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Astronomy. — “*Preliminary investigation into the motion of the pole of the earth in 1907.*” By Dr. H. J. ZWIERS. (Communicated by Prof. E. F. VAN DE SANDE BAKHUYZEN.)

In 1910 I started investigations into the motion of the momentary rotation-pole of the earth since the beginning of the year 1890. Besides the general scientific importance of such an analysis, especially for the future explanation of the rather complicated phenomenon, another reason prompted these investigations, i.e. the desire to arrive at a safe basis for a quick reduction of some observations of declination with the Leyden meridian circle. For often enough starplaces must be reduced soon after the day of observation, even before anything is known about the momentary value of the latitude, which is one of the most important elements of reduction.

The general results of these investigations I hope to publish before long. At present I desire only to communicate some preliminary results about a perturbation in the regular motion of the pole, which must have taken place in the course of 1907.

Early in my investigations I found, that, while the motion of the pole up to the beginning of 1907 could be represented by simple formulae with tolerable accuracy, later observations showed great deviations. Originally only the results of the observations up to 1908.5 were known from Prof. ALBRECHT's different publications, and the lapse of time after the moment of the perturbation was too short to determine accurately its nature and the orbit described afterwards. In N^o. 4414 of the *Astronomische Nachrichten* ALBRECHT gave the polar co-ordinates for the period 1908.0 until 1910.0, and in the first days of June 1911 in *Astron. Nachr.* N^o. 4504 a continuation of the table of these co-ordinates was published by him, as far as the commencement of 1911. This enabled me to investigate more closely into the time and the probable nature of the disturbance.

Without dealing with it in detail, I must first give here some results of my earlier investigations.

It appeared, that from 1890.0 until early in 1907 the motion of the pole could be analysed into a yearly ellipse and an approximately 14-monthly circle, the motion in both being from W. to E. The polar co-ordinates may be thus represented by:

$$\left. \begin{aligned} x &= \xi + x_1 + x_2 \\ y &= \eta + y_1 + y_2 \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which the indices 1 refer to the co-ordinates in the yearly ellipse, the indices 2 to the 14-monthly circle, while ξ and η represent the co-ordinates of the mean pole.

I commenced my calculations by deducing also for the period 1890.0 to 1899.8 the value of the yearly z -term, found afterwards by KIMURA, as far as the published observations would allow it. The separate results came out with considerable uncertainty, but taking the mean of the values, found for z for the corresponding tenth parts of the different years, I arrived at 10 mean values, which were represented as well as possible by the following sinusoid:

$$z = + 0''0043 \sin \psi + 0''0221 \cos \psi$$

in which ψ is being counted from the beginning of the year.

From the values of $\Delta\varphi$, corrected for this z -term, I computed the rectangular polar co-ordinates x and y for that period, so that I obtained a continuous, and homogeneously reduced series of these co-ordinates, from 1890.0 up to 1908.5. From this I deduced in first approximation the elements of the yearly component for three periods:

$$\begin{aligned} a & \text{ from 1890.0 to 1897.0} \\ b & \text{ from 1895.5 to 1902.5} \\ c & \text{ from 1901.0 to 1908.0} \end{aligned}$$

taking each time 7 years together in order to eliminate the second component, whose period was thus in this first approximation supposed to be exactly 14 months.

For these three periods I found a yearly ellipse, showing slight variations in size and shape, and in the position of the axes. This need not to be a cause of wonder, however, when we accept changes of mainly atmospheric nature, e. g. varying distribution of atmospheric pressure, accumulation of snow and ice in winter, as the chief factor in bringing about this component.

A single result of this investigation deserves to be shortly mentioned here.

While for the periods a and b the zero-values of ξ and η proved, that the adopted origin of co-ordinates coincided practically exactly with the real mean pole, I found in this first approximation for the central co-ordinates in the period c :

$$\xi = + 0''009 \quad \eta = + 0''.032.$$

These values indicate, that for the later years the adopted origin deviates sensibly from the mean pole. The observations of this period have all been made at the six international latitude stations, and have been reduced uniformly by Prof. ALBRECHT in his *Resultate* Bnd. I, II and III. The origin of co-ordinates chosen by him, and adopted as "mean pole", coincides fairly accurately with the centre of the orbit of the pole resulting from the observations from 1899.9

until the beginning of the year 1901, which were first discussed. ALBRECHT's method of reduction must then for subsequent years always conduct to the same "mean pole" and could not give any answer to the question of its secular motion. Only an analysis of the total motion can give a criterium for this question in the constancy or otherwise of the co-ordinates of the centre found after subtracting the periodical components.

From the three yearly ellipses, combined with ξ and η , I calculated three series of values of $\xi + x_1$ and $\eta + y_1$ from 0.1 to 0.1 year. The first series was used for the period 1890.0—1893.5, the second for 1899.0—1899.9 and the third for 1904.5 and following years. The values for the intermediate years were obtained by simple interpolation. Subtracting these values from x and y , I obtained a continuous series of values of x_2 and y_2 , which served as a first approximation of the second component. This series I divided in two:

$$A: 1890.0 \text{ to } 1899.8 \qquad B: 1899.9 \text{ to } 1908.0$$

and first of all I deduced the length of the period from transits through the axes of co-ordinates. I found:

$$\begin{aligned} \text{from } A: \quad P_2 &= 1.198 \text{ year} \\ \text{from } B: \quad P_2 &= 1.174 \text{ year.} \end{aligned}$$

Provisionally I decided on adopting a general mean value, and computed from A and B together:

$$P_2 = 1.188 \text{ year} = 434.1 \text{ days.}$$

I examined the shape of the second component for three parts of the whole interval, and found three ellipses, which agreed inter se so closely, that there was no objection to taking them together in one mean orbit:

$$\begin{aligned} x_2 &= + 0."123 \sin \psi_2 - 0."057 \cos \psi_2 \\ y_2 &= + 0.061 \sin \psi_2 + 0.126 \cos \psi_2 \end{aligned}$$

in which ψ_2 has been counted from 1890.198, and increases yearly with $360^\circ : 1.188 = 303."03$.

Taking the two periodical terms together, we find:

$$\begin{aligned} x_2 &= 0."136 \sin (\psi_2 + 335."01) \\ y_2 &= 0."140 \sin (\psi_2 + 64."02) \end{aligned}$$

Practically both amplitudes are equal and the phases differ 90° , so that the second component appears to be a circle with a radius of nearly 0."14.

The co-ordinates x_2 and y_2 , computed from the above formulae, were now used to investigate the yearly component in second approximation. For the present I shall only mention the result I

obtained for the period 1904.6 to 1907.5, which period immediately precedes the perturbation in 1907. As co-ordinates of the mean pole I found:

$$\xi = + 0."001 \quad \eta = + 0."040$$

in good accordance with those mentioned above. The elliptical co-ordinates became

$$\left. \begin{aligned} x_1 &= -0."075 \sin \psi_1 - 0."019 \cos \psi_1 \\ y_1 &= +0."001 \sin \psi_1 - 0."053 \cos \psi_1 \end{aligned} \right\} \dots (2)$$

in which ψ_1 has been counted from the beginning of the year.

This yearly component of the motion was supposed to be constant for the whole period 1904.0 till 1911.0 and I diminished the x and y resulting from the observations with these x_1 and y_1 . From the residual values for $\xi + x_2$ and $\eta + y_2$ I computed, adopting for the length of the period the value found above: 434.1 days, two ellipses, 1° for the period 1904.0—1907.0 and 2° for 1908.0—1911.0. I found:

$$1904.0 - 1907.0 \left\{ \begin{aligned} \xi &= + 0."013 & x_2 &= 0."115 \sin (\psi_2 + 199.^{\circ}2) \\ \eta &= + 0."044 & y_2 &= 0."121 \sin (\psi_2 + 288.^{\circ}3) \end{aligned} \right.$$

ψ_2 being counted from 1904.0

$$1908.0 - 1911.0 \left\{ \begin{aligned} \xi &= + 0."008 & x_2 &= 0."252 \sin (\psi_2 + 286.^{\circ}1) \\ \eta &= + 0."037 & y_2 &= 0."249 \sin (\psi_2 + 110.^{\circ}4) \end{aligned} \right.$$

ψ_2 being counted from 1909.0.

Both orbits are so nearly circular, that I substituted for them the two following circles;

$$\left. \begin{aligned} 1904.0 - 1907.0 &\left\{ \begin{aligned} x_2 &= 0."118 \sin (\psi_2 + 179.^{\circ}3) \\ y_2 &= 0."118 \cos (\psi_2 + 179.^{\circ}3) \end{aligned} \right. \\ 1908.0 - 1911.0 &\left\{ \begin{aligned} x_2 &= 0."250 \sin (\psi_2 + 188.^{\circ}7) \\ y_2 &= 0."250 \cos (\psi_2 + 188.^{\circ}7) \end{aligned} \right. \end{aligned} \right\} \dots (3)$$

ψ_2 being counted for both from 1907.5.

For these formulae (3) I have not yet derived the mean error *Observ.—Comput.*, but when we consider, that ALBRECHT estimates the mean error of each of his polar co-ordinates x and y at $\pm 0".02$, the results I found, justify the following two conclusions:

1. the co-ordinates of the mean pole have remained unaltered since 1904.0;

2. the computed difference in phase of $9^{\circ}.4$ is too slight to be answered for, the more so, as a somewhat smaller value of P_2 ¹⁾ seems not improbable.

Therefore I accepted for both periods:

¹⁾ Cf. the last paragraph of this paper.

T A B L E I.

Epoch	Observation		Computation		O-C	
	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	Δx	Δy
1904.0	- 43	- 144	- 60	- 121	+ 17	- 20
.1	- 146	- 86	- 148	- 72	+ 2	- 14
.2	- 170	+ 5	- 188	+ 13	+ 18	- 8
.3	- 162	+ 97	- 166	+ 105	+ 4	- 8
.4	- 94	+ 183	- 91	+ 179	- 3	+ 4
.5	+ 1	+ 211	+ 15	+ 211	- 14	0
.6	+ 87	+ 175	+ 115	+ 189	- 28	- 14
.7	+ 151	+ 123	+ 180	+ 122	- 29	+ 1
.8	+ 181	+ 55	+ 189	+ 32	- 8	+ 23
.9	+ 148	- 28	+ 141	- 55	+ 7	+ 27
1905.0	+ 92	- 101	+ 52	- 111	+ 40	+ 10
.1	- 11	- 144	- 46	- 120	+ 35	- 24
.2	- 121	- 125	- 125	- 80	+ 4	- 45
.3	- 144	- 54	- 158	- 7	+ 14	- 47
.4	- 131	+ 38	- 141	+ 78	+ 10	- 39
.5	- 82	+ 127	- 79	+ 148	- 3	- 21
.6	+ 1	+ 182	+ 3	+ 182	- 2	0
.7	+ 84	+ 189	+ 80	+ 173	+ 4	+ 16
.8	+ 122	+ 145	+ 128	+ 126	- 6	+ 19
.9	+ 148	+ 68	+ 136	+ 57	+ 12	+ 11
1906.0	+ 105	- 6	+ 105	- 12	0	+ 6
.1	+ 36	- 55	+ 49	- 61	- 13	+ 6
.2	- 35	- 62	- 12	- 77	- 23	+ 15
.3	- 78	- 34	- 60	- 61	- 18	+ 27
.4	- 102	+ 14	- 83	- 18	- 19	+ 32
.5	- 100	+ 72	- 78	+ 36	- 22	+ 36
.6	- 73	+ 112	- 53	+ 85	- 20	+ 27
.7	- 35	+ 133	- 17	+ 117	- 18	+ 16
.8	- 9	+ 133	+ 15	+ 127	- 24	+ 6
.9	+ 14	+ 139	+ 40	+ 115	- 26	+ 24
1907.0	+ 44	+ 122	+ 50	+ 86	- 6	+ 36
1908.0	- 81	+ 193	- 117	+ 215	+ 36	- 22
.1	+ 14	+ 227	- 28	+ 247	+ 42	- 20
.2	+ 118	+ 204	+ 76	+ 227	+ 42	- 23
.3	+ 195	+ 136	+ 170	+ 157	+ 25	- 21
.4	+ 235	+ 41	+ 225	+ 54	+ 10	- 13
.5	+ 209	- 67	+ 225	- 57	- 16	- 10
.6	+ 134	- 141	+ 161	- 149	- 27	+ 8
.7	+ 29	- 165	+ 50	- 198	- 21	+ 28
.8	- 114	- 152	- 82	- 174	- 32	+ 22
.9	- 210	- 89	- 197	- 96	- 13	+ 7
1909.0	- 257	+ 29	- 260	+ 24	+ 3	+ 5
.1	- 242	+ 154	- 248	+ 155	+ 6	- 1
.2	- 162	+ 268	- 160	+ 259	- 2	+ 9
.3	+ 7	+ 327	- 18	+ 303	+ 25	+ 24
.4	+ 189	+ 303	+ 137	+ 275	+ 52	+ 28
.5	+ 289	+ 180	+ 260	+ 178	+ 29	+ 2
.6	+ 318	+ 26	+ 311	+ 37	+ 7	+ 11
.7	+ 266	- 114	+ 272	- 108	- 6	- 6
.8	+ 109	- 207	+ 152	- 213	- 43	+ 6
.9	- 72	- 253	- 14	- 248	- 58	- 5
1910.0	- 184	- 234	- 179	- 200	- 5	- 34
.1	- 262	- 90	- 291	- 80	+ 29	- 10
.2	- 296	+ 78	- 315	+ 78	+ 19	0
.3	- 231	+ 241	- 243	+ 226	+ 12	+ 15
.4	- 73	+ 327	- 97	+ 322	+ 24	+ 5
.5	+ 120	+ 345	+ 82	+ 336	+ 38	+ 9
.6	+ 268	+ 266	+ 237	+ 264	+ 31	+ 2
.7	+ 326	+ 88	+ 323	+ 125	+ 3	- 37
.8	+ 299	- 80	+ 313	- 37	- 14	- 43
.9	+ 195	- 210	+ 214	- 177	- 19	- 33
1911.0	(+ 2)	(- 282)	+ 53	- 254		

$$\left. \begin{aligned} \xi &= +0''.006 & x_2 &= c_2 \sin(\psi_2 + 184^\circ.0) \\ \eta &= +0.040 & y_2 &= c_2 \cos(\psi_2 + 184^\circ.0) \end{aligned} \right\} \dots \dots (4)$$

where again ψ_2 has been counted from 1907.5, and further

from 1904.0 to 1907.0: $c_2 = 0''.118$

from 1908.0 to 1911.0: $c_2 = 0.250$.

From (2) and (4) I now computed, according to (1), the theoretical values of x and y , and compared them with ALBRECHT's results from the observations. Table I gives the result of this comparison, the adopted unit being $1/1000$ second of arc.

The differences O—C (Observation—Computation) give as mean error of my computed values:

$$1904-1907: \quad m_1 = \pm 0''.0204$$

$$1908-1911 \quad \pm 0.0237$$

while, as I said before, the mean error of an observed co-ordinate is estimated by ALBRECHT at $\pm 0''.020$.

I think, that from this I may conclude, that within the admissible limits of error of the observations the polar motion is represented by a yearly and a 14-monthly component, as expressed by the formulae (2) and (4).

From this it follows:

1. that a change in the 14-monthly motion must have taken place in the course of the year 1907;
2. that the perturbation did not cause any appreciable change of phase, or displacement of the mean pole;
3. that the change in the motion is wholly owing to a gradual, or more or less sudden increase of the amplitude of the 14-monthly motion from $0''.12$ to $0''.25$.

In the following manner I have attempted to determine more precisely the very moment and the nature of the perturbation.

From the formulae found for the period 1904—1907 I deduced the co-ordinates for 1907.0—1908.0, such as they ought to have been, had the motion of the pole remained undisturbed, and compared them with the observed positions. The numerical results of this comparison are contained in Table II.

The columns O—C yield as mean error $\pm 0''.0461$, which is far more than the accuracy of the observations allows for. Moreover a glance at these differences makes it clear, that although the agreement up to 1907.3 or 1907.4 may be satisfactory, the differences found afterwards are decidedly not admissible.

Secondly I compared the observed co-ordinates with those which

(217)

T A B L E II.

Epoch	Observation		Computation		O—C	
	x	y	x	y	Δx	Δy
1907 0	+ 41	+ 122	+ 39	+ 95	+ 5	+ 27
.1	+ 63	+ 64	+ 44	+ 64	+ 19	0
.2	+ 67	+ 13	+ 44	+ 32	+ 23	— 19
.3	+ 65	— 28	+ 42	+ 4	+ 23	— 32
.4	+ 37	— 63	+ 35	— 14	+ 2	— 49
.5	+ 3	— 81	+ 23	— 21	— 20	— 60
.6	— 43	— 63	+ 4	— 17	— 47	— 46
7	— 101	— 47	— 22	0	— 79	— 17
8	— 147	+ 45	— 50	+ 27	— 97	+ 18
9	— 137	+ 123	— 70	+ 61	— 67	+ 61
1908.0	— 81	+ 193	— 74	+ 104	— 7	+ 89

ought to follow for 1907 according to the formulae found for 1908—1911. Table III gives the results of this comparison.

TABLE III.

Epoch	Computation		O—C	
	x	y	Δx	Δy
1907.0	+ 140	+ 183	— 96	— 61
.1	+ 180	+ 91	— 117	— 27
.2	+ 179	— 12	— 112	+ 25
.3	+ 140	— 100	— 75	+ 72
.4	+ 71	— 152	— 34	+ 80
.5	— 1	— 157	+ 4	+ 76
.6	— 90	— 115	+ 47	+ 52
.7	— 149	— 36	+ 48	+ 19
.8	— 173	+ 62	+ 26	— 17
.9	— 155	+ 154	+ 18	— 31
1908 0	— 96	+ 219	+ 15	— 26

As mean value of the differences between observation and computation we here find $\pm 0''.0591$, an even less admissible result from chance errors than the value, derived from Table II. Here it is the differences O—C from 1907.0 up to about 1907.6, that reach very abnormal values. From 1907.7 the agreement may be considered satisfactory.

This would seem to lead up to the conclusion, that the change in the second component of the polar motion must have taken place rather rapidly, and somewhere between 1907.3 or 1907.4 and 1907.7. On closer examination, however, it seems to me, that the observations do not sufficiently justify this conclusion. We may rather say, that the real path of the pole during the year 1907 deviates more and more from the former orbit (1904—1907), to approach to that deduced from the elements found for 1908—1911.

Better still than by the Tables II and III, this is shown by the annexed figure, which represents the observed path of the pole, and the two computed ones from Table II and III. The curve drawn continuously shows the displacement of the pole according to the observations; the computed curves have been represented by dotted lines. The inner one results from the elements found for 1904—'07, the outer one from those for 1908—'11.

It had already appeared from Table I, and the mean errors deduced from it, that the elements (4) represent the observations before 1907 and after 1908 with sufficient accuracy, and the figure shows, that the observations in 1907 indicate a gradual rather than a sudden transition from one orbit to another. Thus I simply supposed, that the amplitude c_2 of the 14-monthly circle gradually increased in the course of 1907 from $0''.118$ to $0''.250$, and the computation on this basis corresponds so remarkably well with the observed co-ordinates, that I thought it unnecessary to extend the researches still in other directions.

For a closer investigation moreover, it would have been necessary to go back to the original observations of the several stations. ALBRECHT's co-ordinates have been obtained by a process of adjustment, and this turns even rather sudden changes into smooth transitions. In the first place the time for such an investigation was lacking, and secondly it remains to be questioned, whether the accuracy of the separate results would admit of a decided conclusion.

In formula (4) I substituted therefore for the year 1907:

$$c_2 = 0''.118 + 0''.132 (t - 1907.0) \quad . \quad . \quad . \quad . \quad (5)$$

Combining the resulting values of x_2 and y_2 with the yearly com-

ponent, and the constant co-ordinates of the mean pole, I obtained the computed values of x and y of Table IV.

T A B L E IV.

Epoch	Observation		Computation		O—C	
	x	y	x	y	Δx	Δy
1907.0	+ 44	+ 122	+ 50	+ 87	— 6	+ 35
.1	+ 63	+ 64	+ 64	+ 58	— 1	+ 6
.2	+ 67	+ 43	+ 73	+ 17	— 6	— 4
.3	+ 65	— 28	+ 73	— 30	— 8	+ 2
.4	+ 37	— 63	+ 53	— 70	— 16	+ 7
.5	+ 3	— 81	+ 12	— 91	— 9	+ 10
.6	— 43	— 63	— 46	— 81	+ 3	+ 18
.7	— 101	— 17	— 107	— 35	+ 6	+ 18
.8	— 147	+ 45	— 152	+ 42	+ 5	+ 3
.9	— 137	+ 123	— 159	+ 133	+ 22	— 10
1908.0	— 81	+ 193	— 117	+ 214	+ 36	— 21

If we look at the columns O—C, and compare them with the corresponding ones in Tables II and III, we see, how much the agreement between observation and computation has been improved. As mean error we now find $\pm 0''.0150$, which is even less than the uncertainty (m. e. $= \pm 0''.02$) of the co-ordinates deduced from observation.

The calculated polar curve, obtained by adopting formula (5) has been represented in the figure by dashed lines. In judging the agreement it may be useful to remind, that the unit of $1/1000$ second of arc, in which the numbers of the tables are expressed, represents a length of 31 millimeter on the surface of the earth; the scale of the figure is nearly 1 : 77.5.

Now that we may hold it proved by the foregoing comparisons that the observations of 1904 up to 1911 may all be represented by the same elements with the exception of the increased value of c , the question arises, in how far the same elements also satisfy the former observations. Small variations in the yearly ellipse can be

easily explained by the causes mentioned above. So the question regards more particularly the agreement in length of period and in phase of the 14-monthly component.

In the Archives Néerlandaises, Série II, Tome II p. 479 Dr. E. F. VAN DE SANDE BAKHUYZEN gives a summary of deduced transits through the positive axis of x , i e. through the Greenwich meridian.

From my computations mentioned on p. 213 I have added to this list two new epochs, one being deduced from the observations of 1890.0 up to 1899.8, the other from those of 1899.9 up to 1907. I compared the whole series with the elements obtained by E. F. VAN DE SANDE BAKHUYZEN:

$$\text{Epoch} = \text{J. D. } 2408567 \quad P_s = 431^{\text{d}}.14.$$

Representing the corrections of these elements respectively by u and v , we arrive at the following equations:

1. Washington 1 st vert.	1862—67	$u-14$	$v=-26^{\text{d}}$	$p=2$
2. Pulkowa vert. c., Pol.	1863—70	$u-13$	$v=+72$	2
3. Leyden, Fund. stars	1864—68	$u-12$	$v=0$	2
4. Leyden, Polaris	1864—74	$u-12$	$v=-8$	2
5. Greenw., Trans. circle	1865—72	$u-12$	$v=+41$	1
6. Pulk., vert. c., Fund. st.	1863—75	$u-10$	$v=+23$	4
7. Pulk., vert. c., Pol.	1871—75	$u-8$	$v=+28$	2
8. Pulk., 1 st vert.	1875—82	$u-3$	$v=+16$	2
9. Pulk., vert. c.	1882—91	$u+3$	$v=+6$	4
10. Greenw., trans. c.	1880—91	$u+3$	$v=+9$	1
11. Madison	1883—90	$u+5$	$v=-18$	1
12. Lyons	1885—93	$u+6$	$v=-2$	2
13. Albrecht-Zwiers	1890—99	$u+10$	$v=-3$	6
14. Albrecht-Zwiers	1900—07	$u+19$	$v=+30$	8

The solution of these equations by the method of least squares, taking the weights p into consideration, gives:

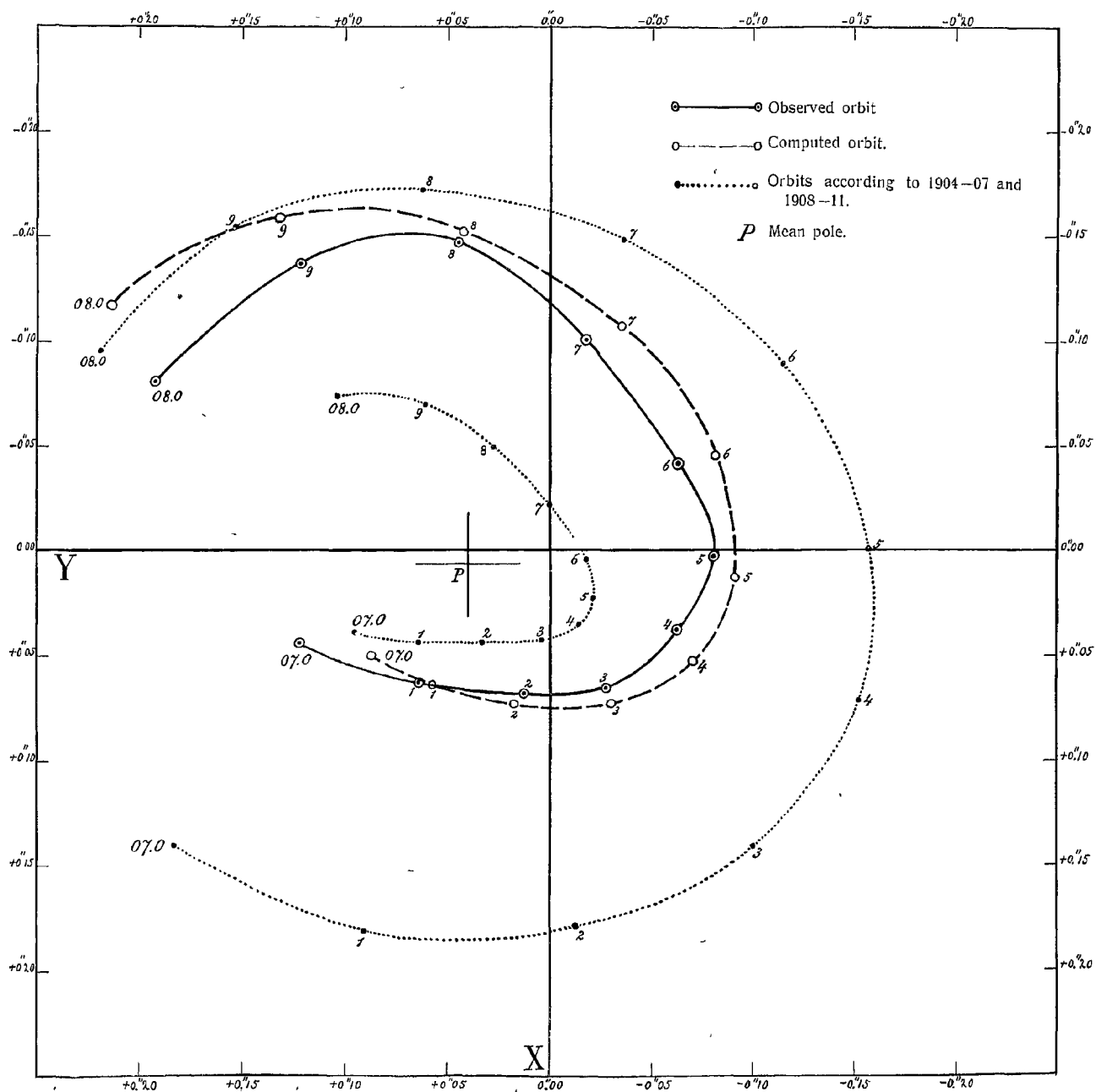
$$u = +13^{\text{d}}.42 \quad v = +0^{\text{d}}.097$$

so that the new elements become:

$$\text{Epoch} = \text{J. D. } 2408580 \quad P_s = 431^{\text{d}}.24.$$

With these elements I find the following residuals:

1. — 38 days	8. + 3 days
2. + 60 „	9. — 8 „
3. — 12 „	10. — 5 „
4. — 20 „	11. — 32 „
5. + 29 „	12. — 16 „
6. + 11 „	13. — 17 „
7. + 15 „	14. + 15 „



Although this computation of P_2 has only a preliminary character, still I think I am free to conclude, that the agreement between the new and the old observations is as satisfactory as may be expected from the delicate nature of this research. The period we have now found for the so-called 14-monthly motion, corresponding with a yearly increase of phase of $304^{\circ}.95$, even brings out a closer agreement between the phases for 1907.5 as deduced from the motion before 1907 and after 1908. With $P_2 = 434^{\circ}.1$ we had found a difference of $9^{\circ}.4$; reducing the phases found for 1905.5 and for 1909.5 with the above mentioned value of $\frac{d\psi_2}{dt}$ to 1907.5 we obtain:

from 1904—1907: ψ_2 for 1907.5 = $183^{\circ}.14$

from 1908—1911: ψ_2 for 1907.5 = $184^{\circ}.89$.

This result strengthens the conclusion arrived at on p. 216, that the perturbation of 1907 did not cause any appreciable change of phase.

When we try to find the cause of the decidedly rather sudden change in the 14-monthly motion, it seems natural to seek it in the influence of rapid displacements of mass caused for instance by volcanic eruptions or earthquakes. Prof. HELMHOLTZ has already developed their influence in his "*Höhere Geodäsie*", II Teil, S. 416—418, but the details of the observed phenomenon are not in accordance with his results. After his analysis a sudden displacement of mass must chiefly cause a change in the direction of the axis of greatest moment of inertia, the direction of the instantaneous axis of rotation remaining unchanged. The angular distance between the two axes is altered, and the axis of rotation continues regularly its motion in a circular cone around the new principal axis of inertia i.e. around the new mean pole. A phenomenon of this character would therefore alter the position of the mean pole and the mean value of the geographical latitudes, while the analysis of the observed facts did on the contrary indicate an unaltered position of the mean pole, and a spiral displacement of the pole of rotation.

So the cause of the phenomenon must be sought elsewhere, and there is another problem to be solved by the dynamical theory of the polar motion.

Leyden Observatory, June 1911.