θ_0 is the angle between the radius vector and the north-axis at noon, T is the moment at which the radius vector has the direction α of the great axis or, whenever the ellipse is flattened down to a straight line, the moment when it attains its maximum value.

It appears from these tables that the gradient ellipses, for both stations and in all seasons, approach to a straight line, so that a graphical representation could only be given on a large scale.

It would not be difficult to proffer an explanation of the somewhat startling result that the angle of deviation varies with the different seasons. Such an explanation could be based only on a premised conception concerning the mechanical meaning of the friction coefficient, as introduced in the calculation, and would be premature before the results obtained have been put to the test by application of the method indicated in this paper to other series of observations made at many and differently situated stations.

Mathematics. — *"The pentagonal projections of the regular fivecell and its semiregular offspring."* Communicated by Prof. SCHOUTE.

1. *Fundamental theorem.* If in two circles (fig. 1) with radius ϱ situated in the planes $O(X_1 X_2)$, $O(X_3 X_4)$ of a rectangular system of coordinates in space S_4 we describe two regular pentagons $(1, 2, 3, 4, 5)$, $(1', 2', 3', 4', 5')$, of which the first is convex while the other is star shaped, the five points $P_1, P_2, \ldots, P_5$, whose projections are the vertices of these pentagons indicated by corresponding numbers, form the vertices of a regular fivecell with $\varrho\sqrt{5}$ as length of edge. [1])

[1]) This theorem is not new. Probably it was given for the first time by Dr. S. L. VAN OSS in his dissertation (Utrecht, 1894). Compare also my paper: "Les projections régulières des polytopes réguliers" (*Archives Teyler*, Haarlem, 1904).

We repeat here the simple proof. If (P_{12}, P_{34}) and (Q_{12}, Q_{34}) are the projections of the points P and Q with the coordinates x_i and y_i $(i=1, 2, 3, 4)$ on the planes $O(X_1 X_2)$, $O(X_3 X_4)$, we have

$$\overline{P_{12}Q}^2_{12} = (x_1-y_1)^2 + (x_2-y_2)^2 \ , \ \overline{P_{34}Q}^2_{34} = (x_3-y_3)^2 + (x_4-y_4)^2$$

and therefore if d denotes the distance PQ

$$\overline{P_{12}Q}^2_{12} + \overline{P_{34}Q}^2_{34} = d^2.$$

Now the projections $P_{12}Q_{12}$ and $P_{34}Q_{34}$ of each of the ten edges $12, \ldots, 45$ of the fivepoint $P_1P_2P_3P_4P_5$ are either side and diagonal or diagonal and side of the same regular pentagon, etc.

Which position has the regular simplex $S(5)$ with respect to the planes of projection $O(X_1X_2)$ and $O(X_3X_4)$? Evidently this projection is characterized by the fact that each of the five pairs of non intersecting edges

$$(25)\,(34) \ , \ (13)\,(45) \ , \ (24)\,(15) \ , \ (35)\,(12) \ , \ (14)\,(23)$$

We indicate the projection of the regular $S(5)$ obtained in fig. 1 as the "pentagonal projection" of that polytope, and we try to show in the following pages how easily the corresponding projections of

has *parallel* projections on *either* of the two planes, i. e. that the five lines at infinity cutting these pairs of non intersecting edges have the lines at infinity of the two planes of projection for common transversals.

Now there are altogether fifteen pairs of non intersecting edges and therefore also fifteen lines at infinity each of which cuts a pair of non intersecting edges. Moreover it can be shown easily that these fifteen lines at infinity lie on a cubic surface. For, in barycentric coordinates with respect to the regular fivecell as simplex of coordinates these fifteen lines at infinity, for which the relation $\sum_{i=1}^{5} x_i = 1$ changes into $\sum_{i=1}^{5} x_i = 0$, are represented by the equations

$$x_i + x_k = 0, \quad x_l + x_m = 0, \quad x_n = 0,$$

where i, k, l, m, n stands for any permutation of $1, 2, 3, 4, 5$, and these relations satisfy the equation $\sum_{i=1}^{5} x_i^3 = 0$ of the diagonal surface of CLEBSCH. So the SCHLÄFLI double six completing the fifteen lines mentioned above to the 27 lines of that surface $\sum_{i=1}^{5} x_i^3 = 0$ consists of the lines at infinity of six pairs of planes $O(X_1X_2)$ and $O(X_3X_4)$ corresponding to the six pairs of circular permutations

$$\left.\begin{matrix}(12345)\\(13524)\end{matrix}\right\}, \quad \left.\begin{matrix}(12354)\\(13425)\end{matrix}\right\}, \quad \left.\begin{matrix}(12435)\\(14523)\end{matrix}\right\}, \quad \left.\begin{matrix}(12453)\\(14325)\end{matrix}\right\}, \quad \left.\begin{matrix}(12534)\\(15423)\end{matrix}\right\}, \quad \left.\begin{matrix}(12543)\\(15324)\end{matrix}\right\}$$

with the property that in each pair any digit has in the two constituents different adjacent digits. Each of these six pairs consists of two reciprocal polars with respect to the sphere $\sum_{i=1}^{5} x_i^2 = 0$ at infinity common to all the spherical spaces of S_4, as the two planes of each pair are perfectly normal to each other. According to a known property, found for the first time by F. SCHUR, the six pairs of lines of a SCHLÄFLI double six are really always reciprocal polars with respect to a quadratic surface (compare TH. REYE "Beziehungen der allgemeinen Fläche dritter Ordnung zu einer covarianten Fläche dritter Classe", *Math. Annalen* vol. 55, p. 257, and G. KOHN "Ueber einige Eigenschaften der allgemeinen Fläche dritter Ordnung", *Wiener Sitzungsberichte*, vol. 117, p. 66).

If we deduce in the ordinary way the projection $O(X_2X_3)$ from the projections $O(X_1X_2)$, $O(X_3X_4)$ after having rotated each of the two regular pentagons over an arbitrary angle, we obtain the projection of the fivecell on any plane the line at infinity of which cuts the lines at infinity of $O(X_1X_2)$, $O(X_3X_4)$. This shows that the projection on an arbitrary plane can only be got in two tempos. i. e. by passing first to two arbitrary projections $O(X_2X_3)$, $O(X_4X_1)$ and by deducing a new projection $O(X_1X_2)$ after having rotated each of the projections $O(X_2X_3)$, $O(X_4X_1)$ over an arbitrary angle. Or otherwise: if l, l' are the lines at infinity of the planes $O(X_1X_2)$, $O(X_3X_4)$ and m, m those of an other pair of planes perfectly normal to each other, there are always two real lines n, n' intersecting l, l', m, m' and reprethe semiregular polytopes derived by Mrs. A. BOOLE STOTT [1]) from the regular fivecell by means of the geometrical operations of expansion and contraction can be constructed.

But it will be useful to develop first some general laws.

2. We consider the projection of the fivecell $S(5)$ more closely which leads us to the following remarks:

a. In pentagonal projection the *ten* edges of $S(5)$ present themselves in *five* directions only, any diagonal of the pentagon being parallel to one of its sides.

b. Though all the edges of $S(5)$ have the same length we find in projection two different lengths, with the proportion $s:d$, where s and d indicate side and diagonal of the pentagon.

If we wish to take into consideration the length of the edge itself we can use a very well known rectangular triangle of plane geometry, saying that when r is the radius of any circle and s_{10} and s_5 denote the sides of the regular decagon and pentagon described in it, s_5 is the length of the edge itself, s_{10} and r being the projections.

It goes without saying that the difference in length of projection is a consequence of difference in inclination; five edges of $S(5)$ make with the plane of projection an angle φ for which $tg\varphi = \frac{1}{2}(\sqrt{5}-1)$, the five others the complementary angle with $\frac{1}{2}(\sqrt{5}+1)$ as tangent.

c. In projection the ten equilateral faces of $S(5)$ split up into two quintuples of isosceles triangles, one group $(2s, d)$ with an obtuse, one group $(s, 2d)$ with an acute vertex angle.

d. In projection the five limiting tetrahedra present the same trapezoidal form (fig. 2). We show that this is of great importance with respect to our aim by saying that a rotation of the projection (2345) of the tetrahedron in the sense of the hands of a watch around the centre C indicated in fig. 1^a to an amount of one, two, three, four times 72° bring this projection succesively into coincidence with the projections (3451), (4512), (5123), (1234) of the other four limiting tetrahedra.

In order to give some relief to the single tetrahedron of fig. 2 we have dotted one of the two diagonals of the trapezoid; by doing

senting therefore the lines at infinity of the planes $O(X_2X_3)$, $O(X_4X_1)$ to be used; unless any plane through m (or m') makes with $O(X_1X_2)$ two equal angles and the lines l, l', m, m' form a hyperboloidical quadruple, in which case the planes $O(X_2X_3)$, $O(X_4X_1)$ may be selected from a singly infinite system.

1) "Geometrical deduction of semiregular from regular polytopes and space fillings" (this Academy, *Verhandelingen*, vol. 9, n°. 1). In the following we suppose the results obtained there to be known.

so we tacitly represent that limiting body considered as lying in its own threedimensional space. For, in the projection of a fourdimensional polytope on a plane the question of visibleness has no sense, as fourdimensional space *surrounds* a plane situated in it in the same way as threedimensional space surrounds a line situated in it.

3. We now examine what we have to expect *in general* as to the pentagonal projection of the semiregular polytopes deduced from the $S(5)$ by expansion and contraction. For shortness we introduce for the group of these polytopes the symbol $\bar{S}(5)$; moreover we make use in future of the symbols T, O, tT, CO, tO, P_3, P_6 for the limiting bodies of these polytopes.

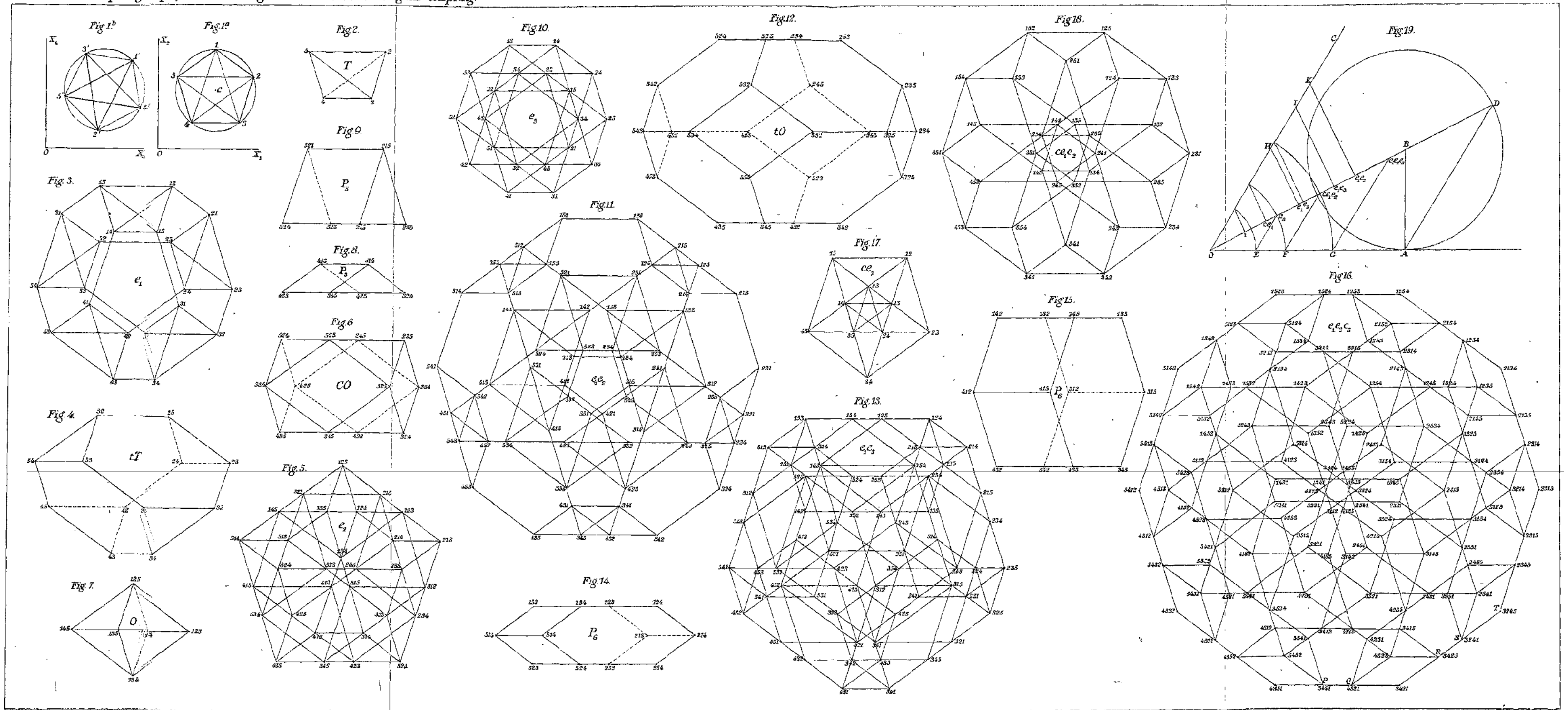
a. The polytopes $\bar{S}(5)$ partake with $S(5)$ the property of presenting in pentagonal projection edges of five directions only. For it is easy to prove that the three operations e_1, e_2, e_3, taken either separately or in combination, can introduce only new edges *parallel to the original ones.*

b. All the edges of $\bar{S}(5)$ being of the same length we find here in projection once more two different lengths with the proportion $s:d$ and the two different complementary angles of inclination obtained above.

c. As the ten faces of $S(5)$ split up in projection into two quintuples of different form, the equivalent faces of $\bar{S}(5)$ must do so likewise. We shall even experience in the treatment of the different particular cases that square faces always present a third form of projection.

d. In projection the limiting bodies of $\bar{S}(5)$ behave differently according to their *import.* The general rule that equivalent limiting bodies correspond in projection only holds for polyhedra of vertex and of body import: while both the group of edges and the group of faces of $\bar{S}(5)$ admit two different projections, the limiting bodies of edge and of face import must do so likewise.

But what is of the greatest value with respect to the construction of the projections desired is that all the limiting bodies of $\bar{S}(5)$ are "arranged pentagonally" around the projection of the centre of the original fivecell, i. e. that the four rotations indicated under *d.* of the preceding article bring any one of these limiting bodies successively into coincidence with four others. If we assert moreover that the effect of the operations of expansion and contraction are extremely easily obtained in pentagonal projection, it must be clear that the execution of what was planned with respect to the polytopes $\bar{S}(5)$ is mere children's play.



J. Bijl lith. P.J. Mulder impr. Leiden

4. We now pass to the systematic treatment of the different particular cases, putting together under the different headings, containing the expansion and contraction symbol, the symbol with the numbers of vertices, edges, faces, limiting bodies, and the symbol with the limiting bodies in the order of body, face, edge, vertex import, several remarks pertaining to facilitate the interpretation of the drawings.

$$e_1\,S(5) — (20, 40, 30, 10) — (5tT, —, —, 5T).$$

The result is given in fig. 3. By the operation e_1 of the moving out of the edges the T of fig. 2 becomes a tT (fig. 4) with four hexagons of face import and four triangles of vertex import. As each vertex of T assumes three different positions if it moves out with each edge passing through it, the vertices of this tT must bear two digits, the first indicating the original vertex of T, the first in combination with the second the edge of T moved out. By retracing in fig. 3 the same pairs of digits one easily finds again the tT deduced from (2345), though for the reason stated above no dotted lines have been admitted. If we rotate this tT around the centre of fig. 3 to an amount of 72° in the indicated sense it is brought into the position with (54, 45) as bottom-edge and (13, 31) as top-edge in coincidence with a second tT, having in common with the first — in its original position — the hexagon (54, 53, 35, 34, 43, 45), deduced by the e_1-operation from the triangle (345) common to the tetrahedra (2345), (3451) of fig. 1^a. Or rather: the centre of fig. 3 is found by drawing the tT of fig. 4 twice and by putting these two tT in such a way upon each other as to get a limiting hexagon in common; then this centre is the point of intersection of the lines bisecting orthogonally the two edges (43, 34) and (54, 45). Or still otherwise: the limiting polygon of the projection is a semiregular decagon with sides alternately equal to z and d and from this fact the circumcentre can be deduced [1]).

It goes without saying that the vertices of each following tT bear pairs of digits deduced from those at the corresponding vertices of the preceding tT by adding unity to each digit, in which process the 5 becomes 0.

The four different positions 12, 13, 14, 15 of the original vertex 1 form the vertices of a T of vertex import.

It is easily verified that the ten limiting bodies $5tT, 5T$, now

[1]) From fig. 2 upward we use in all the diagrams for s and d the same measures in order to show by the projection the swelling of the polytopes corresponding to the operations of expansion.

accounted for, want as limiting faces exactly all the faces shown in projection, each face counted twice over, a triangle always being common to a tT and a T, a hexagon to two tT.

The 20 vertices present themselves in two wreaths (10, 10).

$$e_2 S(5) — (30, 90, 80, 20) — (5CO, —, 10P_3, 5O).$$

For the result fig. 5 may be consulted. By application of the e_2-operation the T of fig. 2 passes into the CO of fig. 6, each edge of T being broadened out into a square, the sides of which are parallel to that edge and to the opposite one. Here the particularity enters that two of the six squares project themselves as line segments, which is due to the fact that in pentagonal projection the edges 25, 34 of the T of fig. 2 are parallel. Here we have to indicate the vertices of the CO by three digits, the first indicating the original vertex of T and the two others, in irrelevant order, in combination with the first, the face which is moved out. This CO in indicated in fig. 5 by the same triplets of digits placed at the vertices. By reproducing it four times by means of the rotations indicated above, fig. 5 is completed; here any two CO have to be placed upon each other in such a way as to have a triangle in common.

After having inscribed all the triplets of digits at the vertices according to the rule given above about the augmentation with unity for each rotation in the right sense to an amount of 72° we find that the 1 is foremost in six triplets, corresponding (fig. 7) to the vertices of an O, i. e. we find $5O$ as limiting bodies of vertex import. Farthermore the notation shows that the edge (34) of the T presents itself in fig. 5 in three positions, the triplets of digits of the endpoints of which are found by putting behind 34 and 43 successively one of the three remaining digits 1, 2, 5, passing — if we rearrange the second and the third figure according to their value — into 314, 324, 345 and 413, 423, 435. So we get the P_3 of fig. 8, occurring in five different positions, and likewise the edge 25 leads to the differently projected P_3 of fig. 9, occurring also in five positions. So the ten P_3 of edge import are accounted for.

Here the circumpolygon is a regular pentagon with sides $s + d$; the 30 vertices appear in four wreaths (5, 10, 10, 5).

$$e_3 S(5) — (20, 60, 70, 30) — (5T, 10P_3, 10P_3, 5T).$$

The pentagonal projection (fig. 10) exhibits central symmetry as does $e_3\,S(5)$ itself. Here (21, 31, 41, 51) is the T of fig. 2 moved out, by which remark the $5T$ of body import are accounted for, whilst the four positions 12, 13, 14, 15 of the original vertex 1 are the vertices of a tetrahedron of opposite orientation, the rotation of which provides us with the $5T$ of vertex import. The relation between these two sets of $5T$ can be indicated by saying, that two T of the same set have nothing at all and that two T of different sets can have only one vertex in common.

If we had followed here the notation indicated under e_1 and e_2 each vertex would have had to bear four digits, the digit of the original vertex of $S(5)$ followed by the digits of the other vertices of the T with which the vertex is moved out; however, for shortness we have placed after the digit of the vertex the only digit which does not occur at the vertices of the T moved out.

In this new notation of pairs of digits, where — at variance with the notation applied under e_1 — the order of succession is of influence, the ten P_3 of edge import present themselves in two quintuples, which can be obtained by putting after each of the digits of the pair of digits of an edge successively each of the three remaining digits; so 43 gives the three edges (41,31), (42, 32), (45, 35) of the P_3 of fig. 8 turned upside down, while 52 leads in the same way to (51, 21), (53, 23), (54, 24), the parallel edges of the P_3 of fig. 9 turned upside down. Similarly the ten P_3 of face import are found by putting after each of the three digits of a face of T successively one of the two remaining digits; so 125 gives the two endplanes (13, 23, 53), (14, 24, 54) of the P_3 of fig. 8, 134 the two endplanes (12, 32, 42), (15, 35, 45) of the P_3 of fig. 9.

The limiting polygon is a regular decagon with side s; from this ensues the possibility of drawing the ten T immediately in position. The 20 vertices are arranged in two wreaths (10, 10), of regular decagons.

$$e_1 e_2 S(5) — (60, 120, 80, 20) — (5t\,O, —, 10P_3, 5tT).$$

In this case, for which fig. 11 represents the result, the T of fig. 2 is transformed into a tO (fig. 12); of the triplet of digits placed at each vertex of this tO the first indicates the original vertex of T, the second with the first the edge moved out, the third with the two preceding ones the face moved out. This notation with triplets of digits differs again from that applied in fig. 5 in this that the order of succession of the second and third digits, of no consequence there, is of influence here.

If we have traced in fig. 11 the tO of fig. 12, rotation about the centre, accompanied by an addition of unity to all the digits, gives

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the vertices and the triplets of digits of each following tO. It goes without saying that here too the centre is found by drawing tO twice over and putting these tO in such a manner upon each other as to have — with a difference in orientation of 72° — a hexagon in common. In the case of the two tO deduced from (2345) and (3451), which T of fig. 1^a have the face (345) in common, this common hexagon is characterized by this that the six vertices bear the digits 3, 4, 5 in all possible permutations.

The digit 1 stands foremost at the triplets of twelve vertices, the vertices of a tT of vertex import; by omitting from these triplets the 1 we get not only in position but also in notation the tT of fig. 4. So the five tT of vertex import are accounted for. Moreover, as to the ten P_3 of edge import we can refer to the development given under e_2.

Circumpolygon a semiregular decagon with sides alternately s and d. Six wreaths of ten vertices, all of them semiregular decagons.

$$e_1 e_3\, S(5) — (60, 150, 120, 30) — (5tT, 10P_6, 10P_3, 5CO).$$

In this case — for the result compare fig. 13 — the T of fig. 2 is transformed by the e_1-operation into the tT of fig. 4, after which this tT is moved out as a whole; as by this process each vertex of $e_1 S(5)$ assumes three different positions we must follow once more the notation of the triplets of digits, which can be done here by placing after each pair of digits of fig. 4 the digit 1 not occurring at the vertices of the tetrahedron (2345) moved out. If these triplets have been inscribed in fig. 13, rotation about the centre and augmentation of the digits by unity gives all that is wanted, as soon as the centre has been constructed. We arrive as soon as possible at the construction of this centre by determining the prisms of face import first. In the case e_3 they were the prisms P_3 represented by fig. 8 and 9; by applying to the T the e_1-operation, the triangles of the T pass into hexagons, which includes that the P_3 are transformed into P_6, which can be drawn immediately. By applying to the endplanes (13, 23, 53), (14, 24, 54) of the upper prism P_3 of fig. 10 the e_1-operation we obtain the upper prism P_6 of fig. 13 represented separately by fig. 14. Consideration of this prism P_6 shows that the limiting polygon is a semiregular decagon, the sides of which are alternately s and $s + d$; from this the centre can be deduced. In the same way the prism P_3 of fig. 10 with the endplanes (12, 32, 42), (15, 35, 45) passes into the P_6 represented by fig. 15. Farthermore the two P_3 with the pairs of endplanes (341, 342, 345), (431, 432, 435) and (521, 523, 524), (251, 253, 254) represent two P_3 of edge import, the prisms of fig. 8 and 9 upside down.

The vertices with the triplets of digits where the 1 is foremost form the vertices of a limiting body of vertex import, a CO in the position of fig. 6.

Six wreaths of ten vertices, semiregular decagons.

$$e_1 e_2 e_3\, S(5) — (120, 240, 150, 30) — (5tO, 10P_6, 10P_6, 5tO).$$

This most inflated of the polytopes $S(5)$ is represented in projection in fig. 16. According to the number of vertices [1]) we have to place at each vertex four of the five digits, each of them with a meaning as to the order of succession; of these four digits the first indicates the original vertex of $S(5)$, the second the new endpoint of the edge moved out, the third the new vertex of the face moved out and the fourth — according to what was stipulated under $e_3 S(5)$ — the digit not occurring at the vertices of the tetrahedron moved out: so 1234 denotes the position of the vertex 1, after this point has been moved out with the edge 12, with the face 123, with the tetrahedron 1235.

Likewise as in the case $e_1 e_2\, S(5)$ the T of fig. 2 passes here into the tO of fig. 12, traced back easily in fig. 16 if one remarks that the moving out of this tO under the influence of the e_3-operation demands the digit 1 after the triplets of fig. 12. While now the lower side (4351, 3451, 4321, 3421) of the projection of this tO assumes the same length $s + d$ as the upper side of the projection of the P_6 of fig. 16, i. e. the side (1523, 1524, 1253, 1254), which P_6 corresponds in form and position with that of fig. 14, it is clear that the circumpolygon is a regular decagon with side $s + d$. So the projection is once more central symmetric as is the polytope itself. In connexion with this the limiting bodies of vertex import are likewise tO, which is immediately verified by looking for the 24 vertices in whose quadruples of digits the 1 is foremost; likewise, not only the prisms of face import, but also those of edge import, are hexagonal.

Evidently the centre of the figure can be deduced from the side $s + d$ of the regular decagon; moreover it is possible to use to that end the property that two adjacent P_6 of the ten of the form of fig. 14 lying at the rim have in projection a square face projected as a lozenge in common.

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[1]) It is easily verified that in each of the cases treated the notation corresponds to the number of vertices, i. e. that the number of possible pairs, triplet, quadruples of figures is always equal to the number of the vertices.

$ce_1\ S(5) - (10, 30, 30, 10) - (5O, -, -, 5T)$.

This figure can be deduced from fig. 3 by moving the limiting bodies of vertex import, i. e. the $5T$ projected as trapezoids (12, 13, 14, 15), (23, 24, 25, 21), etc. in such a way towards the centre, that the ten original edges of $S(5)$, i. e. the five edges (12, 21), (23, 32) etc. and the five edges (52, 25), (13, 31), etc. disappear. It is easily shown that these two conditions do not collide; for, if we suppose that the trapezoid (12, 13, 14, 15) remains where it is, whilst of the two adjacent trapezoids (21, 23, 24, 25), (51, 52, 53, 54) the first experiences a rectilinear translation 21, 12, the second a rectilinear translation 51, 15, the vertices 52 and 25 will coincide in the point of intersection of the projections (12, 14), (13 15). So we get the simple result of fig. 17, where the limiting polygon is a pentagon with side d, oppositely orientated with respect to fig. 1^a. In fig. 17 the six points where the digit 1 is lacking form the vertices of an O of body import, the four points where the digit 1 occurs a T of vertex import, etc.

$ce_1e_2\ S(5) - (30, 60, 40, 10) - (5tT, -, -, 5tT)$.

This figure can be derived from fig. 11 by moving the $5tT$ of vertex import towards the centre in such a way that the ten prisms P_3 of edge import disappear. Then the triplets of parallel edges of these P_3 disappear and only the two coinciding endplanes remain. But this implies that the five tO of body import are reduced to tT by the annihilation of these edges; so in the case of fig. 12 the square (532, 352, 354, 534) is reduced by the coincidence of the vertices 532, 352 and of the vertices 354, 534 to an edge with the direction (532, 534) and the hexagon (523, 253, 235, 325, 352, 532) passes into a triangle, while the adjacent hexagons do not change in form. So we get fig. 18, where each vertex bears a triplet of digits, of which the order of succession of the first and the second is irrelevant, while e.g. 345 results from the coincidence of the vertices 345 and 435 of fig. 11. In this figure the tT of vertex import, remained unaltered, are recognized by the property that at their vertices the same digit occurs under the first two of the three digits, whilst the five other tT of body import lie in projection symmetrically with these with respect to the centre.

It may still be remarked that the centre of the figure can also be found by drawing the tT of fig. 4 twice and by putting these two tT with a difference of 36° in orientation in such a way upon each other that they have in projection a hexagon in common [1]).

The limiting polygon is a regular decagon with side d. The figure is central symmetric as is the polytope itself. The vertices appear in three wreaths of ten regular decagons.

5. Though we have finished what we had proposed to ourselves to do, the plate still contains another diagram. In fig. 19 we have constructed accurately the radii of the circumcircles of the different projections and — for the cases where the limiting polygon is a semiregular decagon — also the side of the regular pentagon inscribed in the circumcircle. So the labour of the pure construction of the figures is reduced to a minimum. This diagram will be clear if we remark that OA is divided in extreme and mean ratio, that on OB measured from O are to be found the radii of the circumcircles and on OC parallel to AD measured likewise from O the sides of the regular pentagons inscribed in the circumcircles of the semiregular decagons. Moreover we have $OE=FG=z$, $OF=EG=d$, whilst in connection with 16

$$OH = QS, \quad OI = QT, \quad OK = PT$$

and the points e_1, e_1e_3, e_1e_2 on OB are obtained by letting down the perpendiculars from H, I, K on OB. Finally Ee_3, Fce_1e_2 and $Ge_1e_2e_3$ are parallel to AD.

Groningen, March 10, 1911.

Physiology. — *"On the irritation-effect in living Organisms."* By Mr. J. L. HOORWEG. (Communicated by Prof. H. ZWAARDEMAKER).

(Communicated in the meeting of April 28, 1911).

1. In this paper I wish to make a few remarks with regard to an essay of HILL (1) intending to give an extension to NERNST's (2) theory about the electric irritation of living organisms.

I may remind my readers of the fact that ever since the year 1890 I have occupied myself with this subject, when I communicated in the *Ned. Tijdschrift voor Geneeskunde* (3) experiments about the contraction of the human muscles by condensator-discharges, and indicated in this paper a simple connection between the capacity C of the condensator used and the potential P, to which the latter was to be charged in order to produce a minimal response. This connection is expressed by the formula

1) We remember that under e_1 the difference in orientation was 72°.