Citation:

Botany. — "A study of temperature-coefficients and van 't Hoff's rule." By C. P. Cohen Stuart. (Communicated by Prof. F. A. F. C. Went).

(Communicated in the meeting of March 30, 1912).

§ 1. Introduction.
In connexion with a number of investigations into the influence of temperature on physiological processes, I have made a study of the available data. At the meeting of the Dutch Botanical Society in October 1911, I already communicated some of the results to which the study of the literature had led me, and although the further deductions are of a somewhat hypothetical character (for which reason I have started an investigation to clear up some doubtful points) I nevertheless feel it may be as well to publish already the main results, because in the discussions which took place they suggest some new points of view.

I may not omit to acknowledge here my indebtedness to Professors Went and Cohen for their advice and interest. 1)

§ 2. What is van 't Hoff's rule?
In the literature of physical and physiological chemistry there is found again and again the phrase "According to van 't Hoff for every 10° rise of temperature the reaction velocity becomes twice to thrice as great" ("R. G. T. rule"). It is the physiologists especially who like to make use of the determination of the temperature-coefficient in order to decide from its magnitude whether a given process is "chemical" or "physical", and who to that end invoke the above erroneously quoted rule. I shall endeavour to show that the formula of van 't Hoff itself makes clear that the usual interpretation of the "R. G. T. rule" with regard to the constancy of the coefficient rests on a misunderstanding.

By temperature-coefficients I mean the quotient (Q) of two reaction-velocities which are separated from each other by a constant interval of temperature, usually 10°. Proceeding on the hypothesis that this temperature-coefficient for every 10° increase equals 2—3 then it is evident that this function may be graphically represented by a curve ascending exponentially. "Bilden die Temperaturen eine arithmetische Reihe, so bilden die Geschwindigkeiten eine geometrische" 2). It is clear, that, when the temperature-coefficients of this function are

1) The dissertation of Miss van Amstel, "On the influence of temperature on physiological processes in yeast", just now appeared, in which she makes a detailed communication. To my regret too late for me to make use of it in this paper.
2) Van 't Hoff-Cohen, Studien z. chem. Dynamik 1896, p. 128.
again plotted against the temperatures as abcissae, the new function is represented by a line parallel to the abcissa, for the coefficient is assumed to be constant.

It has long been known and moreover van't Hoff already stated 1), that this coefficient is not constant, but that it decreases when the temperature is raised and vice versa. But if the coefficient decreases, then this signifies, that the reaction-velocity is diminished and this again indicates, that the curve of the reaction-velocity is no longer exponential.

Trautz and Volkmann 2) were the first to pay full attention to the lowering of the temperature-coefficients. In the saponification of a large number of esters with Ba(OH)₂ and NaOH 3), they obtained acceleration-curves (Pl. II, fig. 1) which agreed very well, and which all possessed the peculiarity of showing a maximum at about 20° and assuming a flat course at a high temperature of 40°.

The investigators then chiefly occupied themselves with the search for a reason for this maximum at 20° and succeeded in finding one, namely, an irregularity in the function \( \frac{d\eta}{dT} \) (in which \( \eta \) = viscosity) and a maximum in \( \frac{d^2\eta}{dT^2} \), both at 20°. I will not here discuss the formula they put forward; and only say that by the introduction of \( \sqrt{\frac{d\eta}{dT}} \) a maximum in the curve calculated was indeed obtained 4).

Here again appears the long suspected connection between reaction-velocity and internal friction, that has been regarded by many as a postulate for formulating in absolute measure the reaction-velocity of chemical processes 5). This however is not the point to which I wish to call attention here.

Our ultimate aim is to establish the shape of the complete curve of the coefficients in connection with the course of the complete curve of reaction-velocity and temperature. Although this aim cannot at present be attained, we may still begin by making some deductions from the formula which already presents some properties of the

2) Zschr. f. physik. Chem. LXIV (1905).
3) Executed by Volkmann, Inaug. Diss., Freiburg i. B., 1908.
desired function, namely, the formula suggested by Van 't Hoff \(^1\)) and simplified by Arrhenius\(^2\))

\[ \frac{d \ln k}{dT} = \frac{A}{T^2} \]  

\[ (1) \]

in which \( k \) = the reaction-velocity at a constant (abs) temperature \( T' \), on integration from \( T_1 \) to \( T_2 \) this becomes

\[ \ln \frac{k_2}{k_1} = A \cdot \frac{T_1 - T_2}{T_1 T_2} \]  

\[ (2) \]

or, if \( T_1 - T_2 = 10^2 \) and \( \frac{k_2}{k_1} = Q \) (coefficient)

\[ \ln Q = \frac{10A}{T_1 T_2} \]  

\[ (3) \]

In the formula \( A \) is equal to \( \frac{q}{2} \) of Van 't Hoff\(^3\)) or \( \frac{q}{2} \) of Arrhenius\(^4\)) and represents half the heat change (e.g. heat of dissociation). This quantity is a function of temperature, but since \( q \) to a much smaller extent than the coefficient \( Q \) and since I only aim at a first approximation, it will be supposed constant and 5500 cal. will be substituted, which number is about the average of the values obtained by Volkmann.

The \( Q \)-curve of course begins with \( T_1 = 10^8 \) absolute temperature. Thus

\[ \log Q_{10} = \frac{10A}{2.3 \times 10 \times 10} = \infty \] \( Q_{10} = \infty \)

\[ \log Q_{19} = 119 \]

\[ \log Q_{29} = 40 \]

\[ \log Q_{39} = 12 \]

\[ \log Q_{49} = 1.67 \]

\[ \log Q_{59} = 0.628 \] \( Q_{59} = 4.25 \)

\[ \log Q_{69} = 0.309 \] \( Q_{69} = 2.04 \)

\[ \log Q_{79} = 0.222 \] \( Q_{79} = 1.57 \)

\[ \log Q_{89} = 0.047 \] \( Q_{89} = 1.13 \)

\[ \log Q_{99} = \frac{10A}{\infty} = 0. \] \( Q_{99} = 1.00 \)

The coefficient \( Q_{100} \) for \( 78^\circ \) approaches very closely the value

\[ \frac{A}{88^\circ} \]

\( \)\(^2\) Études de dynam. chim. 1884, p. 115. Studien, p. 127, 152.

\( \)\(^3\) Zschr. f. physik. Chem. IV (1889), p. 234.


\( \)\(^5\) Immunochemie 1907.

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6.2 found by Plotnikow 1), but this is quite intelligible when it is remembered that for this reaction and this temperature \( A \) equals about 6000. In the same way at 640°—719° abs. temp. Kooy’s 2) coefficient which is 1.17 at 700°, is connected with the value of \( A = 7790 \) at that temperature. Further it is clear that the \( Q \)-curve must take an asymptotic course with respect to the ordinate as well as to the abscissa.

The above figures seem to me to render obvious the meaning of the critical passage 3): “Bei der grössten Zahl der bis jetzt in dieser Richtung untersuchten Falle, welche sich auf das Temperaturintervall 0°—184° beziehen, ist nun sehr auffallend das Geschwindigkeitsverhältniss für 10 Grad Temperaturerhöhung etwa 2 bis 3 bei gewöhnlicher Temperatur, m. a. W.: eine Temperaturerhöhung um 10 Grad verdoppelt, resp. verdreifacht die Reaktionsgeschwindigkeit”. Emphasis must be placed, not on “10 Grad Temperaturerhöhung” but on “etwa 2 bis 3 bei gewöhnlicher Temperatur”. It is indeed very curious that in curves of the most varied processes in which \( A \) can have widely different values, and therefore in the most diverse curves by far the larger portion of lines pass through the rectangle which is formed by the abscissae 0° and 50° C. and the ordinates (coefficients) 2 and 3. Whether the lines within the rectangle descend quickly or slowly, whether they run through the whole of its length or only touch one point of it, does not matter! It is but a general principle, a rule, not a law.

Now it would be of great importance to obtain an exact mathematical definition of the \( Q \)-curve.

If we for the moment imagine the simplest case, namely, that the \( k \) line (reaction-velocity) is a straight line — which opposition, it must be here stated emphatically, is purely mathematical and completely independent of any physical or physiological-chemical hypothesis — then the \( Q \)-curve is also a simple function of temperature.

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1) Diss. Leipzig 1905. — Zschr. f. physik. Chem. LII (1905), p. 630. — If the formula of Arrhenius is used instead of that of Berthelot, then \( Q = 5.77 \) is obtained.
3) VAN 't Hoff—Cohen, Studien, p. 128.
For, in general, if
\[ Q = \frac{k_1}{k_2} = \frac{f(x + h)}{f(x)} \] (see fig. 1 in the text)
then
\[ Q = \frac{f(x + h)}{f(x)} = 1 + \frac{f(x + h) - f(x)}{f(x)} \]
\[ = 1 + \frac{h}{f(x)} \cdot f'(x) \ldots \ldots \ldots \ldots \ldots \ldots (4) \]

If one bears in mind that \( h \) is always supposed = 10° and that \( f(v) \) is the smallest of the ordinates under consideration and may be represented by \( k_{T-10} \), the latter general equation changes to
\[ Q = 1 + \frac{10}{k_{T-10}} \cdot f'(x) \ldots \ldots \ldots \ldots \ldots (5) \]

It is now clear what formula (5) tells us with respect to the shape of the \( Q \) curve, when \( k \) is a linear function. For since \( f'(x) = ky = m \) constant, hence
\[ Q = 1 + \frac{10m}{k_{T-10}} \]
therefore \( (Q-1) \cdot k_{T-10} = 10m = \text{constant} \).

Now since the function is according to hypothesis linear and since, as is actually the case, it passes through the origin (i.e. at 0° abs. temp. \( k = 0 \)), \( k_{T-10} \) is proportionate to the absolute temperature, and we may also write:
\[ (Q-1) \cdot T = \text{constant} \ldots \ldots \ldots \ldots (6) \]
and from this again it at once follows that the \( Q \) curve is a rectangular hyperbola, of which the one limb asymptotically approaches the y-axis (coefficient) and the other at distance 1 runs parallel to the abscissa (abs. temp.). If both sides of equation (5) are multiplied by \( k_{T-10} \), it is at once seen that this hyperbolic shape is nothing peculiar and only signifies that in a straight line the difference between two successive \( k \)-values is constant.

The application of the result will appear from the following section.

2) Instead of the elementary deduction given above, use can be made of the Taylor's theorem (Dr. M. J. VAN UVER very kindly pointed it out to me) or of the method in Cauwolsohn Lehrb. d. Physik III (1905), p. 12.
I have already stated that the $Q$ function for an exponential $k$ curve is a horizontal straight line.

If we attempt to establish the $Q$ function for an arbitrary reaction, the formula of van 't Hoff-Arrhenius does not take us further.

What is the reason for this? The sole and only reason is our ignorance concerning the quantity $A$. For not until this quantity becomes known as $f(T)$, is a conclusive opinion possible with respect to the geometrical meaning of the formula in question. Therefore it is above all necessary that the formula of van 't Hoff-Arrhenius should not be used to interpolate velocity-constants or coefficients (for these should be obtained experimentally) but in order to discover the dependence of $A$ on the temperature. The determinations directed to this end must be made at regular intervals (preferably from $1^\circ$ to $5^\circ$) because at the present time determinations at arbitrary temperatures and interpolations with the formula of Arrhenius or Barthelot are of no value.

Now there are two standpoints from which this problem can be viewed. In the first place the formula can be considered as purely theoretical (founded on thermodynamics) and then $A$ can be identified with half the heat of dissociation; if it turns out that the reaction-velocity is governed by still more factors, then these must be introduced into the formula. I have here in mind, for example, the viscosity. Since the second law of thermodynamics only pronounces the possibility of energy transformations and not on the way in which energy transformation comes about, and therefore not on the loss of energy through friction, it is therefore-intelligible that there should be this omission in a thermodynamical formula. If, on the other hand, we consider van 't Hoff's formula as an interpolation formula, viz. as an expression which agrees as closely as possible with the observations, in which van 't Hoff's formula, rather a note of interrogation than a "non-variable", then it must be our aim to break up this quantity into several others, among which for instance the dissociation and the friction might occur.

It seems to me that there is much to be said in practice for the latter view. In that case, however, there is absolutely no sense in giving a single $n$-value for a reaction (at an entirely arbitrary temperature) as has been frequently done of late on the example of Arrhenius; indeed, it is a very rash generalization. This is at once obvious if we construct for the experiments of Volkman for instance, the corresponding $A$-curves. It will be also clear from the next section.

As a matter of fact one may therefore say that any reaction (and
not even necessarily a chemical process!) may be represented by Van 't Hoff's formula: it is only necessary to give suitable values to the interpolation constant A. But in that case there is little value in pointing out this "applicability", as for instance in serum reactions (experiments of Madsen and others, and of Arrhenius). 1).

The results of our considerations may be summarized thus:

I. The curve representing for chemical reactions the relationship between reaction-velocity and temperature is not an exponential one; the relationship between temperature-coefficients and temperature is hence not a horizontal line.

II. If the function of reaction-velocity is represented by a straight line the curve of coefficients is a rectangular hyperbola.

III. Nothing definite can be said with regard to the true form of the desired functions before the quantity A has been completely defined.

§ 3. The temperature-coefficients of vital processes.

After all that has been said above with regard to the fact that temperature-coefficients of chemical processes generally diminish with rise of temperature, it can hardly create surprise when it is found that the processes in the living organism are equally subject to this rule. Kantz 2) was the first to notice this and evidently regarded it as an essential relationship, but it is obvious from discussions occurring in the literature after 1905, that his remarks have remained quite unnoticed, as (just to quote two recent instances) in the papers of Miss van Amstel and Professor van Iterson 3), and the research of Mlle Pilon 4). The reasoning is as follows: "the rule of van 't Hoff is not applicable to living processes, because the coefficient diminishes and is too high at low temperatures and too low at high ones". It is hardly necessary to say that this conception should have been already entirely abandoned. But, e.g., against Rutgers's 5), argument (namely, that Trautz and Volkman had also found diminishing coefficients — with which he did indeed hit the nail on the head) van Amstel and van Iterson defend themselves as follows 6):

Admitting that for physiological processes, also at harmless temper-

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1) Immunochemie 1907.
3) These Proceedings 1910, p. 227 and 508.
6) L. c. p. 605.
atures, the rule of van 't Hoff is of no consequence, what then remains of Blackman's theory?" Here all depends on what is understood by van 't Hoff's rule, which these processes are obliged to obey!

Recently a paper by Snyder ¹) appeared "On the meaning of variation in the magnitude of temperature-coefficients of physiological processes". In this the principle of diminution is plainly stated and supported by many numerical data. It does, however, seem to one that all these figures, taken from the most complicated processes of animal physiology, do not form any trustworthy foundation on which to erect even a small structure of theoretical deductions. What, however, deserves great credit is that, in collaboration with Todo ²), he at once made an attempt (in connection with Volkmanh's hypothesis) to measure the change of viscosity of the body-fluids (dog's peptone plasma) under the influence of temperature. He finds a diminution of η with rise of temperature parallel to \( \frac{d\eta}{dT} \) for water.

As usual, the observations are here also restricted "within physiological limits" while it is exactly the study of the phenomena of death that can furnish most important data with respect to the condition in which the unharmed protoplasm finds itself ³). The extension of the experiments to higher and lower temperatures, which is a comparatively small difficulty in the course of an investigation, should make these experiments much more productive.

In view of the slight results which the above research has yielded (Blackman's theory with all its appurtenances is clearly still unknown in animal physiology ⁴)) I may leave this and proceed to a survey of the botanical publications from which data can be gathered for our object. I add below a list of the literature I have used.

In my calculations and constructions I have proceeded from the hypothesis that the figures given for the reaction velocities are free from error. I am aware that I am thus guilty of a heresy. My excuse is in the first place that a critical selection is impossible because in almost all cases the sources of error are not given, both as regards number and magnitude. Moreover I wished to see what an exact calculation would yield from data, which in general could indeed

²) Ibid. p. 161.
⁴) Höhne in his comprehensive work (Physik Chem. d. Zelle u. d. Gewebe, 3. Aufl. 1911) still devotes only a single chapter to physiological-chemical kinetics, and in this only a few words to the question of optima.
vary in all directions. I believe I may say that the result which is represented in the subjoined Q-curves, justifies to some extent my procedure.

And this for three reasons especially: firstly because the course of the curves is very regular — secondly because in those investigations, especially those of Kuyper, in which account is taken of the duration of warming (for which in animal experiments all data are wanting) the curves for one and the same duration of warming take an accurately parallel course, — and finally there is the more remarkable quality, that the course of the coefficient curves are of one general type. This type can perhaps be best-characterized by comparing it to the well-known inflected gas-isothermals.

Although the following considerations apply to a greater or less extent to all constructed curves, I shall always pay special attention to Kuyper's curves (Plate I) since his figures (especially those for Pisum) are presumably the most accurate which we possess with respect to temperature-coefficients in physiology.

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Fig. 2. A-curves for the respiration of Pisum sativum (Kuyper).

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- 1 hours' — 2 hours' —- 3 hours' warming.
The following peculiarities may now be noticed:

1. Many curves have a maximum at $15^\circ$.

2. In successive hours the curves of the coefficients show a tendency to pass from a gradual to a steep descent.

3. All curves show a tendency up to 35 or $40^\circ$ to approach the coefficient 1 asymptotically. Afterwards the direction changes to one of rapid descent.

4. The curves of Nägeli, Velten, van Rijsselberge and Rutgers descend much more steeply than the others.

5. All curves descend much more rapidly than those of Volkman).

Further I have constructed from the five $Q$-curves for Pisum the corresponding $A$-curves with the aid of Arrhenius' formula, for from every value of $Q$ a value of $A$ can be calculated; from the latter curves it is obvious that $A$ is by no means constant and therefore here also a previous remark on the rash generalization of $A$-values applies. In fig. 2 of the text I have indicated the place where according to Arrhenius "the" $A$-value lies, namely at $u = 2A = 14800$ (calculated according to Clausen). It will be seen that there is no question of a real "mean", not even if we limit ourselves to "innocuous" temperatures. It is also seen that for supra-optimal temperatures $A$ soon acquires very large negative values. This results from the formula $\ln Q = \frac{10A}{T_1 T_2}$, since, when $Q = 1$, $A = 0$, and when $Q = 0$, $A = -\infty$.

The latter result of course means that the respiration has stopped and that death has occurred. The coefficient $Q$ itself never becomes negative.

We now proceed to a discussion of the property mentioned above sub 1. Is the maximum which appears in many $Q$-curves a real maximum? It occurs so frequently that we must consider it appears according to a rule and we are at once reminded of the maximum which Trautz and Volkman always found at $20^\circ$. It should further be noted that these authors leave open the possibility of the maximum being somewhat lower, since they made determinations at intervals

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1) The rate of descent could of course be again expressed by $\frac{Q^2}{Q^2}$. In practice this, however, introduces large errors (compare v. Halban, l.c.).

2) Immunochemie 1907, p. 88.

C. P. COHEN STUART. "A study of temperature-coefficients and van 't Hoff's rule". Plate I.

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Plate II

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Fig. 1

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Fig. 2

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Fig. 3

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Fig. 4

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5' — 10' — 15' — 20' warming.

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of $10^0$). It would be worth while to investigate this maximum also for physiological processes, in order to demonstrate a possible connexion with \( \frac{dy}{dT} \). Too much importance may, however, not yet be attached to this maximum, since at $0^0$ the experimental errors may be considerable, and these errors influence the quotient $\frac{10^0}{0^0}$.

As regards the second point, i.e. the decrease of the coefficients in successive hours, we may note that the line of 1 hour’s warming often shows (irregular?) oscillations, whereas the other lines generally succeed one another more or less regularly. If we now bear in mind, that VOLKMANN’s lines were very slightly inclined, we obtain the impression that the line for 1 hour’s warming approximates to those for reactions in vitro \(^1\) and that the subsequent lines deviate more and more from this. This is exactly what BLACKMAN has asserted for reaction velocities and with him we may assume that the correspondence would be complete if observations could be made after an infinitely short time of preliminary warming.

It should be noted in passing that the construction of the $Q$-curves affords a means of demonstrating almost imperceptible irregularities of the $k$-curve. Since the various $Q$-curves are moreover directly comparable (assuming that the units are in constant equal proportions), in contradistinction to the $k$-curves, which change their slope in accordance with a change in the choice of the values of the units, the construction of the $Q$-curves is much more correct and important than that of the $k$-curves.

The third point deals with the asymptotic course when $Q = 1$. In the preceding section we said that in reactions which are linear functions of temperature the coefficient at 1 runs asymptotically, and in agreement with this is the behaviour of the physiological coefficients at “harmless temperatures”. The upper limit of these temperatures varies between $25^0$ and $40^0$ according to the process and the object. Starting from this temperature one could draw further a “theoretical” asymptotic line. (The same is seen in the $A$-curves.) The explanation of this phenomenon is of course simply that at infra-optimal temperatures the $k$-curves frequently differ very little.


\(^2\) Later physico-chemical investigations will be required to show whether my generalisation from VOLKMANN’s experiments is justified.
from straight lines. In that case the Q-curve must necessarily resemble a hyperbola.

There are finally the fourth and fifth points, concerning the more or less rapid diminution. The curve of Nägeli shows that the "abnormally" high coefficient of Rutgers for $10^0_0$°, namely 6.75, no longer stands by itself but is even surpassed by the enormously high figure 9.33 for Nitella (which figure, it should be noted, was obtained by extrapolation from $10^0_0$° and is therefore probably too low). Rutgers 1) attributed the high value of his coefficient to the cessation of growth at $0^0_0$°. We are here at once reminded of the cessation of protoplasmic streaming and the very slight diosmosis at $0^0_0$°. What is the cause of this "Kältestarrre", which one is inclined to neglect as "freezing"? What is more characteristic in the cooling of an emulsion-colloid than the enormous increase in the increment $f'(x)$, therefore also in $Q$, of the viscosity? 2)

I am indeed convinced that by the investigation of the influence of temperature on the viscosity of emulsoids, the mystery which still surrounds heat and cold-rigor, will be solved, at least in part. It is impossible to discuss this further, but I only desire to record my scepticism as to the "chemicity" of a process possessing a high temperature coefficient. The real distinction between a Q-curve under purely physical influence and a purely chemico-physiological one would perhaps become perfectly clear at once if one could compare the two kinds of curves. It is, however, very remarkable, that hitherto not a single case has come to light in which the coefficients throughout have "physical" values 3) not even there, where a purely physical influence might have been expected (e.g. protoplasmic streaming, diosmosis). Now is it legitimate to refer this result without further to a (chemical) metabolic influence? That is indeed somewhat too simple. I entirely concur with Sutherland 4) in his severe judgment: "In order that the chemical theory of nerve-impulse may prove helpful it must show in some reasonable way how the velocity of propagation of a chemical reaction along a nerve can be proportional

2) W. Ostwald. Kolloidchemie 1911, p. 204, 221.
to the metaphorically so-called velocity of reaction at any point in
the nerve."

In my opinion it is legitimate to "deduce" the chemical nature
of a process from the coefficient $2$ to $3$ only in such cases... of
which one can be certain in advance (respiration, assimilation).

Finally I wish to indicate very briefly the conclusions to be
drawn from the above with regard to BLACKMAN’s theory $^1$), which
interprets the temperature optimum in physiological processes as the
resultant of a favourable and a harmful influence. This theory is
entirely based on the assumption (found to be erroneous) that the
reaction velocity ($k$-)curve according to VAN ’t HOFF is an exponen-
tial function; to this curve ("the line of zero time") all the observed
reaction velocities are extrapolated, and if these extrapolated curves
end in the points "required by theory" the matter is sound, according
to BLACKMAN, and VAN ’t HOFF’s rule also applies to harmful tem-
peratures.

BLACKMAN’s theory has been controverted by VAN AMSTEEL and VAN
ITERSSON (loc. cit.). These authors start with a totally different ex-
trapolation method and with its aid obtain results entirely opposed to
BLACKMAN’s This question cannot here be discussed further, and the
correctness of VAN AMSTEEL and ITERSSON’s extrapolation method is
therefore also left out of consideration.

A different question, however, is, whether BLACKMAN’s method has
any use. Of course it has not. Since BLACKMAN extrapolates his heating
curves to a line which is not "chemical", but quite imaginary and
arbitrary $^2$), the end-points of these curves are as inaccurate as the
conclusions, which are drawn from the supposed agreement between
fact and theory, and the proof of his theory, based on these con-
clusions, is worthless. Is therefore his theory also worthless?

No, at least I do not think so; only the validity of BLACKMAN’s
theory cannot at present be proved. If the true curve were known
(but this awaits a chemical definition of $A$) then, and then only,
something might be attained by extrapolating. Meanwhile one can
only adduce grounds of probability. Some of these have already
been mentioned. A further argument results from the following con-
sideration. If the last portion of the $k$-curve, before the concave
bend, is produced rectilinearly and if we extrapolate to this, much
smoother curves are obtained than with BLACKMAN’s exponential line.
This is for instance well shown by KUYPER’s curves for Triticum;
with Pisum a line is required intermediate between a rectilinear

$^2$) KANITZ, Biol. Cbl. XXVII (1907), p. 20.
and an exponential lengthening. Hence it follows that the sharp an-
tagomism between the lines of zero time of Blackman and of van
Amstel and van Horn is considerably lessened. It is clear, however,
that this gives scope for a most arbitrary procedure. There is no
positive proof.

II. It should be expressly pointed out that the above results in no
way diminish Blackman’s merit — a merit based on his opposition
to the general misuse of the terms “Reiz” and “Anlösung” as
meaningless principles), thought to explain everyting, and more
especially on the manner in which he traced a clear and logical con-
nection between the theory of reaction velocity in chemical and in
physiological systems, the theory of limiting factors and the theory
of the origin of the optimum. —

The above considerations may be summarized as follows.

I. The falling of temperature-coefficients with rise of temperature
is also the rule in physiological processes.

II. Blackman’s theory, which is thus deprived of its “chemical
curve” and its extrapolation method, is at present not susceptible
of proof.

III. Probably the study of temperature-coefficients will furnish
a better insight into the mode of action of temperature than that of
reaction velocity.

IV. Both the velocity curves and the coefficients point to a close
correspondence between physiological and chemical processes.

V. This correspondence is, however, rendered less evident by the
influence of viscosity, which plays presumably a very important
part in the heterogeneous, colloidal protoplasm.

VI. The study of the influence of temperature on the viscosity
of colloidal systems will probably explain a large proportion of the
apparent deviations of vital processes from the course of chemical
reactions in homogeneous systems.

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**EXPLANATION OF THE FIGURES**

**PLATE I.**

**Fig. 1.** Q-curves for the respiration of Pisum sativum (1—5 hours’


**Fig. 2.** The same for Triticum vulgare. According to Kuypers, Proc. 1909.

**Fig. 3.** The same for Lupinus luteus. According to Kuypers, Proc. p 225

Rec. p. 190.

Mathematics. — "On a congruence of the second order and the first class formed by conics." By Prof. Jan de Vries.

The theory of the linear congruences of conics (congruences of the first order) has been completely developed by Montesano 1) (1892—95). For the determination of the relations between the characteristic numbers this author among others made use of two auxiliary surfaces: the locus $A$ of the conics meeting a given line and the locus $B$ of the conics the planes of which pass through a given point. An extension of this method to congruences of higher order remains infertile by the fact that in these cases a point common to two conics is not necessarily a singular point. Until now general considerations on congruences of higher order have not been published.

In these pages we consider a special congruence of order two and class one, showing a family-likeness with the congruence $(2,1)$ described in 1908 by L. Godaux 2).

1. The congruence $\Gamma$ under consideration consists of the conics $\gamma^2$ passing through a given point $O$, having three points in common with a given twisted cubic $\nu^2$ and meeting a given line $r$.

The locus of the conics passing through $O$ and through any point $P$ and meeting $\nu^2$ thrice is a quadratic surface $\Sigma^2$. By the points common to $r$ and $\Sigma^2$ two conics $\gamma^2$ of $\Gamma$ are individualized; so $P$ lies on two $\gamma^2$ and $\Gamma$ is of the second order.


2) Sur une congruence $(2,1)$ de coniques. (Mem. Soc. des Sciences du Hainaut, 6me série, t. X).

In 1904 we dealt with a special congruence $(2,2)$. (These Proceedings Vol. VII, p. 311).