Physics. — "*The magnetic separation of absorption lines in connection with Sun-spot spectra*". (Second Part)[1]). By Prof. P. ZEEMAN and Dr. B. WINAWER.

(Communicated in the meeting of April 29, 1910).

21. The outer components of a magnetically divided line, if observed in a direction inclined to the lines of force under an angle ϑ, are elliptically polarized.

In our experiments of §§ 12--17 we frequently referred to this elliptical polarization. In § 12 were resumed the simple rules, which relate to the ellipses characterizing the state of polarization of the outer components, if *very narrow* spectral lines are observed in a *strong* field.

The linear vibrations of the central component of a triplet lie according to the elementary theory in the plane passing through the ray and a line of force, and the amplitude is proportional to $\sin\vartheta$.

RIGHI's theoretical considerations in his paper cited in § 1 also agree with this conclusion.

22. In VOIGT's[2]) theoretical investigation of the magnetic effect in a direction inclined to the lines of force, the remarkable conclusion is drawn that also the central component of a triplet may execute an elliptical vibration. This result is most closely connected with the taking into account of the mutual action between neighbouring molecules.

LORENTZ's considerations concerning our present subject (cf. § 1 above) give results which we may be permitted to summarize here briefly.

For arbitrarily chosen values of the angle ϑ between the ray and the magnetic force for every frequency two elliptical vibrations of opposite directions can be transmitted.

In the case of the *outer* components of a sharp triplet one of the two elliptic vibrations is absorbed.

If we have not to deal with a sharp triplet, i.e. three absorption bands that are completely separated, we can still say something about the vibration ellipses of the outer components.

Let axes OY and OX' be chosen, the one normal to the plane

[1]) Continuation of the paper published in these Proceedings. Vol. XII. p. 584. 1910.

[2]) W. VOIGT, Weiteres zur Theorie der magneto-optischen Wirkungen. Ann. d. Phys. I. (1900) p. 389.

3*

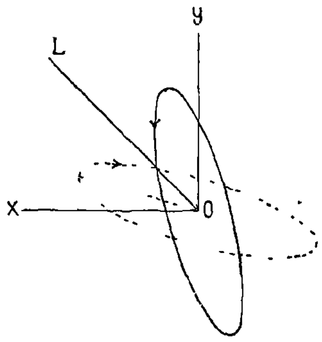

Fig. 1.

passing through the ray and the magnetic force, the other perpendicular to the ray and lying in the plane just mentioned. Then one of the characteristic vibration ellipses can be considered as the reflected image of the other with respect to a line bisecting the angle $X'OY$. This rule also applies to the direction of motion in the two ellipses.

The nature of the phenomena that will be observed for rays of a frequency corresponding to the *central* line of the triplet, depends upon the value of ϑ being greater or smaller than a certain angle ϑ_1. This latter is determined by the equation

$$tang\ \vartheta_1\ sin\ \vartheta_1 = \frac{g}{v}.$$

The quantity g may be regarded as a measure of the width of an absorption line and depends upon the constants of the vapour; v is determined by the change of the frequency of the free vibrations of the electrons and has a value proportional to the strength of the field.

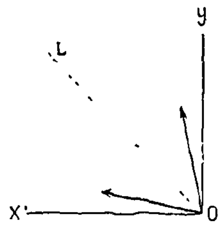

Fig. 2.

If $\vartheta > \vartheta_1$, then two linearly polarized beams with equal indices of refraction and different absorption indices can be propagated. The rectilinear vibrations make equal angles with the line OL bisecting the angle XOY'. The absorption is strongest for the beam whose vibrations make the smaller angle with the direction of the field. In the figure the most strongly absorbed vibration is indicated by a thicker arrow.

As ϑ decreases the vibrations of the two principal beams approach more and more to OL, so that for $\vartheta = \vartheta_1$ both directions coincide with the bisectrix. The two principal beams are now equally absorbed also.

When $\vartheta < \vartheta_1$, the state of things is wholly different.

In this case two elliptically polarized beams can be propagated; they are equally absorbed, but have different velocities of propagation. For both beams the characteristic ellipses are the same, but described in opposite directions. One of the axes of the ellipses coincides with the line OL in Fig. 2.

The ellipses become less and less eccentric as the wave becomes less inclined to the direction of the field. For $\vartheta = 0$ the ellipses become circles described in opposite directions.

A further approximation for $\vartheta = \vartheta_1$ shows, that in this case the two vibrations do not coincide exactly. As in the general case there are two distinct beams with different characteristic ellipses, both deviating somewhat from the line OL of Fig. 2.

The regions of the longitudinal and the transverse magnetic effect overlap to a certain extent and are not sharply separated from each other at the angle ϑ_1.

23. There are three results of LORENTZ's theory that probably admit of experimental verification.

Let us imagine the absorbing vapour placed in such circumstances that the elementary theory cannot be applied. The components of a divided line are now not neatly separated by practically transparent regions. The vapour density must be chosen relatively great and the magnetic intensity rather small. As always in the present paper we suppose the lines of force to be horizontal; we examine the propagation of the light also in a horizontal plane.

The three predictions referred to and which apply, if we exclude the cases of the true longitudinal and transverse effects, are:

1st. the major axes of the vibration ellipses of the outer components deviate from the vertical line.

2nd. the vibrations of the middle component (c.q. components) are, depending on circumstances, either linear and not horizontal or elliptic, the axes of the ellipse being inclined to the horizon.

3rd. there exists an angle ϑ_1 separating the regions of the longitudinal and the transverse magnetic effect.

Oblique position of the vibration ellipses of the outer components.

24. We succeeded establishing experimentally the oblique position of the vibration ellipses in the inverse magnetic effect of the D-lines; the amount of the slope of the axes we could measure.

The obliquity is far from striking.

When ϑ was already such that the ellipticity was very marked, we first only after some difficulty could make sure of the obliquity.

Some details of a definite case may be given. With $\vartheta = 69°$ and a field of about 18000 gauss the first observations were made.

Attention was given to D_2, the vapour density being regulated so that the outer components of the sextet could not be seen separately. Before the slit of the spectroscope a Nicol was placed with its plane of vibration under an azimut of say 35° with the horizon. The central part of the resolution figure is now very dark, the outer components of the pseudo-triplet however are only faintly visible. This has the advantage to increase the visibility of small changes of the intensity of the outer components.

The direction of the field we denote as field direction 1.

With the reversed field direction 2, the outer components became darker.

This experiment was repeated several times with the same result.

The Nicol then was placed in a position symmetrical to the one just mentioned. Now with field direction 1, the outer components were darker. From these experiments we must conclude that a vertical line is not an axis of symmetry of the vibration ellipses of the outer components, hence that the position of these ellipses is oblique.

25. The direction of the smaller axis of the vibration ellipse we measured for $\vartheta = 69°$, the vapour density being between the first and second phase (§ 13). Before the slit of the spectroscope a Nicol was introduced, mounted upon a divided circle, which gives the rotation of the Nicol in degrees. The vanishing or reappearing of the outer components gave a good criterion for the determination of the smaller and therefore of the major axis of the vibration ellipse.

The measurements gave the result that under the circumstances of the experiment the major axis made an angle of 5 degrees with the vertical. The obliquity was the same in amount and direction for the components towards the red and towards the violet. The diagram Fig. 3 illustrates the relation between the slope of the ellipses and the direction of the field.

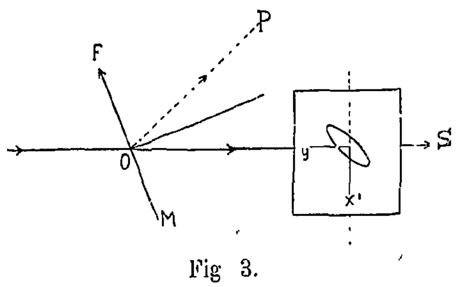

Fig 3.

Let OS be the beam, which traverses the source of light placed in O and OF the direction of the magnetic force. For an observer looking in the direction SO, the upper part of the vibration ellipse is inclined towards the right. The plane YX' containing the ellipse is normal to the ray and in the figure has been rotated round the dotted line until brought into coincidence with the plane SOM. That side of the plane which was visible from S, can now be seen. Both the ellipse towards the red and the ellipse, described in opposite direction, towards the violet have the same slope with a given direction of the magnetic field, as was remarked above.

26. *The same* sodium flame investigated as to the inverse effect in the direction OS, we studied in the direction OP (i.e, for an angle $FOP = MOS = 180° - \vartheta$) for the phenomenon of partial polarization, discovered by EGOROFF and GEORGIEWSKY. A small telescope focussed upon the flame was used and provided with a SAVART plate and a Nicol. This polariscope is mounted upon a divided circle graduated in degrees. The direction in which the fringes were most brilliant was determined in order to detect a possible deviation of the plane of maximum polarization from the vertical. It was easily seen that there was such a deviation. The fringes were most clear if for the observer in P their direction was from the upper left to the lower right quadrant, the direction of the field being always as indicated in the figure. After reversal of the magnetic field the fringes became indistinct. They became distinct again if the principal direction of the polariscope was from the upper right to the lower left quadrant. The result of these observations at least proves that the whole phenomenon is asymmetrical with respect to the vertical and hence proves the presence of oblique vibrations. In a conversation with one of the authors Prof. LORENTZ had kindly communicated that he observed phenomena of the kind described in this §.

27. In the experiment of the last § the axis of the telescope must be placed carefully in a horizontal plane passing through the poles of the electromagnet. If the observation is made in a plane which is not horizontal an apparent slope of the axes of the vibration ellipses becomes manifest, as is easily seen from a geometrical consideration.

28. The position of the plane of maximum polarization can be determined rather accurately. The obliquity of the major axis of the outer ellipses of sextet and quartet in one experiment was 5°; with the very same vapour density and the same strength of field the plane of partial polarization made an angle of 21° with the vertical.

At first sight it seems rather startling that the polariscope of SAVART is so sensitive to the obliquity of the ellipses.

The phenomenon of the partial polarization of the emitted light is very complicated and the complete theory still outstanding.

It seems not doubtful, however, in what direction we have to look for the explanation of the remarkable difference between the indications of the two instruments. They measure different quantities.

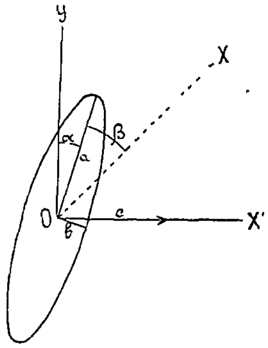

Fig. 4.

As long as the inclination of the vibration ellipses of the emitted light is zero, the total light also vibrates symmetrically relatively to the vertical.

If the inclination is not zero, however, but has the value α, the plane of maximum resultant luminous motion is inclined at an angle $\alpha + \beta$, which may be occasionally much greater.

The light emitted by the sodium flame contains:

1. horizontal vibrations of intensity c^2. (We neglect here a change mentioned in § 30 below).

2. elliptic vibrations, the major axes of which form an angle α with the vertical. Let the principal axes of these ellipses be a and b.

The intensity I_x in a direction $O\ X$ becomes

$$I_x = c^2 \sin^2 (\alpha + \beta) + a^2 \cos^2 \beta + b^2 \sin^2 \beta \quad . \quad . \quad . \quad . \quad (1)$$

This expression becomes a maximum for a value of β satisfying

$$c^2 \sin 2(\alpha + \beta) + (b^2 - a^2) \sin 2\beta = O. \quad . \quad . \quad . \quad . \quad (2)$$

Hence it follows already that β cannot be zero, for otherwise α ought to be zero also.

From (2) we obtain

$$\frac{\sin 2(\alpha + \beta)}{\sin 2\beta} = \frac{a^2 - b^2}{c^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence the value of β depends upon the intensities of the horizontal and vertical vibrations. Always $a > b$; in the emitted light the vertical vibrations generally preponderate hence also $a > c$. We conclude that β can only be positive.

If we take $\alpha = 5^\circ$ $\alpha + \beta = 21^\circ$ $b = 0{,}3\ a$ (cf. § 29) equation (3) gives

$$\frac{a^2}{c^2} = 1{,}4.$$

This is a plausible value. Hence there is no contradiction between the observations made with the polariscope and the results obtained with the Nicol alone.

29. We made, with the inverse effect, some measurements of the ellipticity of the outer components at different angles of incidence. We used for this investigation the well known method of the quarter wave plate and Nicol. The axes of the quarter wave

plate being placed parallel to the axes of the original ellipse, the resulting light is plane polarized. Let b and a be the horizontal and vertical or the nearly horizontal and the nearly vertical axes then

$$\frac{b}{a} = tang\ \alpha.$$

The mica quarter plate used proved to be very accurate for light of the refrangibility of the sodium lines, when tried by the method described on a former occasion [1] Three determinations gave for the deviation from an exact quarter wave plate the values 1,8, 0,1, 1.0 %.

For our present determinations this accuracy of the plate is quite superfluous. The measurements are very difficult, relating as they do to the mean of the outer components of the sextet, hence to an extremely narrow part of the spectrum. Moreover the density of the vapour can be defined only approximately (10).

The following table embodies the results concerning the ellipticity of the outer components of the sextet obtained in a somewhat extended series of measurements.

ϑ	b/a		Remarks
69° ½	0.31 0.31 0.28	0.30	Vapour of intermediate density (§ 10)
47°	0.45 0,45	0.45	”
	0.47 0.50	0.47	Vapour somewhat denser
39°	0.67 0.70 0.70 0.60 0.64 0.67 0.63 0.65 0.65	0.66	Very dilute vapour (§ 10)

The ratio of the axes at a certain angle undoubtedly somewhat depends upon the vapour density. Part of the oscillations of the results obtained at the same angle must be described to this cause.

At $\vartheta = 69°\,{}^1/_2$ and with dense vapour the inclination of the major

[1]) ZEEMAN, These Proceedings October 30. 1909.

axis of the ellipse was 6°; with very dilute vapour the value zero was obtained.

At $\vartheta = 47°$ and with vapour of intermediate density the inclination was 4° ½. The SAVART fringes then made an angle of 28° with the vertical.

Oblique position of the vibrations of the middle components.

30. Whereas the inclination of the vibration ellipses of the outer components could be demonstrated first for the sextet, it was for the quartet on the contrary we first succeeded in verifying the second of LORENTZ's above mentioned conclusions (23).

The deviation of the vibrations of the middle components of the quartet from the horizontal line can be shown in the same manner as the inclination of the ellipses (24).

The principal section of the Nicol before the slit was placed at an angle of about 30° with the horizon. The outer components of the quartet of D_1 are then hardly visible. The inner components are rather dark. The direction of the field be indicated as direction 1. Under the influence of the reverse field 2, the middle components become more black. If the Nicol be placed in the symmetrical position then it is with the field direction 1 that the middle components are most distinct.

The angle ϑ in this experiment was 47°.

Two different attempts to measure the angle between the vibration and the horizon gave the results 4° ½, resp. 5° ½. These measurements are very difficult, however and perhaps indicate only the order of magnitude of the inclination. The vicinity of the outer components largely interferes with the accuracy of the adjustment of the Nicol, for while it is moved about near the position of extinction and approaches to a vertical direction the greater intensity of the outer components distracts the eye.

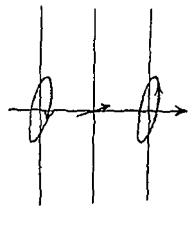

Fig. 5.

31. We have made yet another experiment which confirms the result of (30) for both the sodium lines and also exhibits the relation between the inclinations of the different components. This connection is for a triplet diagrammatically shown in Fig. 5. For the result obtained with the middle components of the quartet and the sextet certainly can be applied qualitatively to the triplet.

The experiment was the following: the principal section of the Nicol made an angle of +40° with the vertical; the positive direction in Fig. 5 be anti-clockwise. Then the Nicol was placed at 320° (i.e. in the symmetrical position). The last position be indicated as position *B*, the first mentioned as position *A*.

The direction of the field remains unchanged.

In position *A* all lines were weaker than in position *B*.

Hence we conclude that the ellipses as well as the vibrations of the middle components are inclined, moreover that the relative position of the vibrations must be that shown in Fig. 5.

32. In the important paper already frequently mentioned RIGHI (Note p. 291 of the paper cited § 1 above) says that VOIGT's theoretical investigation of the general case of propagation of light in a direction inclined to the lines of force was published too late to guide him in his investigation. RIGHI expresses the opinion that it is rather improbable that in the course of his numerous observations particulars in the behaviour of the middle components as indicated by VOIGT could have escaped him and that LORENTZ's elementary theory is in accordance with all the observed phenomena.

This seems in contradiction with our experiments. This contradiction vanishes, however, if we assume that the vapour in RIGHI's experiments was very dilute, or the field so intense that the components were neatly separated. Under such circumstances also our observations are in complete accordance with the elementary theory, at least as to the polarization of the components and the direction of the vibrations.

Neither was it in RIGHI's experiments a matter of course to reverse the direction of the magnetic field, the procedure which most easily exhibits any obliquity of the vibrations.

Application of the results of §§ 24—31 to the interpretation of sunspot spectra.

33. The vibrations of the middle component of a triplet are parallel to the lines of force. The outer components vibrate linearly at right angles to the field. These rules also apply to dense vapours, if only the pure transverse magnetic effect be under consideration. If we assume that the direction of observation is oblique to the lines of force then only in the case of very dilute vapours the projection of the magnetic force on a plane normal to the line of vision can be found according to the rules of the elementary theory from the direction of the vibrations. If, however, the components of an inverse triplet are not neatly separated by practically transparent parts, — and the sun-spot lines seem to belong to this class of lines, — the particulars diagrammatically illustrated by Fig. 5 are to be taken into consideration.

In drawing charts of the magnetic fields in sun-spots, showing the intensity, the direction and the polarity of the magnetic force, the determination of the direction of the force will give some difficulties.

The value of the correction to the indications of the elementary theory necessary in some cases shall be given on another occasion.

The rule, which determines the direction of the deviation, may be indicated here.

The direction of rotation in the vibration ellipses of the outer components towards the red and towards the violet shows whether ϑ is acute or obtuse. If ϑ is obtuse (Fig. 3), then the relative position of the directions of the magnetic force, of the major axis of the vibration ellipses and of the vibration of the middle component is shown in Fig. 5.

From any point O draw a line OB parallel to the major axis of the vibration ellipses of the outer components and a line OM parallel to the vibration of the middle component, the angle BOM being always chosen acute. The projection OF of the magnetic force on a plane normal to the line of sight then makes a positive acute angle with OB, the angle BOF being greater than BOM, the positive direction being reckoned from OB to OM.

By ascertaining whether or not the major axes of the ellipses and the vibrations of the middle component are perpendicular to each other we can make sure whether the elementary theory may be applied or not.

Mathematics. — *"On linear polar groups belonging to a biquadratic plane curve".* By Prof. JAN DE VRIES.

(Communicated in the meeting of April 29, 1910).

1. With respect to a biquadratic curve γ^4, with the symbolic equation $a_x{}^4 = 0$, the points X, Y, Z, W form a polarquadruple when the relation $a_x a_y a_z a_w = 0$ is satisfied. If we take X, Y, Z arbitrarily on a line r and if we take as fourth point W the point of intersection of r with the "triple polar line" p_{xyz} of X, Y, Z, we get a "linear" polarquadruple. The linear polarquadruples on r evidently form an involution $I_3{}^4$, its *"principal points"* (fourfold elements) are the points of intersection of γ^4 with r.

If we assume for the points on r such a parameter representation that two of its principal points are indicated by $\lambda = 0$ and $\lambda = \infty$ we find for the groups of the $I_3{}^4$ the relation

$$\sum_4 \lambda_1\lambda_2\lambda_3 + p\sum_6 \lambda_1\lambda_2 + q\sum_4 \lambda_1 = 0, \quad \ldots \quad (1)$$