Meteorology. — *"On the diurnal variation of the wind and the atmospheric pressure and their relation to the variation of the gradient."* By Dr. J. P. van der Stok.

(Communicated in the meeting of April 28, 1911).

1. The diurnal variation of the wind, a phenomenon well known near coast stations as land- and seabreeze, is also observed, and generally in a well marked degree, at land stations.

On account of the laborious work of calculating the diurnal variation from observations made hourly or at fixed hours, this phenomenon has but rarely been investigated and, after all, the knowledge acquired is hardly proportionate to the labour.

The influence of the earth's rotation and of friction gives rise to a rather complicated relation between cause and effect, i.e. the variation of the pressure gradient on the one hand and that of the wind on the other, so that it is not possible to arrive at well founded conclusions based on the results of observations only, without the help of some theory.

For many places, as e.g. Helder, situated at the top of a land-tongue, it would he difficult to tell *a priori* how and to what degree the gradient varies in the course of a day, and neither would it be possible to formulate simple and probable assumptions as to the gradient in the case of a land station as de Bilt, surrounded by regions of very different heat-absorption and radiation.

The variability of the periodic gradient in different seasons is a still more difficult problem as, near coast stations, the seacurrents and the temperature of the seawater in the surroundings, and near land stations, the difference of physical properties of the adjacent regions play an important part.

Only for a station situated in an extensive and homogeneous region the simple assumption of a heat wave propagating from East towards West, with the sun, would be permissible and only in such a case would it be possible to deduce the variation of the gradient from the diurnal variation of the barometric height, as observed *in loco*, provided the law of variation depending upon geographical latitude were known as well.

In most cases, however, the variation of the gradient will be smallest where the change of pressure is greatest and conversely, the mechanism thus rather corresponding to an interchange of two *stationary* sources of periodic pressure variation, situated at a fixed distance the one from the other, than to a propagating wave passing at constant velocity.

In general it is therefore not possible to derive any conclusions concerning the gradient from the diurnal variation of barometric pressure as observed at a given place, and neither would it be possible to increase the number of stations and the accuracy of the observations to such a degree, that the gradient variation could be experimentally deduced.

The inverse way is therefore indicated and from the *known* variation of the wind we must try to derive the *unknown* value and variation of the gradient, which of course is possible only with the help of a suitable mechanical theory of the air motion.

2. If, neglecting possible and probable vertical motions, the rotation of the earth is taken into account, and the influence of friction is assumed to be proportional to the velocity, the relation between gradient and air motion may be represented by the following expressions, the same as used e.g. by OBERBECK [1]) in his well known paper on cyclonic motion:

$$\left.\begin{aligned}\frac{\partial v}{\partial t} + nau + lv &= -\frac{1}{\varrho}\frac{\partial p}{\partial y}\\ \frac{\partial u}{\partial t} - nav + lu &= -\frac{1}{\varrho}\frac{\partial p}{\partial x}\end{aligned}\right\} \quad \ldots \ldots \quad (1)$$

In these formulae y and v are considered to be directed towards North, x and u towards East.

$a = 2 \cos \varphi$

$\varphi =$ polar distance.

$n =$ angular velocity of the earth.

If we put:

$$l = kn$$

then, after division by n and for the case of a periodically varying gradient, (1) becomes

$$\left.\begin{aligned}\frac{1}{n}\frac{\partial v}{\partial t} + au + kv &= -\frac{1}{\varrho n}\frac{\partial p}{\partial y} = H_1 \cos(qnt - \lambda_1)\\ \frac{1}{n}\frac{\partial u}{\partial t} - av + ku &= -\frac{1}{\varrho n}\frac{\partial p}{\partial x} = H_2 \cos(qnt - \lambda_2)\end{aligned}\right\} \quad \ldots \quad (2)$$

The amplitudes H_1 and H_2 are proportionate to the gradient directed towards North and East, q is the order of the period under consideration.

Representing the components of the velocity of the airparticles by:

$$\left.\begin{aligned}v &= A \cos(qnt - C_1)\\ u &= B \cos(qnt - C_2)\end{aligned}\right\} \quad \ldots \ldots \ldots \quad (3)$$

1) Ann. d. Phys. u. Ch. 1882, 17, (128—148).

we find from (2) and (3):

$$\left.\begin{aligned} H_1 \sin \lambda_1 &= -qA \cos C_1 + aB \sin C_2 + kA \sin C_1 \\ H_1 \cos \lambda_1 &= qA \sin C_1 + aB \cos C_2 + kA \cos C_1 \\ H_2 \sin \lambda_2 &= -qB \cos C_2 - aA \sin C_1 + kB \sin C_2 \\ H_2 \cos \lambda_2 &= qB \sin C_2 - aA \cos C_1 + kB \cos C_2 \end{aligned}\right\} \quad . \; . \quad (4)$$

The quantities H and λ can then easily be calculated by means of the following relatively simple formulae, provided the wind variation and friction coëfficient be known:

$$\left.\begin{aligned} H_1 \sin (\lambda_1 - C_1) &= -qA - aB \sin \Delta \\ H_1 \cos (\lambda_1 - C_1) &= kA + aB \cos \Delta \\ H_2 \sin (\lambda_2 - C_2) &= -qB - aA \sin \Delta \\ H_2 \cos (\lambda_2 - C_2) &= kB - aA \cos \Delta \\ \Delta &= C_1 - C_2 \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (5)$$

and further:

$$\left.\begin{aligned} H_1^2 - H_2^2 &= (k^2 + q^2 - a^2)(A^2 - B^2) + 4\,ka\,AB \cos \Delta \\ H_1^2 + H_2^2 &= (k^2 + q^2 + a^2)(A^2 + B^2) + 4\,qa\,AB \sin \Delta \end{aligned}\right\} \quad . \quad (6)$$

Form. (3) can be represented by an ellipse, the radius vector being the resultant velocity, and the great axis forming with the y, v (North) direction an angle α determined by the expression:

$$tang\; 2\,\alpha = \frac{2\,AB \cos \Delta}{A^2 - B^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

Likewise the gradient vector can, according to magnitude and direction, be represented by an ellipse, its great axis making with the North direction an angle α', determined by the form.

$$tang\; 2\,\alpha' = \frac{2\,H_1 H_2 \cos (\lambda_1 - \lambda_2)}{H_1^2 - H_2^2} \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

From (4) follows:

$$\left.\begin{aligned} H_1 H_2 \sin (\lambda_1 - \lambda_2) &= (k^2 + q^2 + a^2)\,AB \sin \Delta + qa\,(A^2 + B^2) \\ H_1 H_2 \cos (\lambda_1 - \lambda_2) &= (k^2 + q^2 - a^2)\,AB \cos \Delta - ka\,(A^2 - B^2) \end{aligned}\right\} \quad (9)$$

If then we put:

$$tang\; 2\,m = \frac{2\,ak}{k^2 + q^2 - a^2}, \quad . \; . \; . \; . \; . \; . \; . \quad (10)$$

$$tang\; 2\,\alpha' = tang\,(2\alpha - 2m) \quad . \; . \; . \; . \; . \; . \; . \quad (11)$$

$$m = \alpha - \alpha'.$$

Although, therefore, for the case of a periodically varying gradient, the angle of deviation between gradient and wind direction does not assume a simple form and is a rather complicated function of the time, there is (if we admit the form. (1) as suitable for the purpose) a constant difference in direction m between the maximum values of those quantities, as defined by (10) and therefore dependent upon the friction (k) and variable with the order of the period (q).

If the angle of deviation were known, the value of k and hence the friction coefficient kn might be deduced from (10).

3. If we calculate the diurnal variation of the barometric height for different places on the earth's surface, it appears that the monodiurnal variation widely differs, as to amplitude and phase, in the different seasons and that it shows great divergences for places as near to each other as de Bilt, Helder, and Vlissingen.

TABLE I.

Diurnal variation of barometric height (1902—1910).

de Bilt.

	mm.		mm.	
Winter	0.0447	$\cos(nt - 178°)$ +	0.1805	$\cos(2\,nt - 295°)$
Spring	0.0097	$\cos(nt - 177°)$ +	0.2049	$\cos(2\,nt - 301°)$
Summer	0.0238	$\cos(nt - 79°)$ +	0.1665	$\cos(2\,nt - 311°)$
Autumn	0.0305	$\cos(nt - 135°)$ +	0.2001	$\cos(2\,nt - 296°)$
Year	0.0214	$\cos(nt - 147°)$ +	0.1864	$\cos(2\,nt - 300°)$

Helder.

Winter	0.0431	$\cos(nt - 93°)$ +	0.1976	$\cos(2\,nt - 309°)$
Spring	0.1942	$\cos(nt - 59°)$ +	0.2087	$\cos(2\,nt - 313°)$
Summer	0.2200	$\cos(nt - 59°)$ +	0.1856	$\cos(2\,nt - 324°)$
Autumn	0.1391	$\cos(nt - 72°)$ +	0.2255	$\cos(2\,nt - 304°)$
Year	0.1472	$\cos(nt - 65°)$ +	0.2038	$\cos(2\,nt - 313°)$

Vlissingen.

Winter	0.1321	$\cos(nt - 190°)$ +	0.2079	$\cos(2\,nt - 315°)$
Spring	0.0369	$\cos(nt - 49°)$ +	0.2268	$\cos(2\,nt - 308°)$
Summer	0.0994	$\cos(nt - 55°)$ +	0.2109	$\cos(2\,nt - 317°)$
Autumn	0.0267	$\cos(nt - 122°)$ +	0.2366	$\cos(2\,nt - 307°)$
Year	0.0314	$\cos(nt - 121°)$ +	0.2196	$\cos(2\,nt - 312°)$

The semidiurnal variation on the contrary, extensively investigated in well known memoirs by HANN and ANGOT, shows a remarkable uniformity, the amplitude regularly decreasing from about 1 mm. near the equator to the pole; at Batavia the daily variation can be represented by the expression:

4*

$$0.63 \cos (nt - 64^\circ.5) + 1.00 \cos (2 nt - 290^\circ)$$ [1])

As appears from Table I the phase shows small differences in the different seasons, and at Batavia it is somewhat smaller than at the three Dutch stations, but nowhere does it differ much from 300°, the greatest height occurring everywhere about 10^h a. m. and. p.m. According to a theory, first suggested by KELVIN, afterwards mathematically founded by MARGULES, this regular variation can be ascribed to a free oscillation of the atmosphere as a whole in its own period of very nearly 12^h, which again finds its origin in the semi-diurnal term of the daily variation of the air temperature.

Such an oscillation of the whole atmosphere can be regarded as a pressure wave, propagating from East towards West, with a velocity c of the heat wave and can be represented by the expression:

$$p = E \cos \left(2nt - C + \frac{2nx}{c} \right)$$

If, according to the observations [2]), we assume that

$$E = E_0 \sin^3 \varphi ,$$

then

$$\left(\frac{\partial p}{\partial y}\right)_{x=0} = - \frac{\partial p}{R \partial \varphi} = - \frac{3E \cos \varphi}{R \sin \varphi} \cos (2nt - C)$$

$$\left(\frac{\partial p}{\partial x}\right)_{x=0} = - \frac{2En}{c} \sin (2nt - C)$$

and as

$$\frac{n}{c} = \frac{1}{R \sin \varphi},$$

$$- \left(\frac{\partial p}{\partial y}\right)_{x=0} = \frac{3E \cos \varphi}{R \sin \varphi} \cos (2nt - C)$$

$$- \left(\frac{\partial p}{\partial x}\right)_{x=0} = \frac{2E}{R \sin \varphi} \cos (2nt - C - 90^\circ).$$

According to this theory therefore:

$$\lambda_1 - \lambda_2 = 270^\circ$$

and

$$H_1 = {}^3/_2\, H_2 \cos \varphi .$$

For all places situated in higher latitudes than 41° 49′, therefore, $H_1 > H_2$, and, according to (8):

$$a' = 0.$$

It appears then that, at places situated in latitudes higher than

1) Observations. Vol. XXVIII (1866—1905). p. 97.

2) JAERISCH. Zur Theorie der Luftschwankung. Meteor. Zeitschr. **24**, 1907, p. 481.

42°, the great axis of the gradient ellipse is directed towards (N—S); for a latitude of 42° $H_1 = H_2$, and the ellipse becomes a circle; more southerly the great axis points towards (E—W); at the equator $H_1 = 0$ and the ellipse is flattened down to a straight line directed E—W.

If E — as is probable — is not exactly proportionate to the third power of $\sin\varphi$, the limit 42° varies of course accordingly.

4. In two recent publications GOLD [1]) has compared this theory with the results of the observations; for this purpose he chooses the following method: the gradient variation, as deduced from the semidiurnal variation of the atmospheric pressure is assumed to be known; from form. (1) or, what comes to the same, form. (4) the four wind constants A, B, C_1 and C_2 are calculated and the results are compared with the wind variation, as deduced from the observations by giving k the values 0, ½ and 1.

Obviously this method leads to very complicated and almost unmanageable expressions for the wind variation and the inverse way, namely taking the wind variation as a known quantity and then calculating the barometric variation, would be a simpler and equally suitable method.

Calculating GOLD's formulae for a latitude of 52°, the average latitude of the Netherlands, we find,

	v	u
$k = 0$	$45 \cos(2nt - 33°)$	$44 \cos(2nt - 123°)$
$k = 1$	$28 \cos(2nt - 343°)$	$27 \cos(2nt - 72°)$
$k = \frac{1}{2}$	$17 \cos(2nt - 328°)$	$16^5 \cos(2nt - 55°)$,

In calculating these expressions, the semidiurnal variation of atmospheric pressure as found for de Bilt has been used; the amplitudes are expressed in cm. p. sec.

On the average, taken over the whole year, the result of wind observations made at two Dutch stations is:

	v	u
de Bilt	$16.8 \cos(2nt - 333°)$	$17.4 \cos(2nt - 65°)$
Terschellinger bank	$11.1 \cos(2nt - 334°)$	$11.9 \cos(2nt - 66°)$

The agreement between theory and observation is very satisfactory, much better than for the two cases as calculated by GOLD,

[1]) E. GOLD. The relation between periodic variations of pressure, temperature and wind in the atmosphere. Phil Mag. 1909, p. 26—109.

Id. Note on the connexion between the periodic variations of windvelocity and of atmospheric pressure. Publ. Meteor. Office, 203, 1910.

namely the mountain station Säntis and St. Helena, which perhaps is due to the peculiar situation of these stations.

At St. Helena, where the windvector ought to turn with an anticlockwise motion, (to the left) the veering is to the right, as if St. Helena were situated in northerly latitude. At this small latitude (16° S) the windellipse is, as has been noticed above, very flat so that the veering of the wind is a rather uncertain factor.

For the Dutch stations the theory appears to be so well in accordance with the observation that the purpose of this investigation, namely determining the friction coefficient from the *semidiurnal* variation of wind and barometric height, and then, with the help of this value, deducing the *monodiurnal* gradient variation from the corresponding wind variation, seems attainable.

For such an inquiry the wind observations made during 25 consecutive years on board the lightship Terschellingerbank in the North of the country, offer an excellent material, and it is interesting to investigate in what respect the gradient variation differs for two places so differently situated as de Bilt and Terschellingerbank.

5. Table II shows the diurnal variation of the wind for both places.

TABLE II, diurnal variation of the Wind
in cm. p. sec., $t = 0 =$ Noon.

de Bilt, N. Lat. 52°6′, L. G. 5°11′ hourly observations (1903—1908).

Winter
$$v = 15.1 \cos(nt - 260°) + 10.4 \cos(2\,nt - 279°)$$
$$u = 11.5 \cos(nt - 17°) + 12.1 \cos(2\,nt - 33°)$$
$$\Delta = 243° \qquad \Delta = 246°$$

Spring
$$v = 71.4 \cos(nt - 256°) + 21.8 \cos(2\,nt - 337°)$$
$$u = 52.7 \cos(nt - 14°) + 21.8 \cos(2\,nt - 74°)$$
$$\Delta = 242° \qquad \Delta = 263°$$

Summer
$$v = 95.6 \cos(nt - 261°) + 27.7 \cos(2\,nt - 358°)$$
$$u = 86.1 \cos(nt - 25°) + 22.5 \cos(2\,nt - 79°)$$
$$\Delta = 236 \qquad \Delta = 279°$$

Autumn
$$v = 40.7 \cos(nt - 264°) + 14.9 \cos(2\,nt - 316°)$$
$$u = 30.3 \cos(nt - 19°) + 16.2 \cos(2\,nt - 56°)$$
$$\Delta = 245° \qquad \Delta = 260°$$

Year
$$v = 55.6 \cos(nt - 260°) + 16.8 \cos(2\,nt - 333°)$$
$$u = 45.0 \cos(nt - 21°) + 17.4 \cos(2\,nt - 65°)$$
$$\Delta = 239° \qquad \Delta = 268°$$

Lightship Terschellingerbank, N. Lat. 53°27′, L. G. 4°52′

watch-observ. (6 times daily), magnetic., (1884—1908).

Winter
$$v = 3.9 \cos(nt - 344°) + 11.8 \cos(2\,nt - 319°)$$
$$u = 3.1 \cos(nt - 293°) + 11.1 \cos(2\,nt - 56°)$$
$$\Delta = 51° \qquad \Delta = 263°$$

Spring
$$v = 20.3 \cos(nt - 249°) + 12.6 \cos(2\,nt - 344°)$$
$$u = 18.9 \cos(nt - 350°) + 10.1 \cos(2\,nt - 75°)$$
$$\Delta = 259° \qquad \Delta = 269°$$

Summer
$$v = 21.6 \cos(nt - 240°) + 9.3 \cos(2\,nt - 348°)$$
$$u = 30.9 \cos(nt - 354°) + 11.5 \cos(2\,nt - 94°)$$
$$\Delta = 246° \qquad \Delta = 254°$$

Autumn
$$v = 8.0 \cos(nt - 248°) + 11.9 \cos(2\,nt - 326°)$$
$$u = 13.1 \cos(nt - 358°) + 17.6 \cos(2\,nt - 52°)$$
$$\Delta = 250° \qquad \Delta\ 274°$$

Year
$$v = 12.3 \cos(nt - 249°) + 11.1 \cos(2\,nt - 334°)$$
$$u = 16.1 \cos(nt - 351°) + 11.9 \cos(2\,nt - 66°)$$
$$\Delta = 258° \qquad \Delta = 268°$$

If, for the present, we leave the monodiurnal motion out of consideration, it appears that, whereas the angular values of the semidiurnal variation show a close agreement, the amplitudes do not agree in so far that sometimes the north- and sometimes the east-component is the greatest; on the average the east component is somewhat greater, but the difference is so small and variable, that a serious objection arises against calculating the friction coefficient by means of formulae (10) and (11), when a', according to theory is equalized to zero because

$$\lambda_1 - \lambda_2 = 270°.$$

If $A = B$ the windellipse approaches to a circle because $\cos \Delta$ is also a small quantity, according to theory as well as to observation, and the angle of deviation becomes undetermined.

Different other methods however can be chosen for calculating the value of k by means of form. (5), (6) and (9), as it is sufficient if only one quantity or relation be assumed to be equal to its theoretical value.

We might e.g. assume that the theoretical value

$$H_1 = {}^3/_2\ H_2 \cos \varphi$$

were accurately true; then, putting

$$\beta = \frac{2\,AB\,sin\,\Delta}{A^2 + B^2} \qquad \gamma = \frac{2\,q\,a}{k^2 + q^2 + a^2} \quad . \; . \; . \; . \quad (12)$$

we find from (6) and (9)

$$M = \frac{12\ cos\ \varphi}{9\ cos^2\ \varphi + 4} = -\frac{\beta + \gamma}{1 + \beta\,\gamma} . \quad . \; . \; . \; . \quad (13)$$

In these formulae the values of A—B and $cos\ \Delta$ do not appear, and (12) offers the advantage that it enables us to calculate the friction coefficient without the intervention of a quadratic equation, as when (10) is used.

This advantage however is only apparent, as, for $A = B$ and $\Delta = 270°$, $\beta = -1$, and the quantity γ disappears from the formula; in fact, for a latitude of 52° M approaches very nearly to unity (0.986), as required by theory.

It is therefore necessary to have recourse to another method and as, without doubt, angular values can be determined with a greater degree of accuracy than amplitudes and the relation

$$H_1 = {}^2/_3\, H_2\, cos\ \varphi$$

can be only approximatively true, we assume that:

$$\lambda_1 = C,\ \lambda_2 = C + 90°$$

or, i. o. w. the problem is thus formulated: what is the value to be given to k in order to ensure agreement between the angular values of the windvariation (C_1 and C_2) on the one hand, and of the barometer variation (C) on the other, for the semidiurnal variation. As in this way not one but actually two relations are derived from theory, obviously two values of k may be deduced from (5); taking these together, the relation

$$k = (2 - a)\ \frac{B\ sin\ \delta_2 - A\ cos\ \delta_1}{A\ sin\ \delta_1 + B\ cos\ \delta_2}$$

$$\delta_1 = C - C_1,\ \delta_2 = C - C_2$$

is easily found.

If k is known, then, with the help of form. (10), three different values for the angle of deviation m may be derived, namely

m_2 ($q = 2$) for semidiurnal periodic winds
m_1 ($q = 1$) „ monodiurnal „ „
m_0 ($q = 0$) „ constant non „ „

For the last named quantity form. (10) gives:

$$tang\ m_0 = \frac{a}{k},$$

the same value as, according to Oberbeck's theory, obtains for the outer part of a cyclone and also — as follows from (2) — for constant winds and straight isobars.

In Table III the values of k and m, calculated in this way are given.

TABLE III.

Values of the friction k and the angle of deviation m.

de Bilt.

	Winter	Spring	Summer	Autumn	Year
k =	0.834	0.514	0.455	0.900	0.627
m_2 =	20°.0	21°.2	19°.9	25°.3	23°.0
m_1 =	53.°4	63°.5	65°.9	51°.7	59°.5
m_0 =	62°.1	73°.0	74°.9	60°.3	68°.3

Terschellingerbank.

	Winter	Spring	Summer	Autumn	Year
k =	1.648	0.657	0.608	1.101	0,940
m_2 =	26°.0	24°.4	23°.8	26°.7	26°.3
m_1 =	39°.0	59°.3	61°.0	47°.5	50°.6
m_0 =	44°.3	67°.8	69°.3	55°.6	59°.7

The friction coefficient $l = nk = k \times 7.3 \times 10^{-5}$ then becomes:

de Bilt	6.09	3.75	3.32	7.57	4.58	$\times 10^{-5}$
Terschellingerbank	12.03	4.80	4.44	8.05	6.86	

OBERBECK assumes arbitrarily $l = 12 \times 10^{-5}$, WIEN[1]) $l = 8 \times 10^{-5}$.

It is, of course, doubtful whether a friction coefficient, as calculated for the case of feeble periodic winds may be used also for non periodic winds of any force; but the fact that VAN EVERDINGEN[2]) on determining the angle of deviation directly from weather charts and for strong gradients (one mm, and more) for de Bilt and taken over the whole year, comes to almost exactly the same value (69°) as that given in Table III (68°), can hardly be considered as a merely accidental coincidence.

6. With the aid of form. (5) the quantities H and λ, k being known, can easily be calculated: this calculation has been made only for the mono-diurnal variation. Putting

$$-\frac{\partial p}{\partial y} = G_y \text{ (gradient)}, \quad -\frac{\partial p}{\partial x} = G_x$$

further

1) WIEN. Lehrbuch der Hydrodynamik, 1900, p. 294.

2) Phys. Z. S. 9, 1908. Verh. d. D. Phys. Ges., 1908.

$$\frac{1}{\varrho} = \frac{KT}{p_0}$$

where ϱ is the density of the air, K the gasconstant, T the absolute temperature of the air and p_0 the average atmospheric pressure expressed in mm. mercury-pressure, and taking:

$K = 2.87 \times 10^6$, $T = 283°$. $p_0 = 760$, $n = 7.3 \times 10^{-5}$

we have the quantities H to divide by

$$1.464 \times 10^{10}$$

in order to obtain the gradient expressed in mm. mercury-pressure per cm., or by 1319 if we wish to express the gradient, as usual, per 111 km. In order to avoid the use of very small quantities, this division has not been made and the quantities H are considered as gradients.

Table IV shows the diurnal variation of the gradient as calculated in this way. As the wind observations made on board of the light-ship Terschellingerbank are made with respect to the magnetic meridian, the direction of the resulting gradient ought be counted from the same.

TABLE IV.

Monodiurnal variation of the Gradient, time counted from noon, H_1 to North, H_2 to South.

R. L. Veering to right and left.

	de Bilt.	Terschellingerbank. (magnetical)
Winter	$H_1 = 4.5 \cos(nt - 273°.3)$ *R.*	$H_1 = 12.3 \cos(nt - 305°.1)$ *L.*
	$H_2 = 22.5 \cos(nt - 42°.5)$	$H_2 = 8.1 \cos(nt - 211°.3)$
	$\Delta = 230°.8$	$\Delta = 93°.8$
Spring	$H_1 = 3.1 \cos(nt - 35°.0)$ *R.*	$H_1 = 12.2 \cos(nt = 300°.7)$ *R.*
	$H_2 = 92.5 \cos(nt - 44°.3)$	$H_2 = 23.3 \cos(nt - 25°.2)$
	$\Delta = 350°.7$	$\Delta = 275°.5$
Summer	$H_1 = 36.5 \cos(nt - 53°.4)$ *L.*	$H_1 = 23.5 \cos(nt - 346°.2)$ *R.*
	$H_2 = 129.3 \cos(nt - 42°.5)$	$H_2 = 32.4 \cos(nt - 355°.5)$
	$\Delta = 10°.9$	$\Delta = 350°.7$
Autumn	$H_1 = 16.6 \cos(nt - 273°.1)$ *R.*	$H_1 = 11.9 \cos(nt - 330°.2)$ *R.*
	$H_2 = 61.0 \cos(nt - 46°.1)$	$H_2 = 18.9 \cos(nt - 354°.9)$
	$227°.0$	$\Delta = 335°.3$
Year	$H_1 = 5.5 \cos(nt - 7°.8)$ *R.*	$H_1 = 14.6 \cos(nt - 312°.3)$ *R.*
	$H_2 = 79.3 \cos(nt - 43°.4)$	$H_2 = 20.0 \cos(nt - 0°.3)$
	$\Delta = 324°.4$	$\Delta = 312°.0$

From Table II it appears that, in the neighbourhood of the lightship the veering of the wind in wintertime is against sun, whereas in other seasons it is with sun. Near de Bilt the veering is always to the right as, in normal cases, ought to be the case, owing to the influence of the earth's rotation.

This abnormality is not accidental, due to observational errors and the smallness of the amplitudes; near the lightship Haaks and the landstation Helder, the veering in wintertime is also to the left; near Helder not only during the wintermonths but in the autumn as well.

The results for Helder are not given here because the friction coefficient, as deduced in the manner described above, does not show reliable values in the different seasons; evidently the series of observations used for this purpose is of insufficient duration. This abnormal behaviour of the gradient, namely veering to the left, must occur at every coast station where the sea is to the South and the land to the North. If, however, the gradient veers against sun it is quite possible that the wind still veers to the right for, if we put $H_1 = 0$, and thus assume that there is only an East—West gradient, it appears from form. (5) that the wind veers with the sun; the possibility of a gradient veering to the left causing a wind veering to the right is, therefore, not excluded.

In fact this case presents itself in summer time near de Bilt where the gradient turns to the left whereas the wind veers to the right.

At de Bilt also there is an important difference between spring and summer on the one hand, winter and autumm on the other, as shown in the differences of phase Δ; in the former seasons this phase difference is small and the ellipse approaches to a straight line, in the latter the excentricity of the ellipse is much smaller.

Tables V and VI, showing the elements of the wind- and gradient-ellipses, convey a better idea of the results obtained and here they can be easily corrected for the magnetical bearing by subtraction of 14° from the angular values for Terschellingerbank.

TABLE V.

Windellipse of the monodiurnal variation.

de Bilt.

	a	b	α	ϑ_0	T h
Winter	16.5	9.3	151°	103°	2.28
Spring	77.8	42.7	152°	109°	2.65
Summer	123.1	37.3	139°	101°	3.41
Autumn	43.8	25.2	153°	98°	2.34
Year	62.9	34.1	146°	103°	2.84

Terschellingerbank. (geographical).

	a	b	α	θ_0	T h
Winter	4.5	2.1	21°	350°	0.52
Spring	21.5	17.5	131°	83°	2.61
Summer	32.7	18.7	100°	81	0.54
Autumn	13.5	7.3	93°	75°	0.51
Year	16.5	11.7	95°	78°	0.39

TABLE VI.

Gradiëntellipse of the monodiurnal variation.

de Bilt.

	a	b	α	θ_0	T h
Winter	22.7	3.4	97°	89°	2.75
Spring	92.5	2.7	89°	88°	2.94
Summer	134.3	6.5	74°	89°	3.51
Autumn	62.1	12.0	101°	89°	3.23
Year	79.4	3.2	87°	85°	3.27

Terschellingerbank (geographical).

	a	b	α	θ_0	T h
Winter	12.3	8.0	342°	302°	8.43
Spring	23.4	12.1	72°	68°	1.04
Summer	39.9	3.1	40°	41°	23.35
Autumn	21.9	4.3	45°	47°	0.41
Year	22.8	9.5	44°	50°	0.63

The elements of the ellipse are deduced from the expressions:

$$v = A \cos(nt - C_1) \qquad C_1 - C_2 = \Delta$$
$$u = B \cos(nt - C_2)$$

by means of the well known formulae:

$$\operatorname{tang} 2\,\alpha = \frac{2\,AB \cos\Delta}{A^2 - B^2} \qquad a^2 + b^2 = A^2 + B^2$$

$$a^2 - b^2 = \frac{A^2 - B^2}{\cos 2\,\alpha} \qquad \cos\theta_0 = \frac{B \cos C_2}{A \cos C_1}$$

$$\operatorname{tang} T = \operatorname{tang}\left(\frac{C_1 + C_2}{2} + \chi\right) \qquad \operatorname{tang}\chi = \frac{\operatorname{tang}\varphi - 1}{\operatorname{tang}\varphi + 1}\cot\frac{\Delta}{2}$$

$$\operatorname{tang}\varphi = \frac{B}{A}\cot\alpha.$$

θ_0 is the angle between the radius vector and the north-axis at noon, T is the moment at which the radius vector has the direction α of the great axis or, whenever the ellipse is flattened down to a straight line, the moment when it attains its maximum value.

It appears from these tables that the gradient ellipses, for both stations and in all seasons, approach to a straight line, so that a graphical representation could only be given on a large scale.

It would not be difficult to proffer an explanation of the somewhat startling result that the angle of deviation varies with the different seasons. Such an explanation could be based only on a premised conception concerning the mechanical meaning of the friction coefficient, as introduced in the calculation, and would be premature before the results obtained have been put to the test by application of the method indicated in this paper to other series of observations made at many and differently situated stations.

Mathematics. — *"The pentagonal projections of the regular fivecell and its semiregular offspring."* Communicated by Prof. SCHOUTE.

1. *Fundamental theorem.* If in two circles (fig. 1) with radius ϱ situated in the planes $O(X_1X_2)$, $O(X_3X_4)$ of a rectangular system of coordinates in space S_4 we describe two regular pentagons $(1, 2, 3, 4, 5)$, $(1', 2', 3', 4', 5')$, of which the first is convex while the other is star shaped, the five points $P_1, P_2, \ldots, P_5$, whose projections are the vertices of these pentagons indicated by corresponding numbers, form the vertices of a regular fivecell with $\varrho\sqrt{5}$ as length of edge. [1])

[1]) This theorem is not new. Probably it was given for the first time by Dr. S. L. VAN OSS in his dissertation (Utrecht, 1894). Compare also my paper: "Les projections régulières des polytopes réguliers" (*Archives Teyler*, Haarlem, 1904).

We repeat here the simple proof. If (P_{12}, P_{34}) and (Q_{12}, Q_{34}) are the projections of the points P and Q with the coordinates x_i and y_i $(i = 1, 2, 3, 4)$ on the planes $O(X_1X_2)$, $O(X_3X_4)$, we have

$$\overline{P_{12}Q}^2_{12} = (x_1-y_1)^2 + (x_2-y_2)^2 \ , \ \overline{P_{34}Q}^2_{34} = (x_3-y_3)^2 + (x_4-y_4)^2$$

and therefore if d denotes the distance PQ

$$\overline{P_{12}Q}^2_{12} + \overline{P_{34}Q}^2_{34} = d^2.$$

Now the projections $P_{12}Q_{12}$ and $P_{34}Q_{34}$ of each of the ten edges $12, \ldots, 45$ of the fivepoint $P_1P_2P_3P_4P_5$ are either side and diagonal or diagonal and side of the same regular pentagon, etc.

Which position has the regular simplex $S(5)$ with respect to the planes of projection $O(X_1X_2)$ and $O(X_3X_4)$? Evidently this projection is characterized by the fact that each of the five pairs of non intersecting edges

(25) (34) , (13) (45) , (24) (15) , (35) (12) , (14) (23)