Physics. *"Energy and mass."* II[1]). By J. D. VAN DER WAALS Jr. (Communicated by Prof. J. D. VAN DER WAALS.)

HERGLOTZ[2]) has assumed that we can deduce the equations of motion of a system after the method of PLANCK with the aid of the principle of least action from a kinetic potential, and he has investigated what conditions are necessary and sufficient in order that that potential has the property to depend only on the "rest-deformations" after a "LORENTZ-transformation to rest". With rest-deformation of a moving element of volume we mean the deformation which it shows after being transformed to rest, compared with the shape which it shows when it rests and is not subjected to any stress. For this he finds the following conditions: 1st. the tensor of the "absolute stress" must be symmetrical, i. e. $p_{xy} = p_{yx}$, etc., 2nd. the current of energy must be equal to $c^2 \times$ the momentum, 3rd. a set of equations ((77) p. 508 of his treatise), which in the notation used by me may be represented as follows:

$$\left.\begin{array}{l} \mathfrak{S}_x = \mathfrak{v}_x F + \mathfrak{v}_x p_{xx} + \mathfrak{v}_y p_{xy} + \mathfrak{v}_z p_{xz} \\ \mathfrak{S}_y = \mathfrak{v}_y F + \mathfrak{v}_x p_{xy} + \mathfrak{v}_y p_{yy} + \mathfrak{v}_z p_{yz} \\ \mathfrak{S}_z = \mathfrak{v}_z F + \mathfrak{v}_x p_{xz} + \mathfrak{v}_y p_{yz} + \mathfrak{v}_z p_{zz} \\ W = F + \frac{1}{c^2} \mathfrak{v}_x \mathfrak{S}_x + \frac{1}{c^2} \mathfrak{v}_y \mathfrak{S}_y + \frac{1}{c^2} \mathfrak{v}_z \mathfrak{S}_z \end{array}\right\} \quad . \quad . \quad . \quad (1)$$

The fourth equation may be considered as the definition of the quantity F.

When these equations are satisfied, the hypothesis of relativity is satisfied. For when we use different coordinate systems moving with different velocities, the equations of motion are always derived in the same way from the kinetic potential, and this potential depends in the same way on the rest-deformations and on the velocities of the elements of mass relative to the coordinate systems. From this it follows that as well the equations of motion as the conditions found by HERGLOTZ are covariant for a LORENTZ-transformation, and that

[1]) In Sept. 1911, when I wrote "Energy and Mass" I, it was not known to me, that investigations of the same kind and with partly the same results had already been published by

D. F. COMSTOCK Phil. Mag. 15, p. 1. Anno 1908.

G. N. LEWIS. Phil. Mag. 16, p. 705. Anno 1908.

G. N. LEWIS and R. C. TOLMAN. Proc. Amer. Akad. of Arts and Sc. 44, p. 711, Anno 1909.

[2]) G. HERGLOTZ. Ann. d. Phys. 36, p. 493. Anno 1911.

they are therefore satisfied in the same way for the different coordinate systems, i.e. that for moving systems x', y', z' depend on t' according to the same laws, as for a stationary system x, y, z depend on t.

So we cannot deduce from the way in which different processes take place, whether the coordinate system which we are using moves or is stationary. In particular, — and this consequence, which is not separately mentioned by HERGLOTZ seems to me to be of enough importance to draw attention to it, — it is possible to conclude that if these conditions are satisfied the LORENTZ-contraction must take place. For we saw, that if these conditions are satisfied the rest-tensions (i. e. the quantities p, which we find in a volume-element after we have transformed it to rest) depend only on the rest-deformations. If therefore for a moving system the relative (elastic) tensions are zero, then the rest-tensions are zero and also the rest deformations; and the volume element shows in a coordinate system in which it rests, its normal shape. In a coordinate system relative to which it moves, it shows then a shape which is shortened in the direction of motion in the well known way.

The equations (1), however, deduced by HERGLOTZ from the postulate of relativity in the way indicated above, are identical with the equations

$$\mathfrak{S}_x (1 + \beta^2) = (W + p_{xx})\, \mathfrak{v}$$
$$\mathfrak{S}_y = p_{xx}\, \mathfrak{v} \text{ [1]}$$

as we see by choosing in (1) the direction of motion as direction of X, i.e. by putting $\mathfrak{v}_y = \mathfrak{v}_z = 0$.

Now these equations had been derived by me l.c. without making use of the hypothesis of relativity, but only basing my deductions on the supposition $m = \frac{1}{c^2} W$. I therefore conclude that we may derive the whole theory of relativity from classical mechanics, when we change the principles of mechanics only in this one point, that we assume the mass of the bodies to vary with their energy according to this formula; and that therefore by working out the idea advanced by POINCARÉ in 1900, that the energy possesses mass, we could have arrived at a theory from which the negative result of the experiments of MICHELSON, etc. might have been predicted.

SOMMERFELD [2]) declares the theory of relativity not to be any more

1) "Energy and mass" I p. 252. The symbol $\mathfrak{S}_{xy}$ occurring there is evidently a printer's error for $\mathfrak{S}_y$.

2) A. SOMMERFELD. Phys. Zeitschr. 12, p. 1057. Anno 1911.

actual. If he means to say that in this theory only details should have been left for further investigation because the principal ideas have been sufficiently established, this assertion seems to me to be inaccurate. In my opinion the present state of the problem could be more aptly compared with the state of planetar mechanics at the time, when some laws of the planetar motion were known, — namely the laws of KEPPLER — but when the causal explanation of these laws with the aid of the principia of mechanics of NEWTON had not yet been furnished. Thus in the theory of relativity we have known up till now some laws, — namely the laws of LORENTZ for the contraction in the direction of motion and for the variation of the mass with the velocity — but an explanation of this variation of mass and shape was not known. I think I have shown here that the principia of NEWTON together with the supposition $m = \frac{1}{c^2} W$ are sufficient to give this explanation.

Yet, and I will state this most emphatically, this is no more than a first step. Many questions are still waiting for a solution. In what way for instance must the kinetic energy be explained, or in other words why does the mass of a body vary when its motion is accelerated; why is an acceleration accompanied with a flow of mass towards the body?

A second question is the following one: how must the equation

$$\frac{1}{c^2}\frac{\partial \mathfrak{S}_x}{\partial t} = \frac{\partial p_{xx}}{\partial x} + \frac{\partial p_{xy}}{\partial y} + \frac{\partial p_{xz}}{\partial z}$$

be interpreted? It has exactly the form of an equation of continuity. A great (perhaps a too great) importance has of late been attached to such like analogies in the ways in which some quantities occur in some equations. But this equation suggests the question whether it is really an equation of continuity and whether it perhaps signifies that the momentum moves continuously through space.

Finally, the question has often been raised whether the theory of electricity must be deduced from mechanics or vice versa. Are we not to consider also a third possibility, namely that they are both to be derived from a still more fundamental law which determines the motion of the energy through space? So we should get a theory which might rightly be called energetics. Moreover the hidden masses which formerly played a part in mechanics, would have to be introduced again, but we should have advanced so much that we now know, that these hidden masses are nothing but the energy residing in the medium.