to one and the same fluid phase. On continued elevation of temperature only incidental intersections in the P-x projection remain.

In the P-T-projection the three-phase line S_A-S_B-Fluid will, therefore, have always positive values for $\frac{dp}{dT}$ [1]; there exists a temperature traject where the pressure at constant temperature is trivalent; the stable-metastable branch is connected with the labile branch by means of a cusp. The connection is indicated in the P-T-projection of fig. 6.

Physics. — "*The magnetic separation of absorption lines in connection with sun-spot spectra.*" (Third Part) [2]. By Prof. P. ZEEMAN and Dr. B. WINAWER.

Demonstration of oblique position of vibrations by means of half wave-length plate.

34. The observations published in our two preceding communications relate to the region between $\vartheta = 90°$ and $\vartheta = 39°$, the two principal directions inclusive. We now intend to describe in this third, conclusive, part of our paper experiments relative to the remaining region between $\vartheta = 39°$ and 0°.

This region seemed very interesting because under suitably chosen circumstances it probably would contain the angle ϑ_1 of LORENTZ, separating the regions of the longitudinal and the transverse magnetic effect. The principal object we had in view in undertaking this third part of our investigation was to prove experimentally the existence of an angle of the kind mentioned. We think we attained our purpose.

Before proceeding to describe these experiments, we shall mention a method for verifying the results (24—32) relating to the oblique position of the vibration ellipses of the outer components and that of the vibrations of the inner components, but without commutation of the current in the electromagnet.

Whereas in our former experiments the *difference* of the intensity of the components by *commutation* of the current gives the proof for the obliquity of the components, the half wave-length plate demonstrates it at once.

A half wave-length plate with one of its principal directions situated horizontally and limited by a horizontal line is placed near the source

[1]) A quite analogous view may be applied to the coincidence of I and K.

[2]) Continued from these Proceedings Vol. XIII p. 35, 1910.

of light. Vibrations from the source, making a definite angle with the edge of the plate, after traversing it are rotated through twice that angle. The plate covers only half of the field of view. The directions of the emergent vibrations make the same angles with the horizontal edge as at first, but upon the further side.

An image of the edge is focussed upon the slit of the spectroscope; before the slit a Nicol is placed.

In one of our experiments, ϑ being 39°, the plane of vibration of the Nicol was under an angle of 35° with the horizon. The magnetic components are now seen unequally dark in the two halves of the field of view. It appeared possible to photograph the phenomenon; small variations of vapour density, which may possibly introduce errors with other methods of observation, are now without influence.

Reversion of the direction of the current, changes the sign of the difference of intensity of the two halves of the field of view.

Connection between the inclination of the ellipses in particular cases.

35. The direction of the magnetic field, and that of propagation of the beam, traversing the magnetized source of light, determine the sense of the inclination of the vibration ellipses (25). If the direction of the field be reversed, the sign of the inclination of the vibration ellipses also changes. In fig. 3 (§ 25) the connection established by our experiments, between the three mentioned directions is given.

Fig. 6.

Let OF be the magnetic force, and let the beam, traversing the magnetized flame O, be propagated in the direction from O to S. The inclination of the ellipses in this case is indicated in fig. 6. The plane normal to the ray and containing the ellipse has been rotated round the dotted line until brought into coincidence with the plane of the paper.

What is the inclination, if the source of light be traversed by the beam in the direction OS'?

This question is easily answered by applying the well-known method of reflected images.

The geometrical outlines of all things composing a given system, together with the physical processes in the system, which we suppose may be all represented by geometrical figures, we imagine reflected at every instant in a plane V. The new system obtained by reflection and which we call the image of the original system is a possible one, as soon as the last mentioned one has an objective existence.

Applying this to our experiment (fig. 7) and placing the plane V parallel to OF and perpendicular to the plane of the paper we obtain from system I, the system II.

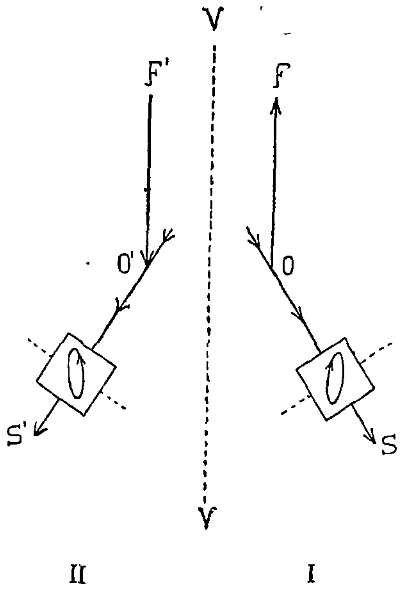

Fig. 7.

The magnetic field in the second system is the inverted image of the field in the first one; indeed, before taking the image of the field we have to substitute it by the equivalent Ampère currents.

Hence in II the arrow $F'O'$ is drawn from F' to O'.

Reversing afterwards the field in system II, the inclination of the ellipse changes its sign.

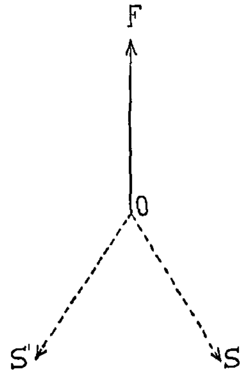

Fig. 8.

Hence we conclude that (fig. 8), if OF be the direction of the magnetic field, the inclination of the major axes of the ellipses, as observed from S as well as from S', is always from the lower left to the upper right quadrant.

By means of SAVART's polariscope all this could be experimentally verified.

We come to the same conclusion by using the experimental result of § 26, concerning the inclination of the ellipses in the beam emitted in the direction OP (see fig. 6).

The close connection, existing between emission and absorption, enables us to predict the phenomena to be seen if light traverses the source in the direction OS'. (cf. § 44).

Investigation concerning the existence of an angle ϑ_1 (§ 36—§ 46).

36. It seems possible to give by different ways experimental proof of the existence of an angle ϑ_1, separating the regions of the longitudinal and of the transverse effect.

The most direct proof would be given, if, with a chosen magnetic force, the vapour density could be changed in such a degree, that at last the direction of the vibrations in the issuing beam were inclined at an angle of 45° with the vertical. Then one would observe at the angle ϑ_1 itself, the values of density (width) and magnetic intensity corresponding. The following up of this plan gives rise however to serious difficulties.

The significance and the particularity of the angle ϑ_1 become however manifest also, if it be possible to establish the existence of the characteristic phenomena only observable for a direction of observation which forms an angle with the lines of force lying between 0° and ϑ_1. We have experimentally verified the theoretical inference.

We made many experiments belonging to each of the two classes of experiments mentioned and intend to give a few examples of each.

37. *Observations at* $\vartheta = 32°$. Soft iron cones with a vertex semiangle of 32° were made and adapted to a DU BOIS-electromagnet. The intensity of the magnetic field proved sufficient to establish the character of the resolution in the first order spectrum of the large ROWLAND grating.

The middle components were especially watched. It is easily established that the vibrations of these components deviate from the horizon. In order to demonstrate an inclination of 45°, a quartz plate, cut perpendicularly to the axis, and exactly 2 mm. thick, was introduced in the beam. This plate rotates the plane of polarization for sodium light $2 \times 21.7 = 43.4°$. Vibrations under azimut 45°, after traversing the plate, become either horizontal or vertical.

Between the plate and the spectroscope slit a calcspar rhomb was inserted and a horizontal slit placed near the source; two contiguous horizontal images of the slit are now formed on the slit. The one contains the vertical, the others the horizontal constituents of the beam.

The middle components, which at the angle ϑ under consideration are rather weak, are, dependent upon the direction of the current, visible either only in the upper or only in the lower of the two stripes, if the vapour density be properly chosen.

This experiment does not prove however definitely that the middle components may vibrate under an angle of 45° with the vertical. The rather limited sensitiveness of the method must be taken into account.

The experiment certainly proves that the vibrations are inclined relatively to the horizon, at an angle of perhaps 20° or 30°.

It is shown by an observation with the calcspar rhomb alone, after removal of the quartz plate, that the vibrations are not performed under 45°. A difference between the upper and the lower image is now manifest. This would be impossible, if the inclination of the vibrations were 45°. The difference of intensity in the two stripes decreases with increased density of the vapour.

All experiments undertaken in order to measure more accurately the inclination gave no decisive results. The weak intensity of the middle components, the feeble separation (just wanted according to theory for the observations in view), the perturbation by the vicinity of the outer components, and also the fact that the vibrations become probably slightly elliptic, account for the difficulty of the measurements.

We also investigated the emitted light without the aid of the spectroscope, with a SAVART polariscope alone; the emitted light appeared to be nearly unpolarized. The fringes in the polariscope were very weak. This is clearly due to the light containing equal portions of right-handed and left-handed nearly circularly polarized light; the intensity of the light of the middle components is relatively very small and therefore scarcely perceptible in the resulting total intensity.

The indistinctness of the fringes made only inaccurate determinations of the position of the plane of polarization possible. An inclination of 42° relatively to the vertical was found.

38. The method of the non-uniform field[1]) seemed to open the possibility of a direct reading of the field intensity corresponding to ϑ_1, the vapor density (i.e. the width of the spectralline) being given. At $\vartheta = 39°$, a diminished image of the cones of the electromagnet was focussed upon the slit plate of the spectroscope. The magnetic separation is different at different heights and in the spectroscope the spindle-shaped resolution figure, a photograph of which was given on a former occasion, is seen; but now, as the inverse effect is under consideration, rather dark lines on a luminous background are seen. A Nicol with its plane of vibration under 45°

[1]) ZEEMAN. These Proceedings April 1906, November 1907.

with the horizon is placed before the slit. If the vibrations occur under 45° somewhere in the divided lines, the components must become black at such a place. Width and field intensity, belonging to the mentioned part of the components, correspond to a value of ϑ_1 equal to 39°.

No clear result was obtained however by means of this method, which was tried with several vapour densities.

The change of the state of polarization in the resolution figure apparently is too gradual to prove the existence of ϑ_1 by direct observation.

Our following experiments (39—46), indeed, seem to leave no doubt as to the real existence of such an angle.

39. In order to extend observations to still smaller angles ϑ, the second order spectrum of the great ROWLAND grating was employed for all following observations. The brightness is still largely sufficient and more details are seen. Even with cones with a vertex semi-angle of 26° the characteristic phenomena may now be advantageously observed. With vapour of intermediate density (10) now only the outer components of the quartet and sextet are visible, the phenomenon closely resembling the pure longitudinal one. Middle components only make their appearance after the density is largely increased. The nature of these components appears (40) however to have changed, as is proved by an examination of their state of polarization.

The latter is more easily ascertained, if the components are more widely separated. This is the case in the experiments described in the next paragraphs and therefore we prefer to give some details of the observations made with the more efficient arrangement.

40. A still smaller angle between the directions of the beam and of the field may be employed and moreover wider separation obtained than in § 39, by looking through axial holes and deviating the beam in the field by means of two small prisms. A remark of Prof. WERTHEIM SALOMONSON induced us to give prisms a trial.

The arrangement for $\vartheta = 16°$ is shown in the next figure.

The prisms are fixed to copper tubes, which are put into the bored cones of a DU BOIS electromagnet and may be turned about their axes. It is therefore possible to adjust the parallelism of the planes of prisms and to arrange vertically the edges.

A drawback inherent to this method is, that after some time the interior surfaces of the prisms become covered with some white

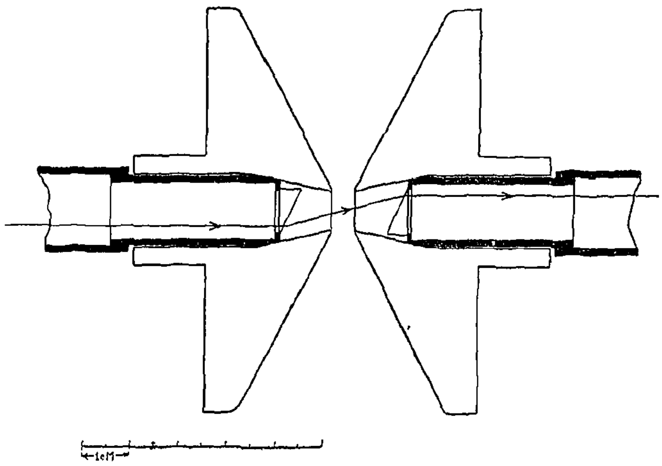

Fig. 9.

precipitate. With very dense vapours this inconvenience is rather troublesome.

Immediately after introduction of the flame into the interferrum aqueous vapour condenses upon the prism faces, soon disappearing, however, when the temperature of the prisms has increased. In order to avoid the danger of cracking, the prisms have been disposed at some distance from terminal planes of the cones.

Even with very dense vapour (third phase of § 10), the field being of the order of 20.000 Gauss, the phenomenon closely resembles the pure longitudinal one. No trace of middle components is visible.

After an increase, however, of the vapour density to the limit obtainable by the introduction of a glass rod, charged with melted salt, into the gas-oxygen flame, two new black lines appeared in the vicinity of D_1; they were clearly visible against the rather dark background formed by the broadened outer components.

These new lines, which have the same period as the middle components, are unpolarized (see 41—44).

41. We have come to this conclusion after trying in vain to detect any trace of polarization phenomena of the new components.

In the first place rotation of a Nicol, placed before the slit of the spectroscope, gave no change of intensity of the lines; only the background formed by the nearly, but not accurately, circularly polarized outer components was slightly changed.

42. After removal of the Nicol a quarter wave plate with its

principal direction under 45° was inserted in the beam and a broad horizontal slit placed near the field. By means of a calcsparrhomb two stripes are obtained, separating the oppositely polarized circular vibrations.

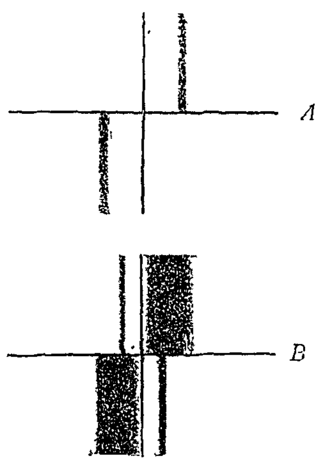

Fig. 10.

With vapour of intermediate density fig. 10^A gives the appearance for D_1. The vertical line represents the reversed line due to the arc light.

With very dense vapour, we get the phenomenon represented in fig. 10^B. New components appear in the initially bright parts of the field of view.

The positions of the new components correspond to those of the inner components of the quartet, at least as far as can be judged by eye observation. This observation is confirmed by measurements made on a photograph of, it must be said, only moderate quality.

As to the polarization of the new lines a few remarks may be made. From an inspection of fig. 10^B alone, one might conclude to a circular polarization of the inner components, of a sign opposite to that of the outer ones.

One might be tempted to infer that, under the circumstances of the experiments, the inner components are due to the motion of positive charges.

There is no need discussing the degree of probability of such a conclusion, as it is refuted by the next observation.

42. If the quarter wave plate be rotated in its own plane so that the principal direction more and more approaches to the horizontal position, the intensity of the outer components decreases. The inner components, which at first are invisible in two of the quadrants, being entirely hidden by the black, broad, outer components, are seen, already soon, as continuous bands crossing at right angles the horizontal separation line.

Finally, when the principal direction of the quarter wave plate has become horizontal, there is, as far as concerns the inner components, no difference at all between the upper and lower fields, and only a slight one as far as concerns the outer components.

43. From the observations recorded in §§ 41 and 42 we cannot

but conclude that under the circumstances of the experiment the inner components of the *new* quartet are unpolarized.

This result seems paradoxical, because one now has become accustomed to expect polarization of all magnetically separated and displaced lines.

The result, however, seems to be in perfect accordance with theory, at least if it be permitted to apply to the middle components of the quartet, the theoretical inference drawn for the central component of the triplet.

LORENTZ has proved that in the case of a triplet for a frequency $n = n_0$ and $\vartheta < \vartheta_1$, two oppositely elliptically polarized beams may be transmitted, having the same index of absorption, but unequal velocities of propagation. The characteristic vibration ellipses for the two beams are the same, but described in opposite directions. (see also § 22 above).

Since the indices of absorption of the two beams are equal, we may expect that, under the circumstances mentioned, a magnetized vapour can produce in a continuous, unpolarized, spectrum only unpolarized absorption lines.

44. The consideration in § 35 of the reflected image of a system, was made in order to show that the inclination of the ellipses remains unaffected by a change of sign of the angle between the line of force and the ray.

45. *Quartet for* $\vartheta = 0$. By increasing still further the vapour density necessary for the § 43 experiment, we were able to observe even in the direction $\vartheta = 0$, the two unpolarized lines, corresponding to the inner components of the quartet. The outer components, however, have become then extremely diffuse.

It is certainly remarkable, that the two new components are still relatively narrow. The theoretical reason for this feature of the phenomenon has still to be worked out.

It is, however, in accordance with theory (always on the supposition that it does apply directly to the quartet) that for $\vartheta = 0°$ the density of the vapour must exceed that for $\vartheta = 16°$, in order to render visible the new lines. Indeed according to the formulae (42) and (26) of LORENTZ's paper (cited in § 1 above) the absorption index decreases with decreasing ϑ.

The experiments (39—43) seem to give conclusive evidence that an angle ϑ_1 really exists.

Indeed, phenomena of the kind described in the last §§ are to be expected in a region only between ϑ_1 and 0°.

The experimental verification of LORENTZ's deductions, formulated in § 23 above, gives a new proof of the rational connexion established by VOIGT's theory of the inverse magnetic effect between diverse phenomena.

A more accurate measurement of ϑ_1, the vapour density and the field being chosen, must be postponed.

46. The new type of magnetic separation, with some components polarized, the other ones unpolarized, which returns to the ordinary separation by decrease of vapour density, we were able to observe also with D_2. Since the density of the vapour must be great in the present experiment, the effects observed with D_2, which splits up into a pseudo-triplet, are less clear and characteristic than with D_1. We, therefore, restricted the detailed description of our observations to the case of D_1.

Mathematics. — *"On continuous vector distributions on surfaces"* (3[rd] communication)[1]). By Dr. L. E. J. BROUWER. (Communicated by Prof. D. J. KORTEWEG).

(Communicated in the meeting of May 28, 1910).

§ 1.

The irrigating field on the sphere.

In order to get an insight into the structure of an arbitrary finite continuous vector field with a finite number of singular points on the sphere over its entire extent, we begin by investigating a particular case characterized by the *absence of simple closed tangent curves.*

In a field which possesses this property, and which we shall call an *irrigating field,* no spirals can appear as tangent curves and no rotation points as singular points. As farthermore a singular point can neither possess elliptic sectors or leaves, it is either a source point without leaves, or a vanishing point without leaves, or it possesses exclusively hyperbolic and parabolic sectors without leaves, in which case we shall speak of a *stroking point.*

The singular points of an irrigating field cannot all be stroking points. This follows from theorem 8 of the second communication [2]) in

1) For the first and second communication see these Proceedings Vol. XI 2, p. 850 and Vol. XII 2, p. 716.

2) l. c. p. 734.