Geophysics. ' — *"On the semi-diurnal lunar tide as deduced from records' of the astatic seismograph at Batavia".* By Dr. C Braak. (Communicated by Dr. J. P. van der Stok).

(Communicated in the meeting of April 29, 1910).

1. Since the beginning of December 1908 an astatic seismograph of the Wiechert pattern with a mass of about 1000 KG. has been in working order at Batavia. This instrument being very sensitive to change of inclination, it was rational to inquire whether the tidal motion of the earth, investigated by Hecker [1]), could be deduced from the records, because tidal forces at Batavia are considerably stronger than at Potsdam.

A preliminary inquiry having afforded satisfactory results, the records obtained during the half year July to December were used for this investigation which, for the present, is restricted to the E-W component of the principal semi-diurnal linear tide.

The seismograph consists of a cylindrical heavy load of 900 KG. weight, resting on a foot of 70 cm. length, and is pivoted on four steel lamels by means of which a Cardanic freedom of motion is ensured. The masonry column is free from the floor and rests on a broad foundation. The building is situated with its longer dimensions in the N-S-direction and is sheltered from the sun's rays by galleries on the east and west side.

The influence of the undulatory motion of the ground due to the sun's heat is evident in the diagrams, principally in those of the E-W-component; this disturbance is so strong that only the records at the hours 7 p.m. to 8 a.m. can be used for this inquiry ; on days of strong disturbance even some of these records had to be omitted.

As there is no basis-line, the distances between the records at 7^h and 10^h, 8^h and 11^h etc. were measured out and, in order to eliminate, at least partially, variations of the zero-point and other disturbances of long duration, the differences were taken between the distances at 13^h to 10^h and 10^h to 7^h, which differences, but for a small correction, correspond to the tidal motion at 10^h with twice the amplitude. The tidal motion is evident in these data even at first sight.

2. For the half year July to December 1909 these data were

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[1]) O. Hecker. Beobachtungen an Horizontalpendeln über die Deformation des Erdkörpers unter dem Einflusz von Sonne und Mond. Veröffentl. des Kön. Preuss. Geod. Inst. Neue Folge n⁰. 32. 1907.

2

arranged according to lunar hours. The records for the months April, May and June might have been used too, but they have been discarded because the determination of the sensitiveness of the instrument during these months seemed to be insufficienttly accurate.

As a control the data were arranged separately for each hour; in this way eight independent series of numbers were obtained for 10^h p.m. to 5 a.m., from which the following results for the lunar tide have been deduced; the amplitude is expressed in mm.

10^u p.m.	0.605	cos $(2 t—253° 52')$	118
11	0.748	cos $(2 t—261° 9')$	134
12	0.501	cos $(2 t—266° 21')$	146
1 a.m.	0.500	cos $(2 t—277° 28')$	159
2	0.451	cos $(2 t—256° 19')$	167
3	0.625	cos $(2 t—239° 41')$	167
4	0.650	cos $(2 t—234° 10')$	154
5	0.732	cos $(2 t- 245° 57')$	98

The numbers behind the formulae denote the corresponding number of observations; taking the weight of each result proportionate to these numbers in calculating the average value of amplitude and argument, we find

$$0.5913 \; cos \; (2t — 254°33').$$

To the amplitude a correction must be applied because the variation of the argument within 3 hours is not 90° but $3 \times 28.°98$; therefore the amplitude must be divided by

$$1 — cos \, (3 \times 28.°98) = 0.9465.$$

As the deflections have been measured out corresponding to the hour-signal, and this is given 5.5 minutes before Batavia mean time, the argument has to be diminished by 2°40', so that the final expression becomes

$$0.6248 \, cos \, (2t — 251°53').$$

3. The sensitiveness of the seismograph has been determined after two different methods.

1. From the weight and the dimensions of the different parts of the instrument the total weight and the height of the centre of gravity above the pivot were deduced, and the deflections of the pointer, caused by small weights being put on the upper side of the bob, were measured out.

2. By determination of the period of vibration and direct measurement of the indicator-magnification.

The deflections were determined twice monthly; for this purpose a weight of 5 grammes was placed at a distance of 342.5 mm. from the vertical passing through the centre of gravity, first on the east side, then on the west side and then again on the east side. The apparatus does not assume immediately its equilibrium position but shows some sluggishness. Therefore in each experiment the weight was left on the bob for half an hour in the same position. In this way the following values were obtained for the double deflection given in mm.

2 July	21.5		17 Oct.	21.0
14 „	22.0		3 Nov.	19.7
29 ..	20.2		16 „	19.7
14 Aug.	20.9		4 Dec.	19.1
3 Sept.	20.8		17 „	19.9
18 „	20.4		3 Jan.	19.4
8 Oct.	21.0			

or, on the average; 20.43 mm.

The height of the centre of gravity was found to be 895 mm., the total weight of the pendulum 985 K.G.; hence the change of inclination corresponding with 1 m.m. on the diagram is

$$\frac{2 \times 5 \times 342.5 \times 206265}{20.43 \times 985000 \times 895} = 0'' 03923$$

For the period of vibration the following determinations were made

July	10.0,	10.0,	10.2,	10.0	mean	10.0⁵	sec.
Aug.	10.0,	10.2,	10.2,	10.4	„	10.1	„
Sept.	10.0,	10.0,	10.0		„	10.0	„
Oct.	10.0,	9.8,	9.0		„	9.6	„
Nov.	9.7,	9.7			„	9.7	„
Dec.	9.7,	9.6			„	9.6⁵	„

from which the average value

9.85 sec.

And the equivalent length of penduluan

24.04 meters.

In order to determine the indicator-magnification each of the horizontal adjusting screws, by means of which the upper side of the pendulum can be clamped, was alternately screwed down so as to ensure good contact with the pendulum and then turned 90° to the right and left. The deflections on the diagram were:

2*

60.3, 60.2, 60.1 and 60.2 mm.

mean value 60.2 mm.

corresponding with 0.25 of the screw's thread

$= 0.3517$ mm.

The adjusting screws are 122.5 c.m., the centre of gravity is 89.5 c.m., above the pivot; hence the indicator-magnification becomes

$$\frac{60.2 \times 122.5}{0.3517 \times 89\ 5} = 234.3$$

and the change of inclination corresponding with 1 mm. :

$$\frac{206265}{234\ 3 \times 24040} = 0''.03661$$

For the N—S component the two methods gave respectively

$0''.04515$ and $0''.04376$

For either component the second method leads to somewhat smaller values than the first, on the whole the results are fairly congruent.

The systematic difference might be ascribed, on the one hand to a remaining influence of sluggishness in determining the deflections, on the other hand to a small increase of the period of vibration with the amplitude. Neither effect could be proved practically.

If the small weight is left for some time on the pendulum, disturbances of different kinds occur which are large in proportion to the small differences to be measured, on the other hand there is a limit to the accuracy to be obtained in determining the time of vibration owing to the pretty rapid decrease of the amplitude; the effect of the two influences is however in an opposite direction. It has further been proved that the deflections for the amplitudes as used are proportionate to the applied weights.

E. g. for 5 grammes the deflections were:

15.4, 15.5, 15.3, 15.2, 15.5, 15.6 mean 15.42 m.m.

for 2.5 gr.

7.7 7.7 7.6 7.6 7.8 7.8 mean $\dfrac{15.40}{2}$ m.m.

Probably of the two methods of determining the sensitiveness the second is to be preferred. The fact that for the E—W pendulum, more sluggish than the N—S pendulum, a difference is found larger than for the latter, seems to indicate that, in applying the first method, the effect of sluggishness was not quite eliminated.

Moreover, the determinations after the second method are the less complicate; therefore the value $0.''03661$ has been taken as a base for the neat calculations.

The E-W component of the semi-diurnal lunar tide is then represented by the formula

$$0.''01144 \; cos \; (2t - 251°53').$$

4. The amplitude of the theoretical tide, on the assumption that the earth is perfectly rigid, is

$$\frac{3m}{2M}\left(\frac{a}{r}\right)^3 cos \; \phi \; cos^4 \frac{I}{2}\left(1 - \frac{5}{2} e^2\right)$$

m and M denoting the mass of moon and earth, a and r the radii of the earth and the moon's orbit, ϕ the latitude, I the obliquity of the moon's orbit to the equator and e the excentricity of the moon's orbit. The assumed values are:

$$\frac{m}{M} = \frac{1}{81.4} \; , \; \frac{a}{r} = \frac{1}{60.27} \; . \; \phi = 6°11' \; , \; I = 25°35'$$

and $e = 0.055$.

The lunar hour 0 corresponds with the time of the moon's upper transit.

Finally we find for the theoretical tide :

$$0.''0155 \; cos \; (2t - 270°)$$

and for the real tide :

$$0.''0114 \; cos \; (2t - 251°53').$$

Mathematics. — "*Infinitesimal iteration of reciprocal functions.*" By M. J. VAN UVEN. (Communicated by Prof. JAN DE VRIES).

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§ 1. A function $\varphi(x)$ will be called a *reciprocal function of order* n, when it satisfies the functional equation

$$\varphi_n(x) = \varphi \left[\varphi \left\{ \ldots \; \varphi(x) \ldots \right\}\right] = x.$$

$$\underbrace{\qquad\qquad}_{n}$$

The solution of this equation is known by the name of "the problem of BABBAGE" [1].

In what follows we shall occupy ourselves exclusively with the reciprocal functions of order 2 which therefore satisfy

$$\varphi_2(x) = \varphi \left\{\varphi(x)\right\} = x, \; \ldots \ldots \ldots \; (1)$$

and which for short we shall call reciprocal functions.

The solution of the problem of BABBAGE shows us that the functional equation (1) must be satisfied by all the functions $y = \varphi(x)$

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[1] See inter alia LAURENT: Traité d'analyse t. VI, Paris 1890, p. 243.