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A better idea of the profit to be gained by the application of these symptomatical indications might be gathered by determining the percentage of success if a forecast were made exclusively on the base of local wind observations and the knowledge of the gradient for *one* given station, *without* consultation of a weather chart.

This has been done for the station Swinemunde and the years 1909 and 1910.

Magnitude and direction of the gradient were computed from the barometric heights at the stations Hamburg, Whisby and Breslau, forming a triangle in the centre of which Swinemunde is nearly situated.

The method of criticizing the degree of success was about the same as applied to the area of a weatherchart; the result was an average success of 65.4 °/₀, which may be considered as a pretty fair result of the method without the application of other means.

As to the signalling of storms, the result was favourable as might have been expected especially for rapidly advancing disturbances, a number of which could be foreseen considerably earlier than if the relative windforce and angles of deviation were not taken into account.

Meteorology. — "*On the angle of deviation between gradient of atmospheric pressure and air motion.*" By J. P. VAN DER STOK.

(Communicated in the meeting of January 27, 1912).

In a previous communication[1]) I have shown how the average friction coefficient, and therefore also the average angle of deviation between gradient of pressure and windvelocity, can be derived from the semidiurnal variation of barometric pressure and wind.

As a result of this inquiry for two sets of observations made at de Bilt and on board the lightvessel Terschellingerbank, it was thus found that the angle of deviation computed according to the well known expression for steady motion

$$tang\ \alpha = \frac{2\ n\ sin\ \varphi}{l} = \frac{a}{k}$$

$a = 2 \sin \varphi, \cdot l = kn$

$\varphi =$ geogr. latitude

$n =$ angular velocity of the earth's rotation

$l =$ frictioncoefficient

in winter and autumn was considerably smaller than in spring and summer.

1) These proceedings: Meeting of May 27, 1911.

If we assume that this result is not due to disturbances in the expressions for the semidiurnal variation or to shortcomings of the theoretical reasoning, if further we take into account that (at least in our country) the angle of deviation strongly depends on the direction of wind and gradient, then two explanations may be examined.

Firstly it is possible that the specific deviation (i. e. the deviation as considered separately for each direction) actually remains the same during the whole year, but that the variability of the mean deviation must be ascribed to the distribution of the winddirections in the *different* seasons.

The fact e. g. that in our country during spring northerly winds prevail and that for this direction the deviation is unusually large, must cause a greater angle of deviation in spring than for the whole year.

Secondly it is possible that the friction coefficient varies with the temperature and possibly also with the turbulence of the air; in this case the specific deviation would be a variable quantity in different seasons.

With a view of putting these results of a theoretical treatment to the test of direct observation, we can make use of the values of the pressure gradient as computed for the hours 7^h and 9^h30^m a.m. of each day from the barometric height at the five Dutch stations which, since March 1904, are printed in the daily weathercharts.

Its mean direction, holding good for de Bilt situated in the centre, is expressed in 16 points and therefore has an uncertainty of $\pm$ 11°.25; in the original computations these directions are, of course, calculated to a much higher degree of precision, but a simple consideration of the weather charts shows clearly that angles of deviation of the most different values are associated with gradients equal as regards magnitude and direction; the use of more accurate values would, therefore, be of little use and it is only from a great number of observations that reliable average values can be derived.

After some trials it likewise did not appear desirable to consider those cases only in which the gradient exceeds a given minimum, as frequently the value of a small gradient is evidently accurate and, conversely, for large gradients the complicated general situation and irregular curvatures of the isobars give rise to unreliable results.

Therefore, in the following investigation, all observations of the angle of deviation computed during the period March 1904 to December 1910 for the five stations are used, without considering the magnitude of the gradient, with the exception only of those rare cases in which the values were greater than 135° or smaller than 0°.

Further it is assumed that the same direction of gradient, holding good for de Bilt, may also be used for the other stations because the radius of curvature of the isobars generally is large in comparison with the distance between the different stations.

In table I the frequencies of the angular values corresponding with different directions of the gradient are given for the central station de Bilt.

TABLE I. Frequencies of angle of deviation, de Bilt, 1904—1910.

Direction gradient	0°	22°.5	45°	67°.5	90°	112°.5	135	Total
N	20	98	**202**	165	67	17	9	578
NNE	19	60	105	**158**	101	17	9	469
NE	10	14	47	**123**	111	46	9	360
ENE	2	4	24	73	**73**	38	4	218
E	—	6	16	46	**60**	23	9	160
ESE	2	3	21	**34**	25	13	3	101
SE	1	7	14	**55**	44	9	1	131
SSE	—	6	28	**84**	27	13	3	161
S	2	13	70	**70**	39	7	6	207
SSW	3	17	58	**73**	39	16	7	213
SW	4	13	39	**62**	51	14	3	186
WSW	2	10	29	**68**	30	14	1	154
W	1	5	75	**85**	34	4	2	206
WNW	4	22	74	**147**	23	4	—	274
NW	8	28	**179**	125	31	5	1	377
NNW	9	80	**246**	195	41	7	1	579
Total	87	386	1227	1563	796	247	68	4374

It appears from this table that the spreading out of the values is considerable so that the simple relation between angle of deviation, earth's rotation and friction, as expressed in the foregoing formula, is but rarely realized. In table II the values of the angle of deviation are given as computed from frequency tables of the same kind as table I for the five stations and 16 gradient directions.

TABLE II. Average angle of deviation for different directions of gradient.

Direction Gradient	Groningen	Helder	de Bilt	Flushing	Maestricht
N	62°.9	68°.6	54°.7	63°.0	50°.8
NNE	69 .4	81 .1	61 .7	64 .9	49 .9*
NE	**82 .4**	93 .1	75 .3	81 .0	54 .8
ENE	81 .4	**94 .5**	80 .2	**94 .0**	70 .3
E	82 .7	93 .7	**82 .3**	92 .3	70 .9
ESE	81 .9	88 .4	73 .5	86 .4	**71 .2**
SE	74 .0	84 .0	73 .3	85 .0	67 .9
SSE	76 .8	78 .1	70 .6	71 .0	59 .0
S	79 .0	77 .1	64 .1*	69 .4	57 .1
SSW	75 .4	70 .5	66 .5	62 .2	52 .8*
SW	72 .8	70 .4	**68 .8**	61 .0*	55 .3
WSW	64 .1	65 .3*	68 .4	65 .7	69 .1
W	64 .0*	68 .5	63 .1	65 .4	**72 .8**
WNW	**68 .1**	**70 .9**	59 .4	70 .2	68 .8
NW	64 .2	66 .6	54 .7	**70 .5**	62 .5
NNW	60 .7*	65 .9*	52 .9*	61 .8	52 .6
Mean	67°.9	77°.3	66°.9	72°.7	61°.6

For the first four stations the principal maximum is situated between the - directions NE and E of the gradient; the principal minimum is spread out over a larger area but mostly associated with N and NNW directions, secondary maxima and minima occur more or less distinctly at all stations. Maestricht shows a considerable divergence from the other stations as there two equivalent well defined maxima occur for the directions ESE and W, and two minima for the NNE and SSW directions.

The differences between the extreme values amount to: for Groningen 22°, Maestricht 23°, Helder and de Bilt 29°, for Flushing 33°.

As has been pointed out in another communication, these large differences can be ascribed to the fact that in our climate steady motions are of comparatively rare occurrence and that then the simple formula is not generally applicable

A westerly gradient often indicates an approaching depression associated with an increase of gradient; an easterly gradient intimates an increase of distance from the centre with a filling up and extinction of the depression.

In the first case the tangent of the angle of deviation will be larger, in the second case smaller than the normal value owing to an apparent increase or decrease of the friction coefficient.

When northerly or southerly gradients obtain, the eastward movement of the centre of depression is associated with a rotation of the gradient with constant magnitude and the effect of this rotation must be equivalent to an apparent decrease or increase of the earth's deviating force; this explains the occurrence of two maxima and minima. The distribution of the different values for Maestricht finds a ready explanation in the fact that this station is situated in the river basin of the Maes where the friction experienced by N and S winds must be considerably less than for E and W winds.

In table III, showing the values of the angle of deviation cor-

TABLE III. Average angle of deviation for different wind directions.

Direction Wind	Groningen	Helder	de Bilt	Flushing	Maestricht	Magnetic 14° West	
						Helder	Flushing
N	**82°.4**	93°.8	77°.4	**92°.5**	**71°.2**	94°.4	93°.6
NNE	77 .5	88 .0	73 .0	86 .2	68 .0	92 .2	90 .1
NE	75 .7	81 .9	71 .0	76 .8	58 .2	85 .9	85 .6
ENE	75 .6	77 .5	64 .4*	69 .6	54 .6	79 .8	70 .6
E	76 .9	71 .7	66 .6	61 .9	54 .3*	77 .2	66 .3
ESE	73 .5	70 .4	**68 .7**	62 .1	59 .7	70 .5	61 .4
SE	64 .1*	65 .6*	68 .4	65 .7	63 .4	68 .8	63 .6
SSE	64 .5	68 .4	62 .2	65 .7	**72 .0**	66 .6	65 .6
S	**68 .0**	**70 .6**	57 .3	69 .7	69 .1	69 .2	67 .3
SSW	63 .6	66 .6	53 .6*	70 .5	58 .6	70 .6	70 .3
SW	61 .1*	66 .1*	54 .0	62 .1*	51 .9	66 .3	67 .0
WSW	62 .6	68 .5	57 .7	63 .3	50 .1*	67 .0	62 .5
W	68 .6	76 .2	63 .9	66 .0	53 .1	71 .2	63 .1
WNW	76 .9	84 .2	72 .4	75 .4	60 .0	79 .3	73 .3
NW	82 .0	92 .0	77 .9	84 .3	69 .2	87 .2	77 .5
NNW	81 .9	**94 .2**	**81 .0**	**92 .5**	70 .8	93 .4	87 .4

responding to different wind-directions, this effect is more clearly visible than in table II; it is derived from the latter by linear interpolation.

This proves that the variability of the gradient in direction and magnitude may be regarded as the principal cause of the spreading out of the angles of deviation, but that local circumstances also play an important part and the friction coefficient *kn* is certainly not the same for different directions.

When N and NNW winds obtain the path of the air particles mostly traverses a sea surface where friction is small; but this is, to some extent, contradicted by the fact that, if the wind is SW, when it also blows over the sea (although to a smaller degree), a minimum rather than a maximum value of the angle of deviation is observed.

TABLE IV. Values of the angle of deviation, de Bilt.

Direction Gradient	Frequencies: Winter	Spring	Summer	Autumn		Angle of deviation: Winter	Spring	Summer	Autumn
N	486	323	439	378		55°.4	61°.7	57°.8*	50°.2
NNE	413	298	388	308		61 .9	71 .2	62 .5	56 .5
NE	282	242	305	218		65 .3	77 .5	70 .6	63 .5
ENE	154	199	234	151		75 .5	80 .1	**80 .1**	75 .8
E	50	147	175	107		82 .4	**80 .8**	78 .0	78 .6
ESE	35	128	131	98		**83 .5**	76 .1	74 .4	**79 .4**
SE	40	145	119	89		74 .2	72 .9	69 .4	74 .1
SSE	77	178	133	111		68 .7	70 .3	66 .3	68 .7
S	121	191	122	147		63 .2	70 .4	67 .1	64 .6
SSW	133	189	96	188		63 .3	70 .6	66 .8	64 .1
SW	121	158	67	207		59 .1	73 .1	70 .8	67 .8
WSW	108	146	69	223		57 .9	70 .7	70 .4	66 .8
W	149	148	99	238		56 .0	66 .4	67 .5	62 .8
WNW	225	205	163	264		59 .9	61 .2	64 .0	54 .3
NW	360	268	274	328		55 .3	58 .3	60 .9	53 .0
NNW	450	323	396	365		54 .2*	58 .0*	58 .6	48 .6*
Total	3204	3288	3210	3420	Mean	64°.7	70°.0	67°.8	64°.3

As might have been expected, the difference between land- and seastations is clearly visible in the general mean values of table II; the most inland station, Maestricht, showing the smallest value viz. 68°, Helder, the most maritime station, the greatest value viz. 77°.

When we calculate the angle of deviation for different gradient directions and different seasons (Table IV), the frequency of occurrence becomes often too small, principally for E—S directions and in winter. Therefore, as has been indicated in Table IV, all frequencies have been taken together for each set of three subsequent directions, so that the computed average values bear relation to an angular area of 67,°5 and not of 22,°5 as those of the foregoing tables.

Even then the number of observations in the SE quadrant is hardly sufficient, but still the average values run in a continuous manner.

TABLE V. Values of the angle of deviation.

	Groningen				Helder			
Direction Gradient	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn
N	62°.3	68°.2	70°.7	61°.7	69°.6	68°.9	69°.1	78°.2
NNE	68 .4	74 .7	69 .3	69 .5	79 .1	76 .0	73 .4	90 .2
NE	75 .2	83 .0	72 .9	75 .6	87 .8	86 .1	82 .2	**100 .0**
ENE	84 .3	88 .0	**77 .2**	80 .0	94 .7	**93 .7**	89 .3	98 .9
E	91 .6	**88 .4**	75 .6	77 .8	98 .6	93 .4	**89 .4**	96 .4
ESE	**92 .9**	84 .4	70 .7	80 .3	**101 .9**	89 .6	84 .1	90 .8
SE	91 .1	80 .1	69 .1	77 .4	98 .0	83 .6	77 .1	80 .3
SSE	88 .8	78 .1	67 .3	78 .5	90 .0	79 .1	72 .8	79 .6
S	80 .1	75 .8	70 .4	**81 .9**	77 .4	75 .5	68 .8	76 .8
SSW	78 .6	74 .3	69 .5	78 .9	77 .6	71 .4	65 .6	74 .5
SW	69 .9	72 .2	66 .8	73 .0	71 .4	69 .3	59 .0*	70 .8
WSW	64 .8	69 .3	68 .2	66 .1	71 .5	66 .9*	66 .2	68 .1
W	59 .8	69 .1*	74 .5	64 .3	64 .8	69 .4	72 .8	69 .2
WNW	59 .3*	69 .5	74 .7	61 .8	62 .1	72 .1	78 .3	65 .1
NW	59 .5	68 .6	69 .5	59 .1	61 .5*	72 .3	73 .3	64 .4*
NNW	60 .0	66 .6	66 .6*	57 .9*	63 .3	69 .0	69 .8	67 .3
Mean	74°.2	75°.6	70°.8	71°.5	79°.3	77°.3	74°.5	79°.4

In the same manner the angles of deviation for the other four stations have been calculated and are given in tables V and VI, for the sake of brevity without the corresponding frequencies.

TABLE VI. Values of the angle of deviation.

	Flushing				Maestricht			
Direction Gradient	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn
N	58°.7	66°.7	65°.0	62°.9	47°.4	52°.3*	57°.4	45°.2
NNE	66 .4	69 .9	67 .2	69 .3	48 .7	55 .8	54 .8*	46 .7
NE	74 .8	79 .4	75 .5	75 .2	54 .9	60 .8	58 .4	49 .1
ENE	86 .5	87 .6	86 .5	88 .5	56 .9	67 .7	65 .0	58 .2
E	**96 .3**	**93 .2**	**88 .6**	94 .2	**66 .5**	**73 .0**	**67 .2**	75 .4
ESE	92 .9	87 .8	82 .2	**96 .6**	62 .7	70 .4	65 .4	**77 .9**
SE	82 .8	78 .1	75 .2	86 .6	58 .3	63 .8	61 .3	76 .1
SSE	77 .0	71 .2	72 .5	78 .7	56 .1	61 .6	59 .0	64 .8
S	65 .2	64 .6	69 .2	70 .7	51 .9*	58 .2	57 .9	55 .4
SSW	63 .0	62 .1	66 .0	66 .5	54 .0	57 .6*	57 .4*	52 .2*
SW	58 .3	60 .1*	64 .2*	66 .4	57 .0	60 .8	58 .3	55 .8
WSW	64 .2	64 .4	64 .6	63 .4	67 .2	69 .1	69 .2	61 .3
W	68 .0	65 .4	73 .4	66 .3	**67 .7**	**75 .6**	**74 .6**	**66 .3**
WNW	64 .7	68 .4	74 .4	63 .3	62 .0	71 .3	74 .5	62 .2
NW	58 .7	68 .4	71 .4	62 .9	53 .7	64 .4	68 .1	52 .4
NNW	56 .5*	68 .2	68 .1	61 .5*	48 .3	60 .2	62 .0	47 .9
Mean	70°.9	72°.2	72°.8	73°.3	57°.1	64°.1	63°.8	59°.2

It appears from the average values for all directions taken together that the specific angle of deviation is greater in spring and summer for the landstations de Bilt and Maestricht but that, at the maritime stations Helder and Flushing, as also at Groningen, this difference is slight or nihil, while for Helder they are of opposite sign. This result cannot be considered as final owing to the small frequencies of some directions, and although for some directions the rule holds good (e.g. for NW and NNW directions, Table VII), it fails for others e.g. the E. direction.

TABLE VII. Mean specific angle of deviation for NW and NNW directions of the gradient

	Winter	Spring	Summer	Autumn
Groningen	59°.8	67°.6	68°.1	58°.5
Helder	62 .4	70 .7	71 .6	65 .9
de Bilt	54 .8	58 .2	59 .8	50 .8
Flushing	57 .6	68 .3	69 .8	62 .2
Maestricht	51 .0	62 .3	65 .1	50 .2

The question in how far the distribution of winddirections in different seasons may cause a difference between the average angles of deviation can be answered by calculating not the specific means, in which every direction is regarded as equivalent, but by giving to each direction the weight of its frequency.

The values thus computed and shown in table VIII apply to the period March 1904—December 1910 and the winddistribution during these years.

TABLE VIII. Mean angle of deviation according to the frequencies of direction, 1904—1910.

	Winter	Spring	Summer	Autumn	Year
Groningen	67°.8	74°.5	70°.1	68°.1	70°.2
Helder	73 .5	76 .6	75 .3	77 .0	75 .6
de Bilt	60 .8	69 .0	65 .8	60 .3	63 .6
Flushing	65 .3	71 .6	72 .3	68 .8	69 .5
Maestricht	53 .5	62 .9	61 .9	55 .4	58 .4

It appears then that, with the only exception of autumn for Helder, the average angles of deviation in spring and summer are actually larger than in winter and autumn and that this phenomenon must be principally ascribed to the distribution of winddirections.

As the meteorological conditions are extremely variable, a period of seven years (1904—1910) is decidedly too short to furnish wind frequencies for different seasons which may be considered as normal values, it is interesting to apply the results of tables V and VI for Helder and Flushing to two series of wind observations extending over 25 years and made on board the lightvessels Terschellingerbank and Schouwenbank situated in the vicinity of the two landstations. Table IX shows the windfrequencies for magnetic directions, the average deviation being 14° (Westerly).

For application to the data of this table, the angles of deviation corresponding with magnetic winddirections are given in Table III for Helder and Flushing.

TABLE IX. Frequencies of winddirection at two lightvessels, pro 1000, magnetic.

	Terschellingerbank 1884—1908				Schouwenbank 1882—1906			
Direction Wind	W	Spr.	Summer	A	W	Spr.	Summer	A
C	24	51	56	27	15	28	36	18
N	37	81	95	63	32	65	78	45
NNE	16	41	46	23	20	62	62	32
NE	36	92	61	36	31	138	101	48
ENE	23	52	40	20	47	73	51	43
E	52	95	61	59	77	64	42	79
ESE	47	32	20	45	57	30	23	56
SE	80	45	29	85	62	36	30	55
SSE	35	22	18	43	36	24	18	34
S	80	42	28	81	71	39	28	61
SSW	64	34	26	53	93	39	28	72
SW	133	122	115	100	139	86	68	111
WSW	64	58	86	62	73	102	132	70
W	138	84	111	125	82	77	130	90
WNW	59	35	55	52	56	37	59	72
NW	77	73	91	84	77	56	67	79
NNW	35	41	62	42	32	44	47	35

The computation leads to the following results:

	Winter	Spring	Summer	Autumn	Year
Terschellingerbank	74°.5	77°.8	78°.5	75°.6	76°.59
Schouwenbank	70 .0	73 .9	73 .6	71 .0	72 .11

It appears then that, although at Helder the specific angles of deviation are greater in winter and autumn than in summer and spring, still the total means follow the general rule, owing to the distribution of the wind in the different seasons.

The angles of deviation as computed in this manner for Terschellingerbank are much larger and probably more accurate than those derived from the semidiurnal variation of wind and barometric height; from which we may conclude that these variations are influenced by various disturbing elements so that a direct application of theoretical reasonings is premature.

Physics. — "*Contribution to the theory of binary mixtures.*" XVIII. By Prof. J. D. VAN DER WAALS.

(Communicated in the meeting of January 27, 1912).

In the preceding contribution some points have been mentioned which deserve a fuller elucidation, and the discussion of others was omitted, which I will now take in hand. In the first place it seems desirable to me to discuss more fully in how far the course of the isobars in connection with well-known properties of the spinodal curve is sufficient to enable us to decide beforehand whether three-phase pressure will occur for a mixture with minimum T_{pl}, so that we need not attribute its existence to other unknown causes, and that accordingly the existence of three-phase pressure must not be considered as an anomalous phenomenon.

In the preceding contribution the question was put as follows: has the spinodal line on the liquid side for mixtures with minimum T_{pl} one value for x where $\frac{dp}{dx}$ is equal to 0 for this line, or are there three values for x where this is the case, always for given T. Thinking that the calculation would not be feasible, I had intended to try and answer this question for myself by a graphical way. And I had come to the conclusion that the existence of 3 values had to be expected 1 if the place where T_{pl} is minimum is close to the side, 2 if the range of temperature for T_k is not too small for the components, and 3 especially if the value of the ratio of the critical pressures of the components is *large*. And strictly speaking, calculation is not feasible yet, and this will continue to be so until the equation of state is known with perfect accuracy. Probably the intricacy of the calculations will then prevent us from obtaining a result. But if we content ourselves with an approximate calculation, and if the quantity b is kept constant in the equation of state, and if quasi-