Physics. — *"On the conception of the current of energy."* By M. LAUE. (Communicated by Prof. J. D. VAN DER WAALS).

The law of the inertia of the energy, which with perfect generality brings the momentum per unit of volume $\mathfrak{g}$ in connection with the energy current $\mathfrak{S}$ according to the formula

$$\mathfrak{g} = \frac{\mathfrak{S}}{c^2}$$

has again drawn the attention to the conception of the current of energy, which at the time was discussed with vivid interest in relation to POYNTING's theorem. The author has given a rule for the transformation of the density of the energy current $\mathfrak{S}$. This rule states that in every department of physics a tensor of stress p exists, which with the three components of the vector $\frac{i}{c}\mathfrak{S}$ and the density of the energy W taken negatively forms the components of a symmetrical "world tensor" T; i. e. we shall have

$$T_{jk} = p_{jk} \qquad \text{if } j, k = x, y, z$$

$$T_{jl} = \frac{i}{c}\mathfrak{S}_j \qquad \text{if } j = x, y, z \text{ and } (l = ict).$$

$$T_{ll} = -W$$

In Electrodynamics the tensor p represents the MAXWELL stresses, in mechanics it is closely connected with the elastic stresses.

Now the conception of the current of energy has been formed in analogy to the conception of the current of a fluid. If we denote the density of the fluid by ϱ, its velocity by $\mathfrak{q}$, then the density of the current is of course $\varrho\,\mathfrak{q}$. In a recent paper [1]) VAN DER WAALS JR. transfers this relation to the energy current, and so he arrives at the conception of velocity of the motion of the energy, which is connected with the energy current $\mathfrak{S}$ and the energy density W according to the relation

$$\mathfrak{w} = \frac{\mathfrak{S}}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This velocity appears to him even to be the more lucid conception, from which the conception of the energy current must be deduced. And in the final remark of his paper [2]) he expresses a doubt whether the above quoted transformation formula for the density of the energy

[1]) VAN DER WAALS JR. Proc. Amsterdam. 1911. 239.

[2]) VAN DER WAALS JR. p 253 last paragraph. The note on this page is undoubtedly the consequence of an oversight, for in formula XXVIII I have explicitly equated to zero the divergence of the sum of all the world tensors as VAN DER WAALS wishes.

current follows from the transformation formula for the density of the energy W and for its velocity $\mathfrak{w}$. He assumes there, if I understand him rightly, that the addition theorem of EINSTEIN applies to $\mathfrak{w}$ as well as to the velocity of a material point.

This, however, is not the case. For if we start from the transformation for $\mathfrak{S}$ and W, we find quite a different law for the transformation formula for $\mathfrak{w}$. It is the question if an objection to that transformation can be derived from this fact.

To me this seems not to be the case. The claim that the addition theorem should apply presupposes that for energy as for matter we can distinguish individually the particles of which it consists. Only on this supposition can the paths of a particle relative to two differently moving coordinate systems be possibly compared with one another, which then leads to the addition theorem of EINSTEIN. This assumption, however, does certainly not hold, for the transformation formula for W, i. e. the equation

$$W = \frac{W' + \beta^2 p'xx + 2\frac{v}{c^2}\mathfrak{S}'_x}{1 - \beta^2},$$

shows, that energy can also then be present in the accentuated system, when in the unaccentuated system no energy of the same kind is to be found.

It is true that in the electromagnetic field in vacuo this case cannot occur. But it can occur for the elastic energy of a body subjected to a tension which is equal in all directions.

If the body rests relatively to the accentuated system, then we have

$$\mathfrak{S}' = 0, \qquad p'_{xx} < 0, \qquad W' > 0$$

and if the body is only little compressible:

$$|p'_{xx}| >> W'.$$

We shall then have $W = 0$ if the relative velocity of translation of the two systems reaches the not very large value

$$v = c\sqrt{-\frac{p'_{xx}}{W'}}$$

If v increases to a still higher value, W will even become negative. In such a case it is certainly impossible to compare the motion of a particle of energy when evaluated with the aid of the two systems.

Perhaps the objection may be raised against this consideration that in the last equation the tensor transformation has been used, whereas its applicability is just to be proved. Therefore I will adduce an instance which shows independently of every special theory, that the velocity of the energy cannot be transformed in the same way as the velocity of a material point. We consider three coordinate systems, K°, K^{+}, K^{-} moving uniformly relative to one another; the latter two will have the velocity $\pm \mathfrak{v}$ relative to K°. A body subjected to a tension (negative pressure) equal in all directions is in rest relative to K°. In the system K^{+} it has the velocity $-\mathfrak{v}$, in K^{-} the velocity $+\mathfrak{v}$. In the same way the elastic energy which is imparted to the body by the tension is in rest relative to K°, but flows in the other systems.

This flow of energy is compounded of the convection current of the energy carried along by the matter and the conduction current occasioned by the tensions. Only the first component agrees in direction with the velocity of the body, the second has on the contrary the opposite direction. If now, as above, we imagine the body to be only little compressible, then the density of the energy W° in the system K° is small compared with p. In this case the conduction current will far exceed the convection current, the velocity of the energy in the system K^{+} will therefore have the direction $+\mathfrak{v}$, in the system K^{-} the direction $-\mathfrak{v}$; this direction is therefore exactly opposite to that of the velocity of a point resting relatively to K°. Now it is true that VAN DER WAALS Jr. tries to evade these difficulties, which he himself, no doubt, has also noticed, by splitting up the energy current for one and the same kind of energy into some components differing in direction and value. It seems to me still doubtful for the present whether this is the way to reach the desired end.

Is the conception of a velocity of the energy, which of course can always be defined and calculated by means of equation (1), after all efficient? In some cases it is doubtless so. O. REYNOLDS [1]) e. g. has calculated the group-velocity for water waves, and the present writer [2]) and in a still more general manner M. ABRAHAM [3]) have done so for light waves according to the electron theory. In both cases we can imagine a closed surface moving with the velocity $\mathfrak{w}$ through which passes no energy. As we can disregard the absorption, this surface always includes the same quantum of mechanical or electromagnetical energy. It has, however, always only its signification for *one* coordi-

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Put in the equation 102 of my book "das Relativitätsprincip" (Braunschweig 1911) $\mathfrak{S} = \mathfrak{w} W$.

[1]) O. REYNOLDS: Nature 6 p. 343, 1877; H. LAMB: Hydrodynamik, p. 446. Leipzig u. Berlin 1907.

[2]) M. LAUE: Ann. d. Phys. 18. 523, 1905.

[3]) M. ABRAHAM. Rendiconti R. Inst, Lomb. d. x. e. lett. (3) 44, 68. 1911.

nate system. For another system the energy flows in general through the surface. (We find an instance for this fact in the outer surface of the body, mentioned in the last paragraph but one, which is in rest relatively to K°. For K° no energy current passes through the surface, it does, however, in K^{+} and K^{-}). But this representation fails altogether when absorption takes place, because then inside such a surface the energy would gradually diminish indefinitely. Therefore it seems to me that no great importance can be attributed to the conception of the velocity of the energy.

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Physics. — *"On the conception of the current of energy"*. By J. D. VAN DER WAALS JR. (Communicated by Prof. J. D. VAN DER WAALS).

In the preceding paper Mr. LAUE advances some objections against the way in which I make use of the conception "current of energy" in my considerations [1]). He was so kind as to send me his remarks in manuscript, in consequence of which I can answer them in this same number. LAUE is of opinion, that we cannot conceive the current of energy as a product of two factors: the density and the velocity of the energy; and more emphatically that in case of a LORENTZ transformation such a velocity must not be transformed according to the ordinary formula for the transformation of velocities.

As a proof for this assertion he points out, that the elastic energy of a moving body can become zero or negative [2]), but that the corresponding current of energy does not become zero or change its sign at the same time. This difficulty, however, is not decisive, if we accept the decomposition of the energy current in components moving with different velocities, as I have indicated, l. c. § 5. And the circumstance, that energy is transferred from one point of the body to

[1]) These proceedings p. 239.

[2]) At first sight it seems to be paradoxal that the elastic energy should become negative. Still it is really possible, as can be explained in the following way. We imagine a stationary body. Now we apply equal and opposite forces at the ends of it. These forces in stretching out the body, do positive work. Then we set the body in motion, in consequence of which it contracts. During this contraction the external forces do a negative amount of work If this negative amount is in absolute value equal to, or larger than the positive work for the extension, the elastic energy of the body can become zero or negative.