow than the outer components and darkest in the centre. All this applies with a slight modification to the case when B does not coincide with the original position of the line.

8. In the second problem, which we will now consider, a vibrating electron is subjected to a *horizontal* magnetic and a *vertical* electric field. Let the magnetic force be parallel to OZ, the electric to OX.

The equations of motion now become:

$$\ddot{\xi} = -a^2\xi + r\dot{\eta} \qquad \ddot{\eta} = -b^2\eta - r\dot{\xi} \quad . \quad . \quad . \quad . \quad (11)$$

$$\ddot{\zeta} = -b^2\zeta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Suppose again $b > a$ and $b - a = s$. s now as before determines the electric, r the magnetic effect.

The frequency of vibrations parallel to OZ is always b.

As in § 5 we put

$$\xi = e^{int}, \quad \eta = qe^{int}$$

By (11) we have

$$-n^2 = -a^2 + inrq \qquad -n^2q = -b^2q - irn \quad . \quad . \quad (13)$$

or

$$(n^2 - a^2)(n^2 - b^2) = n^2r^2.$$

Whence

$$n^2 = \frac{1}{2}(a^2+b^2+r^2) \pm \sqrt{\frac{1}{4}(a^2+b^2+r^2)^2 - a^2b^2}.$$

Replacing b by $a+s$ we may write approximately

$$n^2 = a^2\left(1 + \frac{s \pm \sqrt{r^2+s^2}}{a}\right)$$

or

$$n = a + \frac{1}{2}(s \pm \sqrt{r^2+s^2}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

According to (13) to these frequencies correspond two values of the complex amplitude q

$$q = i\frac{s \pm \sqrt{r^2+s^2}}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

9. We shall again consider three special cases.

Case I. Electric field zero, hence $s = 0$.

Using (14) and (15) we find for the motion in the plane of XY

$$n = a \pm \tfrac{1}{2} r \quad , \quad q = \pm i,$$

equations discussed in § 6 above.

Case II. Magnetic field absent, $r = 0$.

Now, by (14) and (15), if the upper signs are taken

$$n = a + s = b \quad , \quad q = \infty,$$

representing rectilinear vibrations parallel to OY.

If the lower signs are used

$$n = a \quad , \quad q = 0,$$

meaning rectilinear vibrations parallel to OX.

10. *Case III.* Electric field *vertical* and magnetic field *horizontal.* This case is slightly less simple.

Let us write $\sigma = \frac{1}{2}(\sqrt{r^2+s^2} - s)$ and let r be *positive.*

Taking the upper sign in (14) and (15), then

$$n = a + \tfrac{1}{2}(s + \sqrt{r^2+s^2}) = b + \tfrac{1}{2}(\sqrt{r^2+s^2} - s) = b + \sigma.$$

σ being a positive quantity. The coefficient of i in

$$q = i\frac{s + \sqrt{r^2+s^2}}{r}$$

is positive and > 1. This represents an elliptic vibration in the plane of X, Y, the axes being parallel to OX and OY, the major axis parallel to OY. The motion of the electron is right-handed.

Taking now the lower signs in (14) and (15).

We have $n = a + \frac{1}{2}(s - \sqrt{r^2 + s^2}) = a - \sigma$, and

$$q = i \,.\, \frac{s - \sqrt{r^2+s^2}}{r} = -i \,.\, \frac{r}{\sqrt{r^2+s^2}+s}\,.$$

The coefficient of i is positive and < 1. The electron performs an elliptic vibration, left-handed and in the plane of X, Y; the axes are again parallel to OX and OY, but the major axis parallel to OX.

We now get a dissymmetrical triplet.

Let A and B be the two lines of the electric doublet, the electric field alone being present.

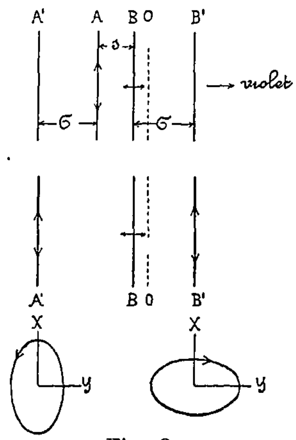

Fig. 2

If now the magnetic field is set up B remains with vibrations parallel to the magnetic force. Two new components are added which one may consider as originating from A and B by a displacement equal to a distance σ.

If the sign of r is *negative*, that is if the magnetic force is reversed, then our considerations still apply with but a small change. To the frequency $n = b + \sigma$ then corresponds

$$q = i\,\frac{s + \sqrt{r^2+s^2}}{r}\,.$$

The coefficient of i is in absolute measure > 1, but negative.

The value of the coefficient of i with the lower sign is the same as above, only the sign of the expression is reversed.

The figure still covers the case, but the motion in the ellipses takes place in the opposite sense.

The product of the two values of q determined by (15) is equal to unity. Hence the product of the horizontal, as well as of the vertical axes of both the vibration ellipses is always equal to unity.

It may be not inappropriate to make here the remark that the lines of the triplet considered in this § exert a kind of "repulsion" upon each other; LORENTZ [1]) proved that such must be generally the case for two spectral lines.

11. If one has to do with an oscillating electric field (cf. § 7

[1]) LORENTZ. Encyclopädie d. math. W. V. 3. Heft 2. Magneto-optische Phänomene No. 36 u. No. 53.

the lines of the triplet are broadened. If the supposition of § 1 concerning the relative position of A, B, and O holds, the width of the middle component becomes $\frac{s}{2}$, but that of the outer ones s. The components are most intense in the centre.

12. The ratio of the axes of the ellipses in § 10 is $1:1+\frac{s}{r}$ resp. $1:1-\frac{s}{r}$ if the electric effect is small relatively to the magnetic one.

Hence the intensities corresponding to vibrations parallel to OX and to OY, differ by an amount proportional to $\frac{2s}{r}$.

If the light is examined parallel to the magnetic lines of force and the separation of the magnetic doublet is 100 times the distance s, then the difference of the intensities of the vibrations parallel to the axes of the ellipses would be $^1/_{50}$ part of the intensity corresponding to the vertical or horizontal vibrations. Moreover the difference of intensity would have opposite sign with both components.

This method seems capable of rendering important services in searching for an electric effect. For differences of intensity of 2 % can be ascertained [1] with certainty by using photographic-photometric measurements. We, therefore, must be able to discover an electric effect one hunderd times smaller than the magnetic effect traceable by means of a spectroscope with the maximum resolving power serviceable under the conditions of the experiments. It does not look now entirely impossible to ascertain under favourable circumstances an electric effect of the order of magnitude estimated by theory.

13. If the electric field is non-uniform, but gradually increasing upwards, then the components bend more and more towards the red (Fig. 3).

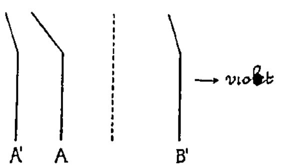

Fig. 3.

The middle component A bends more than the other ones, if the case considered in § 5 is under consideration (parallel electric and magnetic fields).

If, however, the electric force is vertical and the magnetic one horizontal, then the position of the components must become that sketched in the next figure; it is now the middle component which gets the smaller curvature.

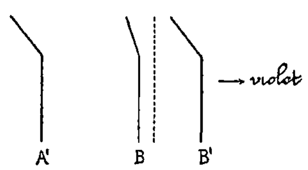

Fig. 4.

1) Comp. P. P. Koch. Ann. d. Phys. Bd. 30. S. 841. 1909.