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We have endeavoured to explain these facts by assuming that the organism is protected by a layer of water through which it has to be reached by the nutrient as well as by the retarding substances. In the case of substances soluble in water it will depend mainly on their solubility in fat whether they will penetrate the organism rapidly and eventually overload the same.

a. If they are absolutely insoluble in water they will have neither a nutrient, nor a toxical action.

b. If they are *very* little soluble in water but fairly so in oil (cetyl alcohol, palmitic acid, naphthalene) they will have a nutrient but no toxical action.

c. If they are considerably soluble in water, but still much more so in oil, they can act as a nutrient in small concentrations only, at higher concentrations they cause retardation.

d. If they are readily soluble in water but very little so in oil, they cannot act as a toxical substance, but only as a nutrient.

Finally, we have drawn from this the conclusion that an antiseptic [1]) must have a large division factor oil: water, also a sufficient solubility in the last solvent.

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Astronomy. — "*Calculations concerning the central line of the solar eclipse of April the 17th.* 1912 *in the Netherlands*". By Mr. J. Weeder. (Communicated by Prof. E. F. van de Sande Bakhuyzen).

(Communicated in the meeting of January 27, 1912).

Although the central line of a solar eclipse is given in the astronomical ephemerides through many points on the surface of the earth, it may be useful for the observation of the approaching eclipse to communicate a few results which have been calculated for Holland in particular. Owing to the small width of the zone of annularity in Dutch Limburg the data given in the almanacs for this eclipse are not sufficient to predict whether or no a particular place will be situated within this zone; this is obvious as the differences between the different calculations surpass the width of the zone.

This disagreement arises principally from the differences between the geocentric places of the moon which have been adopted for the calculations; the employed values of the ellipticity of the earth have had some influence too.

1) No strong acid or strong base is meant here, but a chemically-indifferent substance.

For the basis of my computation I have employed the HANSEN-NEWCOMB values, for the longitude, latitude, and parallax of the moon taken from the Berliner Jahrbuch, but with some corrections added to them. The longitudes have been increased with the sum of 1. an empirical correction deduced by Prof. E. F. v. D. S. BAKHUYZEN from NEWCOMB's results of the occultations and from the Greenwich observations up to 1910 [1]), 2. a few theoretical terms of short period, according to NEWCOMB [2]) RADAU [3]) and HILL [4]), partially modified according to E. F. v. D. S. BAKHUYZEN's results from the observations [5]) and 3. the corrections deduced by the latter, dependent on perigee and eccentricity [6]). The corrections to the latitudes proceed partly from those to the longitudes, partly from those to the node and the inclination of the moon's orbit. The deviation in latitude between the centre of gravity of the moon and its centre of figure to the amount of $-1''.00$, incorporated by HANSEN in his tables, I have kept unaltered according to E. F. v. D. S. BAKHUYZEN's investigation [7]). The parallax constant has been increased with $+0''.37$ according to NEWCOMB and BATTERMANN [8]).

The formulae for the moon's places adopted by me are:

$$l = l\,(B.J.) + (1 + 0.110 \cos g + 0.008 \cos 2g)\,.\,\{+ 7''72 + 1''.69 \sin D$$
$$- 0''.33 \sin 2D - 0''.24 \sin (D + g') + 0''.09 \sin g' + 0''.16 \sin (D - g)$$
$$- 0''.21 \sin (2D - g)\} - 0''.43 \sin g - 0''.17 \cos g + 1''.28 \sin \{g + 217^\circ$$
$$+ 19.36\,(t - 1876.0)\} + 0''.32 \sin \{g + 198^\circ - 9^\circ.67\,(t - 1876.0)\}.$$
$$+ 0''.45 \sin \Omega \cos g = l(B.J.) + 9''.55$$

$$b = b(B.J.) + \Delta i \times \sin u + (9''.55 - \Delta \Omega) \times 0{,}09 \cos u = b(B.J.) - 0''.10.$$

$$\pi = \pi\,(B.J.) + 0''.37.$$

The angles in these formulae, have for the mean time of my computations, 1912 April the 17th $1^h 11^m$ mean time Berlin, the following values:

mean anomaly of the moon	$g = 278^\circ.1$
" " " " sun	$g' = 103.8$
longitude of the ascending node	$\Omega = 21.4$
angle from node to perigee	$\omega = 93.1$
angle from node to the sun's perigee	$\omega' = 260.0$

1) Proc. Acad. Amsterdam **14** 1912 p. 686 et seq.

2) Investigations of Corrections to HANSEN's Tables of the Moon p. 37 (1876).

3) Annales Paris. Mémoires **21**.

4) Papers Americ. Ephemeris **3** Part **2**.

5) Proc. Acad. Amst. **6** 1903 p. 370 et seq. and p. 412 et seq.

6) Ibid.

7) Proc. Acad. Amsterdam **14** 1912 p. 692 et seq.

8) Beobachtungs-Ergebnisse der Königlichen Sternwarte zu Berlin N⁰ **13** p. 12.

mean elongation of the moon from the sun

$$g + \omega - g' - \omega' = D = \quad 7.4$$

mean longitude of the moon ☾ = 32.6

mean argument of latitude ☾ — ☊ = u = 11.2

In the formula for l, t is expressed in years, here 1912.29; in that for b I have adopted as corrections to the inclination and the longitude of the node

$$\Delta i = -0''.10$$

$$\Delta ☊ = +10''.50$$

according to E. F. v. d. S. Bakhuyzen *Proc. Acad. Amst.* 6 p. 426.

For the ellipticity of the earth I adopted the mean of the values according to Helmert [1]) and to Hayford [2]). My computations have been accomplished with $log(1-c) = 9.9985\,385$ corresponding with the ellipticity 1 : 297,65.

For the calculation of the width of the zone of annularity there is also needed the ratio of the radius of the moon to the equatorial radius of the earth. For this ratio I have employed $log\ s = 9.435\,3888$, which, adopting as mean parallax of the moon 57′ 2″.65, corresponds with a radius of the moon of 15′ 32″.68. The last value, which I take from Prof. E. F. v. d. S. Bakhuyzen, is founded on the results of heliometric observations (by Bessel, Wichmann, and Hartwig) and of occultations, (some occultations of the Pleiades and those calculated by L. Struve and Battermann), the former of which gave 32″.75, the latter 32″.65.

The geocentric longitude l', latitude b' and distance of the sun R' and the obliquity of the ecliptic ε have been taken from the Berliner Jahrbuch; further I adopted Auwers' value $\Delta' = 15'59''.63$ for the mean radius of the sun and 8″.80 for the solar parallax.

Calculation of the central line.

According to Hansen [3]) we have the following relations between the co-ordinates of a given place on the surface of the earth and those of the sun and the moon, if these bodies seen from that particular place seem to be in contact with each other.

1) Sitzungsber. Berlin. 1901 p. 328.

2) J. F. Hayford, The figure of the earth and isostasy (1909). Supplementary investigation (1910).

3) P. A. Hansen, Theorie der Sonnenfinsternisse und verwandten Erscheinungen. Abhandl d. K. Sächs. Ges. d. Wissensch. IV (1858) p. 305–334.

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$$\Phi_1 = P \cos h - Q \sin h - \cos \varphi_I \sin (t + \Delta\alpha') = u \sin \theta$$

$$\Phi_2 = P \sin h + Q \cos h - \{(1-c) \sin \varphi_I \cos \delta' - \cos \varphi_I \sin \delta' \cos (t + \Delta\alpha')\} = u \cos O$$

$$u = s \sec f + \{Z - (1-c) \sin \varphi_I \sin \delta' - \cos \varphi_I \cos \delta' \cos (t + \Delta\alpha')\} \, tg f.$$

The factor $1 + x$ which still appears in these formulae of HANSEN and through which the influence of the atmospheric refraction is brought into account, could here be neglected.

In these equations the quantities P, Q, and Z depend in the following way on the co-ordinates of the sun and moon.

$$P = \frac{\cos b \sin (l - \lambda')}{\sin \pi} \quad Q = \frac{\sin (b - \beta')}{\sin \pi} \quad Z = \frac{\cos (b - \beta') \cos (l - \lambda')}{\sin \pi}$$

$$l - \lambda' = (l - l') \left(1 + \frac{\sin 8''.80}{R' \sin \pi}\right) \quad b - \beta' = (b - b') \left(1 + \frac{\sin 8''.80}{R' \sin \pi}\right).$$

The quantities δ', $\Delta\alpha'$, and h refer to the direction of the straight line between the centres of the sun and moon. The declination of this direction is δ' and if to its right ascension α' we add $\Delta\alpha'$, we obtain the right ascension of the sun. For the point on the celestial sphere whose co-ordinates are α' and δ', h is the angle between the hour-circle and the circle of latitude. The quantities δ', $\Delta\alpha'$, and h are calculated from the co-ordinates of the sun and moon and the obliquity of the ecliptic by means of the auxiliary angles da', d', and h_0, in the following way:

$$tg\, a' = \cos \varepsilon \, tg\, l' \qquad \delta' = d' - \frac{P \sin h_0 + Q \cos h_0}{R'} . 8''.80 + b' \cos h_0$$

$$tg\, d' = tg\, \varepsilon \sin a' \qquad \Delta\alpha' = \frac{P \cos h_0 - Q \sin h_0}{R' \cos d'} . 8''.80 + b' \frac{\sin h_0}{\cos d'}$$

$$\sin h_0 = \sin \varepsilon \cos a' \qquad h = h_0 + \sin d' (\Delta\alpha').$$

The quantities φ_I and t depend on the time and the place of observation, since we have

$$tg\, \varphi_I = (1 - c)\, tg\, \varphi \quad \text{and} \quad t = \tau + \lambda$$

where φ and λ denote the geographical latitude and longitude of the place of observation and τ is the true solar time for that meridian from which λ is reckoned as eastern longitude.

The angle f, which appears in the 3rd of HANSEN's equations only, is the semi-angle of the cone which is in contact with the sun and the moon; for an apparent external contact the value of f, which we shall denote by f_e, is determined by:

$$\sin f_e = \frac{\sin \Delta' + s \sin 8''.80}{R' \sin \pi - \sin 8''.80} \sin \pi$$

while for an apparent internal contact, the corresponding value f_i is determined by:

$$\sin f_i = -\frac{\sin \Delta' - s \sin 8''.80}{R' \sin \pi - \sin 8''.80} \sin \pi.$$

For other phases of the eclipse the angle f may be determined, when time and place of observation are known, as a third unknown quantity, together with the angles u and θ, from HANSEN's three fundamental equations [1]). By means of this angle the phase of the eclipse is obtained by calculating the quantity m from:

$$\sin m\Delta' = \frac{\sin f}{\sin \pi} (R' \sin\pi - \sin 8''.80) - s \sin 8'' 80.$$

and the fraction of the apparent diameter of the sun which is eclipsed by the moon [2]) will be

$$\tfrac{1}{2}(1 - m)$$

If we define the angle f as a continuous variable, the three fundamental equations hold good for all phases of the eclipse. The quantities u and θ determine the position of the place of observation relatively to the straight line through the centres of the sun and moon, i. e. to the axis of the shadow-cone; u is the perpendicular distance of the place of observation from this axis and θ is the position-angle of the great circle, parallel to the plane going through the hadow-axis and this place, in the point of the celestial sphere, the co-ordinates of which are α' and δ'.

The central line of an eclipse is the curve on the surface of the earth along which that surface is intersected by the axis of the shadow. Therefore for the points of this line we have $u = 0$, so that the line is determined by the equations $\Phi_1 = 0$ and $\Phi_2 = 0$, i. e. by:

$$\cos \varphi_I \sin (t + \Delta\alpha') = P \cos h - Q \sin h$$

and

$$(1 - c) \sin \varphi_I \cos \delta' - \cos \varphi_I \sin \delta' \cos (t + \Delta \alpha') = P \sin h + Q \cos h.$$

Suppose the perpendicular distances from all points on the surface of the earth to the plane of the equator to be enlarged in the ratio $1 : \frac{1}{1-c}$, then this surface becomes spherical. By giving corresponding displacements to the centres of the sun and moon we somewhat simplify the problems exclusively regarding the central line.

If the declination of the direction of the axis of the shadow,

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1) In this way we always find two values for the angle f, the sum of which is equal to the apparent diameter of the moon with negative sign. We have always to take the greater of the two.

2) For annular as well as for total phases the physical interpretation of the expression $\frac{1}{2}(1-m)$ is no longer the same.

now also altered, is denoted by δ'_I, then we have $tang\,\delta'_I = \frac{tang\,\delta'}{1-c}$.
The right ascensions are not altered by our transformation, neither are the geographical longitudes. The co-ordinate φ_I takes the place of the geocentric latitude.

If we replace $\sqrt{(1-c)^2 \cos^2 \delta' + \sin^2 \delta'}$, by w', then

$$\sin \delta' = w' \sin \delta'_I \quad \text{and} \quad (1-c) \cos \delta' = w' \cos \delta'_I .$$

After the introduction of the auxiliary quantities U and M, which are computed by the formulae:

$$tg\,H = \frac{Q}{P} \quad \text{and} \quad M = P \sec H \quad \ldots\ldots \quad (I)$$

we obtain as equations of the central line:

$$\cos \varphi_I \sin (t + \Delta\alpha') = M \cos (H + h)$$

$$\cos \delta'_I \sin \varphi_I - \sin \delta'_I \cos \varphi_I \cos (t + \Delta\alpha') = \frac{M}{w'} \sin (H + h).$$

By subtracting the sum of the squares of the two members of these equations from unity, we find:

$$\sin \delta'_I \sin \varphi_I + \cos \delta'_I \cos \varphi_I \cos (t + \Delta\alpha') = z_I ,$$

an equation, the 2^{nd} member of which is the positive root of an expression which may be computed by means of the formulae (II).

$$F' = \frac{c(2-c)}{(1-c)^2} \cos^2 \delta'_I ; \quad z_I^2 = 1 - M^2 \{1 + F' \sin^2 (H + h)\} \quad . \quad (II)$$

If further we calculate the auxiliary quantities w', U, and N fulfilling the conditions

$$w' = \frac{1}{\sqrt{1+F'}} ; \quad tg\,U = \frac{M \sin (H+h)}{w' z_I} \quad \text{and} \quad N = z_I \sec U . \quad (III)$$

then the geographical longitude is given by the equations

$$tg\,(t+\Delta\alpha') = \frac{M \cos (H+h)}{z_I \cos (U+\delta'_I)} \quad \text{and} \quad \lambda = (t+\Delta\alpha') - \tau - \Delta\alpha' \quad . \quad (IV)$$

and the geographical latitude by:

$$tg\,\varphi = \frac{1}{1-c}\, tg\,(U+\delta'_I) \cos (t + \Delta\alpha') \quad \ldots\ldots \quad (V)$$

With this set of formulae I calculated the longitude and latitude of two points in the central line for two moments, one 5 min. mean solar time after the other. In the results following below we have $T_0 = 0^h 32^m 26^s.20$ Amsterdam mean time $= 0^h 12^m 54^s 06$

Greenwich mean time, and the geographical longitudes are given relatively to the signal of the Dutch Survey at Ubagsberg [1]).

Times	Geogr. long. E. of Ub.	Geogr. lat.	
$T_0 + 0^m$	$-2° 28' 11''.9$	$49° 41' \ 5''.2$	. (C_0)
$T_0 + 5^m$	$+0 \ 13 \ 56 .5$	$51° \ 7' 37''.8$	. (C_5)

The first point C_0 is still far in Belgium and the second lies already in Germany. In order to determine better the central line for Limburg I have calculated a third point C_4, which comes into the axis of the shadow one minute earlier than C_5, and I obtained

Time	Geogr. long E. of Ub.	Geogr. lat.	
$T_0 + 4^m$	$-0° 19' 53''.3$	$50° 50' 22''.7$	. . (C_4)

This point C_4 is still situated in Belgium, but near the Limburg frontier. By interpolation between these three points the following values were found for the longitudes and latitudes which hold good for the topographical and military map of Holland [2]).

Geogr. long. E. of Amst.	Geogr. lat.
0°.45'	50°.50' 38''.3
50	53 12.8
55	55 46,7
1 0	58 20.1
5	51 0 53.0
10	3 25.2
15	5 56·9

Computation of the place where the vertex of the moon's shadow leaves the earth.

The solar eclipse of April 17 will probably be distinguished by the peculiarity that in the central line at first it is annular, later on it becomes total, then to grow annular again. One of these points of transition, viz. where the total eclipse becomes annular, will be situated in Belgium, if the above mentioned values of the apparent radii of the sun and moon are accurate. First I shall derive the

[1]) The difference in longitude between Berlin and Ubagsberg (7° 26' 34''.9) I took from the determination of the differences of long. between Ubagsberg and the observatories at Bonn, Göttingen and Leyden: Veröff. K. Preuss. Geod Inst Telegraphische Längenbestimmungen in 1890, 1891 und 1893; and Publication de la Commission Géodésique Néerlandaise: Déterminations de la différence de longitude Leyde-Ubagsberg, etc.en 1893.

[2]) From the last mentioned publication I also derived the latitude of Ubagsberg, in order to reduce the computed latitudes to those of the topographical map which are in accordance with the "Meetkunstige beschrijving v. h. Koningrijk d. Nederlanden."

time of this transition; subsequently the point where it takes place.

If $\frac{1}{\sin \pi}$, α and δ are the geocentric spherical coordinates of the moon and $\frac{s}{\sin f_i}$, α' and δ' the selenocentric ones of the vertex of the shadow, both referred to the equator and the equinox, then the rectangular geocentric co-ordinates of this vertex are:

$$\xi = \frac{1}{\sin \pi} \cos \delta \cos \alpha + \frac{s}{\sin f_i} \cos \delta' \cos \alpha'$$

$$\eta = \frac{1}{\sin \pi} \cos \delta \sin \alpha + \frac{s}{\sin f_i} \cos \delta' \sin \alpha'$$

$$\zeta = \frac{1}{\sin \pi} \sin \delta \qquad + \frac{s}{\sin f_i} \sin \delta'$$

In these expressions I substitute for the declinations the quantities δ_I and δ'_I, just as I have done before, so that here the auxiliary quantity $w = \sqrt{(1-c)^2 \cos^2 \delta + \sin^2 \delta}$ has to be introduced.

At the times of transition the vertex of the shadow falls on the transformed surface of the earth, the sphere with radius 1, hence

$$\xi^2 + \eta^2 + \frac{\zeta^2}{(1-c)^2} = 1$$

From this equation we derive

$$(1-c)^2 = \frac{w^2}{\sin^2 \pi} + \frac{s^2 w'^2}{\sin^2 f_i} + 2 \frac{sww'}{\sin \pi \sin f_i} \left\{ \cos \delta_I \cos \delta'_I \cos (\alpha' - \alpha) + \right.$$
$$\left. + \sin \delta_I \sin \delta'_I \right\}.$$

by which the time of transition is determined.

For the angle ψ between the directions ($\alpha\ \delta_I$) and ($\alpha'\ \delta'_I$) the relation

$$\cos \psi = \cos \delta_I \cos \delta'_I \cos (\alpha' - \alpha) + \sin \delta_I \sin \delta'_I,$$

holds good and from this we find

$$\sin^2 \frac{1}{2} \psi = \frac{\left(w + sw' \frac{\sin \pi}{\sin f_i} \right)^2 - \{(1-c) \sin \pi\}^2}{4\, sw\, w' \frac{\sin \pi}{\sin f_i}} \qquad . \ . \ (VI)$$

Another form for the angle ψ is found by using the expressions $P \cos h - Q \sin h = \mathfrak{P}$ and $P \sin h + Q \cos h = \mathfrak{Q}$, which occur in HANSEN's fundamental equations.

$\mathfrak{P}$, $\mathfrak{Q}$, and Z are the rectangular co-ordinates of the centre of the moon referred to axes through the centre of the earth as origin.

The axis of Z has been taken in the direction $(\alpha'\delta')$ and the axis of $\mathfrak{P}$ perpendicular to the plane of the declination-circle α'.

After having applied the transformation described above, which reduces the surface of the earth into a sphere, we can adopt a system of rectangular co-ordinates analogous to the former. The co-ordinates of the displaced centre of the moon with reference to these axes are distinguished from the corresponding co-ordinates of the real centre of the moon by the index I.

So the axis of Z_I is parallel to the direction $(\alpha' \delta'_I)$ and the axis of $\mathfrak{P}_I$ is perpendicular to the plane of the declination circle α'. The radius vector of the centre of the moon after the transformation is denoted by R_I.

The following relations hold between these quantities with and without the index I.

$$\mathfrak{P}_I = \mathfrak{P} \quad ; \quad \mathfrak{Q}_I = \frac{\mathfrak{Q}}{w'} \quad ; \quad R_I = R\frac{w}{1-c} = \frac{w}{(1-c)\sin\pi}.$$

Since $\sin^2\psi = \frac{\mathfrak{P}_I^2 + \mathfrak{Q}_I^2}{R_I^2}$, we thus derive:

$$\sin^2\psi = \left(\mathfrak{P}^2 + \frac{\mathfrak{Q}^2}{w'^2}\right)\frac{(1-c)^2}{w^2}\sin^2\pi$$

As $\mathfrak{P} = M\cos(H+h)$ and $\mathfrak{Q} = M\sin(H+h)$ we find:

$$\mathfrak{P}^2 + \frac{\mathfrak{Q}^2}{w'^2} = \left\{1 + \left(\frac{1}{w'^2} - 1\right)\sin^2(H+h)\right\} M^2 = 1 - z_I^2,$$

so that for the calculation of ψ we can also employ the equation

$$\sin\psi = \frac{(1-c)\sin\pi}{w} \cdot \sqrt{1-z_I^2}. \quad . \quad . \quad . \quad . \quad (VII)$$

The instant at which the transition from totality into annularity takes place is that for which the formulae (VI) and (VII) give the same value for ψ. I calculated $\frac{1}{2}\psi$ for the time T_0, $T_0 + 4$ min. and $T_0 + 5$ min. from both these formulae and found, interpolating for the minutes between T_0 and $T_0 + 4$.

Time	$\frac{\psi}{2}$ from (VI)	$\frac{\psi}{2}$ from (VII)
$T_0 + 0$ min.	0°18′41″.69	0°18′21″.05
„ + 1 „	42 .59	29 .88
„ + 2 „	43 .49	38 .85
„ + 3 „	44 .40	47 .98
„ + 4 „	45 .30	57 .25
„ + 5 „	46 .20	19 6 .67

These values are equal at $T_0 + 2^m33^s.9$.

For this instant I have now calculated from the formulae (I) to (V) the place of C_0 on the central line where the totality passes into annularity, and obtained for the geographical length east of Ubagsberg and for the geographical latitude of this point:

$$\lambda_0 = -1^\circ\ 7'\ 7''.6$$
$$\varphi_0 = 50^\circ 25' 34''.0.$$

Calculation of the angle at which the limiting-lines of the area of internal contact on the surface of the earth intersect each other in C_0.

The equations which are satisfied at the limiting-lines, are:

$$\Phi_1^2 + \Phi_2^2 = u^2 \quad \text{and} \quad \Phi_1 \frac{\partial \Phi_1}{\partial T} + \Phi_2 \frac{\partial \Phi_2}{\partial T} = u \frac{\partial u}{\partial T}$$

if in the expression for u we replace the angle f by f_i.

The first equation follows from HANSEN's fundamental equations. When in that equation the expressions for Φ_1, Φ_2, and u have been substituted, it gives the relation between λ, φ_I and the moment at which the internal contact occurs at the place of observation ($\lambda\varphi_I$). In the limiting-lines this equation will hold for two consecutive moments. From this condition follows the second equation which is derived from the first by differentiation with respect to the time.

At the point C_0 the three functions Φ_1, Φ_2, and u are equal to zero for the time $T_0 + 2^m 33^s.9$. In the vicinity of this point at a small distance L and at a time differing from the first by the small quantity T, it is sufficiently accurate to use linear relations for the computation of the 3 functions. Hence we may put $\Phi_1 = \left(\frac{\partial \Phi_1}{\partial L}\right)_0 L + \left(\frac{\partial \Phi_1}{\partial T}\right)_0 T$ and the same will do for Φ_2 and u.

As all partial derivatives appearing in these equations relate to the point O, I shall henceforth simply write $\frac{\partial \Phi_1}{\partial L}$, etc. instead of $\left(\frac{\partial \Phi_1}{\partial L}\right)_0$, etc.

After the substitution of the linear expressions for φ_1, Φ_2, and u, the equations of the limiting-lines become:

$$\left\{\left(\frac{\partial \Phi_1}{\partial L}\right)^2 + \left(\frac{\partial \Phi_2}{\partial L}\right)^2 - \left(\frac{\partial u}{\partial L}\right)^2\right\} L^2 + 2\left(\frac{\partial \Phi_1}{\partial L}\frac{\partial \Phi_1}{\partial T} + \frac{\partial \Phi_2}{\partial L}\frac{\partial \Phi_2}{\partial T} - \frac{\partial u}{\partial L}\frac{\partial u}{\partial T}\right) LT +$$
$$+ \left\{\left(\frac{\partial \Phi_1}{\partial T}\right)^2 + \left(\frac{\partial \Phi_2}{\partial T}\right)^2 - \left(\frac{\partial u}{\partial T}\right)^2\right\} T^2 = 0$$

and

$$\left(\frac{\partial\Phi_1}{\partial L}\frac{\partial\Phi_1}{\partial T}+\frac{\partial\varphi_2}{\partial L}\frac{\partial\varphi_2}{\partial T}-\frac{\partial u}{\partial L}\frac{\partial u}{\partial T}\right)L+\left\{\left(\frac{\partial\varphi_1}{\partial T}\right)^2+\left(\frac{\partial\varphi_2}{\partial T}\right)^2-\left(\frac{\partial u}{\partial T}\right)^2\right\}T=0$$

We now eliminate from the two equations the relation of L to T and thus we obtain an equation between the differential co-efficients which after reduction becomes:

$$\left(\frac{\partial\Phi_1}{\partial L}\frac{\partial\Phi_2}{\partial T}-\frac{\partial\Phi_2}{\partial L}\frac{\partial\Phi_1}{\partial T}\right)^2=\left(\frac{\partial\Phi_1}{\partial L}\frac{\partial u}{\partial T}-\frac{\partial u}{\partial L}\frac{\partial\Phi_1}{\partial T}\right)^2+\left(\frac{\partial\Phi_2}{\partial L}\frac{\partial u}{\partial T}-\frac{\partial u}{\partial L}\frac{\partial\Phi_2}{\partial T}\right)^2 \quad (VIII)$$

In developing the terms of this equation, I assume the linear element L to lie in an arbitrary azimuth A on the surface of the earth transformed into a sphere.

Then we find:

$$\frac{\partial\varphi_I}{\partial L}=\cos A \quad \text{and} \quad \frac{\partial\lambda}{\partial L}=\sec\varphi_I \sin A$$

Further we have $\frac{\partial(t+\Delta\alpha')}{\partial\lambda}=1$ and for $\frac{\partial(t+\Delta\alpha')}{\partial T}=\frac{d(\tau+\Delta\alpha')}{dT}$ I put $\varkappa$.

By differentiation we find for the derivatives with respect to L:

$$\frac{\partial\Phi_1}{\partial L}=\sin\varphi_I\sin(t+\Delta\alpha')\cos A-\cos(t+\Delta\alpha')\sin A$$

$$\frac{\partial\Phi_2}{\partial L}=-\{(1-c)\cos\delta'\cos\varphi_I+\sin\delta'\sin\varphi_I\cos(t+\Delta\alpha')\}\cos A-\sin\delta'\sin(t+\Delta\alpha')\sin A$$

$$\frac{\partial u}{\partial L}=-tg f_i\{(1-c)\sin\delta'\cos\varphi_I-\cos\delta'\sin\varphi_I\cos(t+\Delta\alpha')\}\cos A+tg f_i\cos\delta'\sin(t+\Delta\alpha')\sin A$$

and for those with respect to T:

$$\frac{\partial\Phi_1}{\partial T}=\frac{d\mathfrak{p}}{dT}-\varkappa\cos\varphi_I\cos(t+\Delta\alpha')$$

$$\frac{\partial\Phi_2}{\partial T}=\frac{d\mathfrak{Q}}{dT}-\varkappa\,\mathfrak{p}\sin\delta'+(Z+s\,\mathrm{cosec}\,f_i)\frac{d\delta'}{dT}$$

$$\frac{\partial u}{\partial T}=tg f_i\left(\frac{dZ}{dT}+\varkappa\,\mathfrak{p}\cos\delta'-\mathfrak{Q}\frac{d\delta'}{dT}\right)-s\,\mathrm{cosec}\,f_i\,\frac{df_i}{dT}.$$

The last two derivatives have been simplified by means of the relations $\Phi_1=0$, $\Phi_2=0$ and $u=0$, which hold good for the point C_0.

Attributing to the linear element L the azimuthal direction of the central line on the sphere A_c, and denoting the derivatives with respect to L in this case by $\frac{\partial\Phi_1}{\partial L_c}$ et seq., we find the following equation for the determination of A_c:

$$\frac{\partial\Phi_1}{\partial L_c}\frac{\partial\Phi_2}{\partial T}-\frac{\partial\varphi_2}{\partial L_c}\frac{\partial\varphi_1}{\partial T}=0 \quad . \; . \; . \; . \; . \; . \; . \; . \quad (IX)$$

Substituting the following relations:

$$\frac{\partial\varphi_1}{\partial T} = -\frac{\partial\varphi_1}{\partial L_c}\frac{L_c}{T} \text{ and } \frac{\partial\varphi_2}{\partial T} = -\frac{\partial\varphi_2}{\partial L_c}\frac{L_c}{T},$$

from which the equation (IX) can be derived, in the expression in brackets in the first member of (VIII), we get

$$\left(\frac{\partial\Phi_1}{\partial L_c}\frac{\partial\Phi_2}{\partial L} - \frac{\partial\Phi_2}{\partial L_c}\frac{\partial\Phi_1}{\partial L}\right)\frac{L_c}{T}$$

Here $\frac{\partial\Phi_1}{\partial L_c}$ and $\frac{\partial\Phi_2}{\partial L_c}$ are the same functions of A_c as $\frac{\partial\Phi_1}{\partial L}$ and $\frac{\partial\Phi_2}{\partial L}$ are of A. We further find by developing:

$$\frac{\partial\Phi_1}{\partial L_c}\frac{\partial\Phi_2}{\partial L} - \frac{\partial\Phi_2}{\partial L_c}\frac{\partial\Phi_1}{\partial L} = \{-\sin\delta' \sin\varphi_I - (1-c)\cos\delta' \cos\varphi_I \cos(t+\Delta\alpha')\}\sin(A-A_c)$$

The expression between $\{\}$ is equal to $-w'z_I$ according to former notations, hence:

$$\frac{\partial\Phi_1}{\partial L}\frac{\partial\Phi_2}{\partial T} - \frac{\partial\Phi_2}{\partial L}\frac{\partial\Phi_1}{\partial T} = -w'z_I \sin(A-A_c)\cdot\frac{L_c}{T} \quad . \; . \; . \quad (X)$$

Employing the numerical values which I needed for calculating the position of the point of transition, I first solved the equation (IX), from which I found:

$$A_c = 50°.5'46''03.$$

Subsequently I replaced in the equation, which followed from (VIII), the angle A by $\Delta = A - A_c$, and obtained for the calculation of Δ an equation of the following form:

$$c_0^2 \sin^2\Delta = (c_1 \sin\Delta + d_1 \cos\Delta)^2 + (c_2 \sin\Delta + d_2 \cos\Delta)^2$$

with the following numerical values of the co-efficients:

$$c_0 = 6138''.9 \qquad c_1 = -7''.24 \qquad c_2 = -10''.24$$
$$d_1 = +9''80 \qquad d_2 = -13''.09$$

For the solution of this equation I first computed the auxiliary quantities $C = c_0^2 - c_1^2 - c_2^2$, $D = d_1^2 + d_2^2$ and $E = 2(c_1 d_1 + c_2 d_2)$; afterwards the angles B and Γ from the formulae:

$$\sin B = +\sqrt{\frac{4CD + E^2}{(C+D)^2 + E^2}} \text{ and } tg\,\Gamma = \frac{E}{C+D}$$

and finally found these two solutions:

$$\Delta_1 = +\tfrac{1}{2}(B+\Gamma) \text{ and } \Delta_2 = -\tfrac{1}{2}(B-\Gamma)$$

The numerical calculation led to:

$$\Delta_1 = +0°9'8''.1 \text{ and } \Delta_2 = -0°9'10''.3$$

So the central line almost bisects the angle between the limiting-lines in C_0. East of this point the north limit deviates a little more from the central line than the south limit. The angle between the two limiting-lines in point C_0 is $\Delta_1 - \Delta_2 = B$; so that for this angle the result of the calculation is: $B = 0°18'18''.4$.

Measured on the surface of the sphere the distances of C_4 and C_5 from C_0 amount to 2338″ and 3985″, so that according to the adopted data the width of the annular zone at these points comes respectively to 12″.4 and 21″.2 or 380 and 650 meter.

From relation (X) we find co-efficient $c_0 = w' z_I \frac{Lc}{T}$; accordingly $\frac{L_c}{T} = \frac{c_0}{w' z_I} =$ the velocity V_0, with which in C_0 the shadow-axis moves along the central line over the surface of the earth. For points in the central line near C_0 the duration of the totality or annularity is given by:

$$\frac{2\left(V_0 \frac{\partial u}{\partial L} + \frac{\partial u}{\partial T}\right)\sqrt{\left(\frac{\partial \varphi_1}{\partial T}\right)^2 + \left(\frac{\partial \varphi_2}{\partial T}\right)^2}}{\left(\frac{\partial \varphi_1}{\partial T}\right)^2 + \left(\frac{\partial \varphi_2}{\partial T}\right)^2 - V_0^2\left(\frac{\partial u}{\partial L}\right)^2} \times T.$$

The value of this expression is in casu: 0.00542 T and from this it follows that the annularity lasts $0^s.47$ in C_4 and $0^s.80$ in C_5.

Calculation of the differential coefficients of the geographical longitude and latitude of the points in the central line with respect to the longitude, latitude, and parallax of the moon.

In these calculations we may neglect the very slight variations of the quantities $\Delta a'$, δ' (or δ'_I) and h in consequence of small variations in the places of the moon. In the differentiation I have therefore treated them as constant quantities.

From formula (IV) for $tg\,(t + \Delta a')$ we deduce

$$\frac{2d\lambda}{\sin 2(t + \Delta a')} = \frac{d\mathfrak{P}}{\mathfrak{P}} - \frac{dY}{Y}, \text{ where } \mathfrak{P} = M\cos(H + h) \text{ (see form. IV)}$$

and $Y = N\cos(U + \delta'_I) = z_I \cos\delta'_I - \frac{\Omega}{w'}\sin\delta'_I$, hence

$$dY = \cos\delta'_I\, dz_I - \frac{\sin\delta'_I}{w'}\, d\Omega$$

From the relation $z_I^2 = 1 - \mathfrak{P}^2 - \frac{\Omega^2}{w'^2}$ we obtain

$dz_I = -\frac{\mathfrak{P}}{z_I}\, d\mathfrak{P} - \frac{\Omega}{w'^2 z_I}\, d\Omega$, and, substituting in the formula for $d\lambda$, the values of the differentials dY and dz_I, we find $d\lambda$ expressed in $d\mathfrak{P}$ and $d\Omega$ as follows:

$$\frac{2\,d\lambda}{\sin 2\,(t + \Delta a')} = \left(\frac{1}{\mathfrak{P}} + \frac{\mathfrak{P}\cos\delta'_I}{Y z_I}\right) d\mathfrak{P} + \frac{X}{Y z_I w'}\, d\Omega$$

where $z_I \sin \delta'_I + \frac{\mathfrak{Q}}{w'} \cos \delta'_I = X = N \sin (U + \delta'_I) = \sin \varphi_I$

In order to profit as much as possible by the quantities already calculated I substitute $tg(U + \delta'_I)$ for $\frac{X}{Y}$ and $tg(t + \Delta\alpha')$ for $\frac{\mathfrak{P}}{Y}$.

By employing the differential relations

$$d\mathfrak{P} = \cos h\, dP - \sin h\, dQ \text{ en } d\mathfrak{Q} = \sin h\, dP + \cos h\, dQ$$

and by introducing the auxiliary angle μ, variable between $-90°$ and $+90°$, which is determined by

$$tg\, \mu = w' \cot g\, (U + \delta'_I) \left\{ \frac{z_I}{\mathfrak{P}} + \cos \delta'_I\, tg\, (t + \Delta\alpha') \right\} \qquad \text{(XIa)}$$

we may reduce the differential expression for $d\lambda$ to:

$$d\lambda = \frac{Sec\, \mu}{2w'z_I} \sin 2(t + \Delta\alpha')\, tg\, (U + \delta'_I) \{\sin (\mu + h)\, dP + \cos (\mu + h)\, dQ\} \qquad (XIb)$$

For the deduction of the corresponding differential expression for the geographical latitude, it is necessary to eliminate $d\varphi_I$, $d\mathfrak{Q}$, and $d\mathfrak{P}$ between the equations

$$\frac{d\varphi}{\sin 2\varphi} = \frac{d\varphi_I}{\sin 2\varphi_I}$$

and

$$\cos \varphi_I d\varphi_I = dX = \left(\frac{\cos \delta'_I}{w} - \frac{\mathfrak{Q} \sin \delta'_I}{w'^2 z_I} \right) d\mathfrak{Q} - \frac{\mathfrak{P} \sin \delta'_I}{z_I}\, d\mathfrak{P}$$

and the expressions formerly found for $d\mathfrak{Q}$ and $d\mathfrak{P}$. To simplify the second equation I employed the relation

$$\frac{\cos \delta'_I}{w'} - \frac{\mathfrak{Q} \sin \delta'_I}{w'^2 z_I} = \frac{Y}{w' z_I}$$

The introduction of the auxiliary angle v variable between $-90°$ and $+90°$, according to the formula

$$tg v = \frac{w' \mathfrak{P} \sin \delta'_I}{Y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XIIa)$$

also simplifies the calculations. We then find:

$$d\varphi = \frac{\sin 2\varphi}{\sin 2\varphi_I} \frac{\cos (t + \Delta\alpha')}{w' z_I \cos v} \{-\sin (v - h)\, dP + \cos (v - h)\, dQ\} \,(XIIb)$$

For the calculation of the differentials dP and dQ I have employed the following approximate formulae

$$dP = Z\, dl - P\, tg\, b\, db - P \cot g\, \pi\, d\pi \quad . \quad . \quad . \quad (XIII)$$

$$dQ = \qquad Z\, db - Q \cot g\, \pi\, d\pi \quad . \quad . \quad . \quad (XIV)$$

The numerical values of the partial derivatives of λ and φ with respect to l, b, and π have been calculated for the two instants $T_0 + 4$ min. and $T_0 + 5$ min. This calculation gave the following result:

for $T_0 + 4$ min.		for $T_0 + 5$ min.	
$\frac{\partial\lambda}{\partial l} = + 94.7$	$\frac{\partial\varphi}{\partial l} = + 25.8$	$\frac{\partial\lambda}{\partial l} = + 96.0$	$\frac{\partial\varphi}{\partial l} = + 25.7$
$\frac{\partial\lambda}{\partial b} = - 18.6$	$\frac{\partial\varphi}{\partial b} = + 73.7$	$\frac{\partial\lambda}{\partial b} = - 17.3$	$\frac{\partial\varphi}{\partial b} = + 74.1$
$\frac{\partial\lambda}{\partial \pi} = - 21.2$	$\frac{\partial\varphi}{\partial \pi} = - 50.2$	$\frac{\partial\lambda}{\partial \pi} = - 23.2$	$\frac{\partial\varphi}{\partial \pi} = - 50.6$

For a given longitude we find for the differential variation of the latitude of the central line $(d\varphi) = d\varphi - d\lambda \cos\varphi \operatorname{cotg} A \frac{\sin^2\varphi}{\sin^2\varphi_I}$, where A denotes the azimuth of that line in the point (λ, φ). In the present case we find for the two instants

$$(d\varphi) = d\varphi - 0.516\, d\lambda \quad \text{and} \quad (d\varphi) = d\varphi - 0.504\, d\lambda$$

From the above mentioned values of the partial derivatives of φ and λ with respect to l, b, and π we finally derive for the time $T_0 + 4$ min.:

$$(d\varphi) = - 23.1\, dl + 83.3\, db - 39.3\, d\pi,$$

and for the time $T_0 + 5$ min.:

$$(d\varphi) = - 22.7\, dl + 82.8\, db - 38.9\, d\pi.$$

As may be seen from these differential formulae, it is in particular the latitude of the moon that influences the situation of the central line. The empirical correction of $-1''.0$, applied by HANSEN to the latitude of the moon diminishes with 1'23" the geographical latitudes of the central line of this eclipse in our country; this corresponds to a transposition of 2,5 K.M. towards the south.

If it appears that corrections are necessary for the places of the moon accepted by me for these calculations, there will be no difficulty in computing their influence on the position of the central line. I myself am not able to give any corrections with any certainty. The only thing would be to take into account the influence of the height above sea-level, owing to which the geographical latitudes for given longitudes are about $2''.1$ per 100 M. smaller.

Postscript.

When I commenced these calculations Prof. E. F. v. D. SANDE BAKHUYZEN's investigations into some points, regarding the longitude and latitude of the moon [1]), were not yet completed. In the meantime the results he arrived at appeared to agree well with the values I employed. The sum of all the terms of short period in the

[1]) Proc Acad. Amst. **14**, 1912, p. 686 et seq.

longitude according to BROWN, which were also calculated, amounts, however, when we add to it the influence of the adopted corrections of perigee and eccentricity, to $+ 0''.67$, i. e. $1''.10$ less than the value I derived from the originally adopted formula. The previous investigation of Prof. BAKHUYZEN into the observations of the moon up to 1902 led him to believe that the co-efficient of *sin g*, which is of special importance here (as we have $g = 278°$) would be in 1912 at least $- 0''.6$ greater than its theoretical value, using also the adopted values for the corrections of perigee and eccentricity. From this remark a value of the longitude results, which is $+ 0''.6$ greater, so that the most probable correction of the longitude I employed would be $- 0''.5$. Thus we should have $dl = - 0''.5$ and $db = - 0''.05$, hence $(d\varphi) = + 7''$. In more than one respect, however, some uncertainty remains.

Anatomy. — *"On the structure of the Dental system of Reptiles."* By Prof. Dr. L. BOLK.

(Communicated in the meeting of January 27, 1912).

If we compare the dental system of mammals with that of reptiles two points of difference come especially to the front. One of them bears a more physiological character and regards the fact that with mammals, as a rule, the dental shelf gives off only two series of buds, one for the milk-set and another for the permanent set. With the majority of reptiles on the contrary a shedding of teeth takes place several times during their life-time, though among this group of vertebrates species are known to us, where the shedding of teeth does not take place at all, or is restricted to a special part of the dental system, as may likewise be the case with mammals. These however are exceptions and in general the dental system of mammals as diphyodont (in some cases monophyodont) is placed over against that of reptiles as polyphyodont. It is pretty well the current view that the diphyodontism of mammals must be derived from the polyphyodontism of the lower vertebrates, and it is supposed that the number of renewals of the dental system was gradually reduced from many to a few, whilst the duration of the existence of the teeth was lengthened. Only a few authors take a different stand-point, and are of opinion that the shedding of teeth of mammals should be a property obtained by that group of animals themselves, and the primitive mammals should consequently have been monophyodont (LECHE).

The second point of difference is of a more morphological nature. The dental system of mammals namely is, save a few exceptions,