Citation:

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Above I have briefly described the structure of the nucleus and the details of the karyokinesis in *Closterium Ehrenbergii*. As appears from what has been said the nucleus, as far as the nucleoli are concerned, does not agree with the nuclei of *Spirogyra*, as earlier investigators have supposed. In this respect the nucleus of *Closterium* differs on an important point from those of *Spirogyra*, namely, it does not possess a nucleolus which may be identified as or compared with a nucleus. The nuclei agree with nuclei, which are generally found among plants, especially the higher plants. Nevertheless they show one peculiarity: the nucleoli which are indeed present in great numbers, form in the middle of the nucleus a conglomeration.

The nucleus divides by karyokinesis or mitosis. All phenomena which generally occur, also take place in *Closterium*. In *Closterium* the nuclear division presents the following particulars: the distribution of the nucleoli in the nucleus and their extrusion into the cytoplasm, the great flat equatorial plate, the great number of chromosomes which is more than 60, the different length of the chromosomes, which in general are short and of which the longer ones only have free ends, protruding sidewards, the wide, feebly developed spindle and the translocation of the daughter-nuclei along the cellwall.

Later I hope to give a more detailed account of the karyokinesis in *Closterium* and to illustrate with figures the above mentioned results. In this paper hardly anything has been said about the cell division and the growth of the cellwall. To this I also hope to refer later.

Mathematics. — *"On the relation between the vertices of a definite sixdimensional polytope and the lines of a cubic surface".* By Prof. P. H. Schoute.

1. In his investigation about semiregular polytopes, and polytopes possessing a higher degree of regularity Mr. E. L. Elte, whose dissertation is to appear shortly has met with a sixdimensional polytope of degree of regularity $\frac{3}{4}$ with 27 vertices. Our aim here is to point out the complete correspondence in relations of position between the 27 vertices of this polytope and the 27 lines of a cubic surface.

The symbol of the characteristic numbers of this polytope is

$$(27,\ 216,\ 720,\ 1080,\ 432 + 216,\ 72 + 27),$$

i. e. the polytope has 27 vertices, 216 edges, 720 faces, 1080 limiting bodies, 648 fourdimensional limiting polytopes and 99 fivedimensional ones. Here the numbers 27, 216, 720, 1080 between the brackets are left undivided, as the corresponding elements are of the same kind: all the edges have the same length and — with respect to the whole figure — the same position, all the faces are equal equilateral triangles in the same position, all the limiting bodies are equal regular tetrahedra in the same position. On the other hand the 648 equal regular fivecells forming the fourdimensional limiting polytopes split up according to their position into two groups, while the 99 fivedimensional limiting polytopes consist of 72 regular simplexes $S(6)$ with six vertices and 27 regular cross polytopes $Cr(10)$ with ten vertices; of the 648 fivecells 432 are common to an $S(6)$ and a $Cr(10)$, the remaining 216 to two $Cr(10)$.

2. In order to be able to enter into our subject immediately we start from the 27 points with the coordinates

$$
\begin{array}{cccccc l}
0 & 0 & 0 & 0 & 0 & -\frac{4}{3}\sqrt{3} & \ldots\ldots a_0, \\
(1 & -1 & -1 & -1 & -1) & -\frac{1}{3}\sqrt{3} & \ldots\ldots 5a_i \\
1 & 1 & 1 & 1 & 1 & -\frac{1}{3}\sqrt{3} & \ldots\ldots b_0 \\
(-1 & -1 & 1 & 1 & 1) & -\frac{1}{3}\sqrt{3} & \ldots\ldots 10c_{ik} \\
(2 & 0 & 0 & 0 & 0) & \frac{2}{3}\sqrt{3} & \ldots\ldots 5b_i \\
(-2 & 0 & 0 & 0 & 0) & \frac{2}{3}\sqrt{3} & \ldots\ldots 5c_{0i}
\end{array}
\left.\begin{array}{l} \\ \\ \\ \\ \\ \\ \end{array}\right\},
$$

In this scheme the symbols $a_0, a_1, \ldots a_5, b_0, c_{12}, \ldots c_{45}, b_1, \ldots b_5, c_{01}, \ldots c_{05}$ of the last column represent the points in a transparent manner; moreover this notation is entirely the same as that generally used for the 27 lines of the cubic surface. Indeed, if — by means of the well known formula for the distance of two points with given coordinates — it has been shown, that any of the 27 points is at distance $2\sqrt{2}$ from 16 and at distance 4 from 10 other points and it has been found for each of the 27 vertices which are the 16 adjacent ones and which the 10 remote ones, it is immediately evident that in using the same symbols a, b, c for the 27 vertices of the polytope and the 27 lines of the cubic surface two *adjacent* vertices (edge distance $= 2\sqrt{2}$) always correspond to two *crossing* lines, two *remote* vertices (diagonal distance $= 4$) always correspond to two intersecting lines. We will show that this correspondence leads to simple geometrical results; but to this end we have to know the projections of the new polytope on different axes of symmetry.

3. All the 27 vertices are at the same distance $\frac{4}{3}\sqrt{3}$ of the origin. So the origin is the centre of the polytope and all its axes of symmetry pass through this point.

The projection of the polytope on the axis OX_6 passing through the vertex a_0 can be deduced immediately from the coordinates. It has been given in the known manner in fig. 1. Moreover the List I gives the names of all the edges, faces, etc.

From this projection $(1, 16, 10)$ it is evident that a limiting cross polytope $Cr(10)$ is oppositely placed to the vertex a_0. We say that these elements are *rightly* opposite to each other, as the line from the vertex to the centre of the polytope passes if produced through the centre of the opposite cross polytope $Cr(10)$.

4. We repeat in fig. 2^a the position of the 27 vertices in the projection $(1, 16, 10)$ and indicate now how the other projections $(2, 10, 10, 5)$, etc. given there have been obtained. We thereby enter into detail with respect to the first new case $(2, 10, 10, 5)$ of 2^b, where the axis passing through the midpoint of the edge a_1a_2 is the axis of projection.

The coordinates of the midpoint of the edge a_1a_2 are

$$0,\quad 0, -1, -1, -1, -\frac{1}{3}\sqrt{3}.$$

So

$$\frac{(x_3 + x_4 + x_5)\sqrt{3} + x_6}{\sqrt{10}} = const.$$

is the standard equation of any space S_5 perpendicular to the axis under consideration. The constant of the second member takes for the groups of vertices (a_1a_2), $(a_0a_3a_4a_5\, c_{03}c_{04}c_{05}c_{34}c_{35}c_{15})$, $(b_1b_2\, c_{01}c_{02}c_{13}c_{14}c_{15}c_{23}c_{24}c_{25})$, $(b_0b_3b_4b_5\, c_{12})$ indicated in fig. 2^b successively the values $-\frac{10}{\sqrt{30}}$, $-\frac{4}{\sqrt{30}}, \frac{2}{\sqrt{30}}, \frac{8}{\sqrt{30}}$; by means of these values the position of the

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points of the axis where the $2, 10, 10, 5$ vertices project themselves, with respect to the origin indicated by the dotted vertical line, is easily found.

The centre of gravity of the regular fivecell $(b_0 b_3 b_4 b_5 c_{12})$, lying opposite to the edge $(a_1 a_2)$, i.e. the point with the coordinates

$$0, 0, \frac{4}{5}, \frac{4}{5}, \frac{4}{5}, \frac{4}{15}\sqrt{3},$$

is situated on the axis of projection. So the edge $(a_1 a_2)$ and the fivecell $(b_0 b_3 b_4 b_5 c_{12})$ are *rightly* opposite to each other. From the number 216 of the edges it follows, that each of the opposite fivecells must be common to two cross polytopes $Cr(10)$; really the fivecell opposite to the edge $(a_1 a_2)$ is common to the two $Cr(10)$ opposite to the vertices a_1, a_2.

5. In an analogous manner the other projections are found.

So fig. 2^c deals with the case of the axis passing through the centre of the face $a_1 a_2 a_3$. The standard equation

$$\frac{x_1 + x_2 + x_3 + 3(x_4 + x_5) + x_6\sqrt{3}}{2\sqrt{6}} = const.$$

corresponding to this case gives for the groups of vertices

$$(a_1 a_2 a_3), (a_0 a_4 a_5 c_{04} c_{05} c_{45}), (c_{01} c_{02} c_{03} c_{14} c_{24} c_{34} c_{15} c_{25} c_{35}), (b_1 b_2 b_3 c_{12} c_{13} c_{23}), (b_0 b_4 b_5)$$

successively the values $-\frac{4}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, 0, \frac{2}{\sqrt{6}}, \frac{4}{\sqrt{6}}$ of the constant. So we find the projection $(3, 6, 9, 6, 3)$, showing that the faces of the polytope are placed in pairs rightly opposite to each other. For the centre of gravity of the triangle $b_0 b_4 b_5$ lies on the axis of projection.

So fig. 2^d treats the case of the axis through the centre of the tetrahedron $a_1 a_2 a_3 a_4$, by means of the standard equation $\frac{(x_1 + x_2 + x_3 + x_4 + 2x_5)\sqrt{3} + 2x_6}{2\sqrt{7}} = const.$ and of the values $-\frac{7}{\sqrt{21}}, -\frac{4}{\sqrt{21}}, -\frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{5}{\sqrt{21}}, \frac{8}{\sqrt{21}}$ of the constant, the projection $(4, 3, 8, 6, 4, 2)$. Here the edge $b_0 b_5$ corresponding to the value $\frac{8}{\sqrt{21}}$ is placed *obliquely* opposite to the tethrahedron $a_1 a_2 a_3 a_4$, for the midpoint of the edge does not lie on the axis of projection. A closer examination shows that in this manner each edge is placed obliquely opposite to *five* limiting tetrahedra, i.e. to the five limiting tetrahedra of the fivecell placed rightly opposite to the edge. In accordance to this the number 1080 of the limiting tetrahedra is five times that of the edges.

Farthermore fig. 2^e gives the projection on the axis passing through the centre of the fivecell $a_1a_2a_2a_4a_5$ common to the simplex $a_0a_1a_2a_3a_4a_5$ and a cross polytope $Cr\,(10)$. The standard equation is

$$\frac{3(x_1+x_2+x_3+x_4+x_5)\sqrt{3}+5x_6}{4\sqrt{10}}=const.,$$

the values of that constant are $-\frac{8}{\sqrt{30}}, -\frac{5}{\sqrt{30}}, -\frac{2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{7}{\sqrt{30}}, \frac{10}{\sqrt{30}}$; the fact that in this arithmetical series the term $\frac{4}{\sqrt{30}}$ is lacking will be accounted for in a natural way later on. The opposite point b_0 lies obliquely opposite to the fivecell from which we started. A closer investigation shows the following. There are — we have already stated this — 216 fivecells, each of which is common to two $Cr(10)$; i.e. of the 27×32 limiting fivecells of the cross polytopes 432 cover each other by pairs, while the 432 remaining ones are covered by the 72×6 limiting fivecells of the $S(6)$. So the 32 limiting fivecells of each $Cr(10)$ are coloured alternately white and black, if we call a fivecell in contact with an $S(6)$ white, a fivecell in contact with a $Cr(10)$ black; now each vertex is obliquely opposite to the 16 white limiting fivecells of the $Cr(10)$ rightly opposite to it. Indeed the number 432 of the fivecells common to two five-dimensional polytopes of different kind is 16 times the number of vertices.

Finally 2^f represents the case of the axis through the centre of gravity of the simplex $a_0a_1a_2a_3a_4a_5$. To this corresponds the standard equation $\frac{x_1+x_2+x_3+x_4+x_5+x_6\sqrt{3}}{2\sqrt{2}} = const.$ with the values $-\sqrt{2}, 0, \sqrt{2}$ of the constant and the simple projection (6, 15, 6) of the points a, c, b given in fig. 3, while the List II gives the names of all the limiting elements[1]). This projection shows us that the 72 limiting $S(6)$ are placed by pairs rightly opposite to each other.

6. Before we consider the obtained projections in connexion with the 27 lines of a cubic surface it will be well to extend our terminology by putting side by side the simpler corresponding properties of the two systems of 27 objects. In this comparison "the polytope"

[1]) Here the subscript 0 has been replaced by 6.

stands for the figure with the 27 vertices, "the configuration" for the 27 lines of the cubic surface.

The polytope has	The configuration has
216 edges and 135 diagonals, lying three by three in 45 planes;	216 twocrossers and 135 twointersecters, the points of intersection of which form 45 triangles;
720 triangular faces, forming 360 pairs of rightly opposite triangles;	720 threecrossers, forming 360 pairs of threecrossers lying on the same quadratic surface;
1080 limiting tetrahedra;	1080 fourcrossers;
648 limiting fivecells;	648 fivecrossers;
72 limiting simplexes $S(6)$, forming 36 pairs of rightly opposite $S(6)$;	72 sixcrossers, forming 36 double-sixers;
27 limiting polytopes $Cr(10)$, placed rightly opposite to the vertices;	27 tenlines admitting one of the other lines as common transversal;
27 fivedimensional sections with sixteen vertices;	27 sixteenlines admitting one of the other lines as common crossing line;
of the 648 fivecells 432 belong to the limits of an $S(6)$.	of the 648 fivecrossers 432 belong to half a double-sixer.

7. We now consider the obtained projections in connexion with the lines of a cubic surface and distinguish the element placed in the diagram at the lefthand side as the "starting element", the element placed at the righthand side as the "end element". In this comparison we immediately find this particularity that the property — following in fig. 2^a from the assumed relation between the vertices of the polytope and the lines of the cubic surface — i.e. that the 10 lines of the end element intersect the line forming the starting element, maintains itself up to fig. 2^e in this form that all the lines of the end element are common transversals of the lines of the starting element, while in fig. 2^f each line of the end element cuts only five of the six lines of the starting element. It is easy to express this by a rule without exception indicating the relations of position much more accurately, if we measure as in fig. 4 on a horizontal line OX from the origin O equal segments, mark the points of division by the row of numbers $0, 1, 2, 3 \ldots$, place under O the lines of the different starting elements [1]) and write under $0, 1, 2, 3 \ldots$

[1]) As to this point, according to the last sentence of this article, the process has to undergo a small amplification, which will be perfecly clear to the reader if he has gone through the whole article.

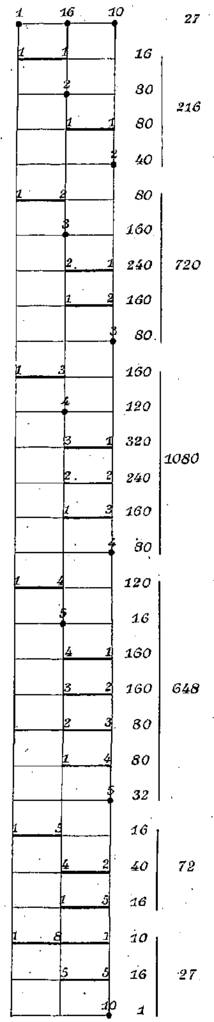

Fig. 1.

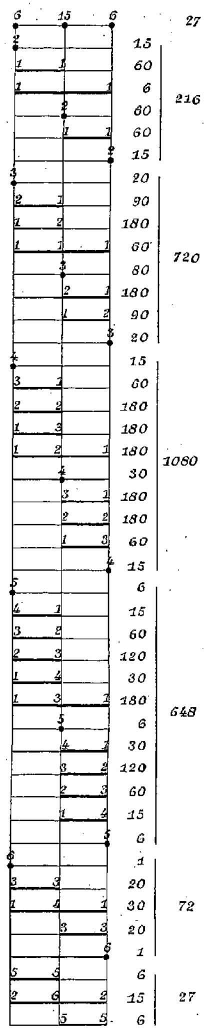

Fig. 3.

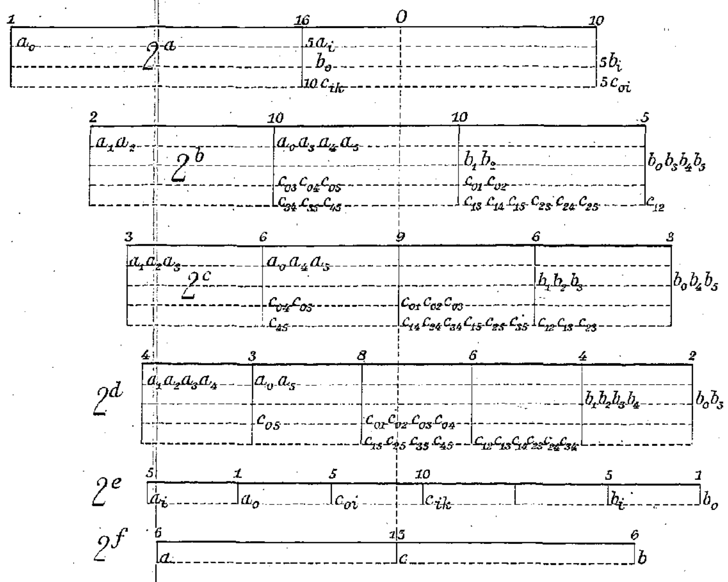

Fig. 2.

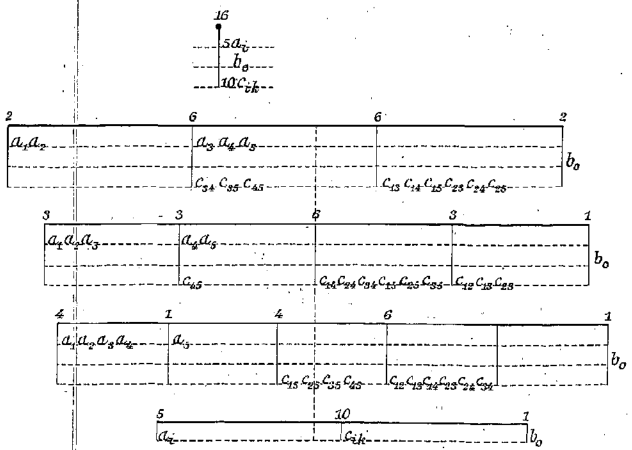

Fig. 4.

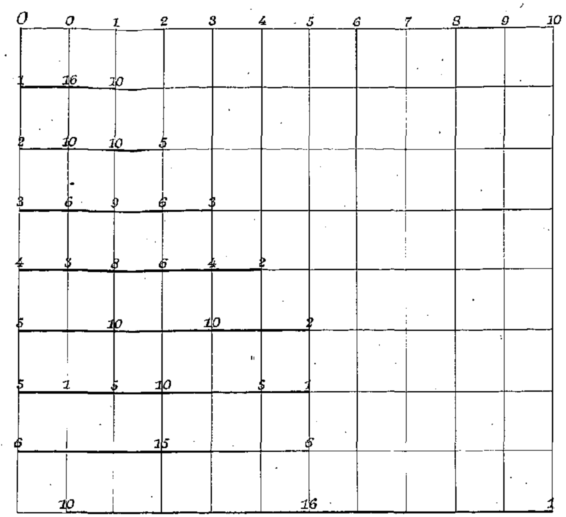

Fig. 5.

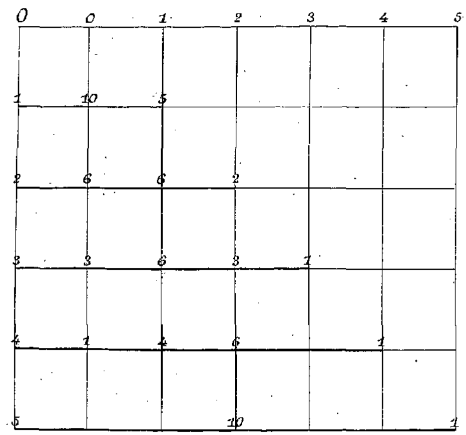

Fig. 6.

J. Bijtel lith. P.J. Mulder impr. Leiden.

the groups of lines, any line of which cuts respectively $0, 1, 2, 3 \ldots$ of the lines forming the starting element. We then really fall back on the projections $(1, 16, 10)$, $(2, 10, 10, 5)$, etc.

The cases in which the starting element contains one, two, three or four lines, give at most rise to the remark, that we find back in fig. 4 the old projections represented on a different scale; for the *mutually* equal segments of each projection have really *different* length for the *different* projections of fig. 2, while *all* the segments have been taken equal to each other in fig. 4.

For the four remaining projections of fig. 4 the starting element is in the language of the configuration successively:

a fivecrosser not belonging to half a double-sixer;

a fivecrosser belonging to half a double-sixer;

a sixcrosser;

a tenline.

Of these four cases still to be discussed the first is that of fig. 2^b, the last that of fig. 2^a, both taken reversely, i.e. with interchange of starting and end element, while the second and the third correspond to fig. 2^e and fig. 2^f.

We treat of the second of the four cases, that of the projection $(5, 1, 5, 10, 0, 5, 1)$ in the first place, in order to fix the attention on the point bearing no projection indicated by the nought. Wherefore has this empty place (fig. 2^c) to present itself? Because the number of lines cutting respectively $0, 1, 2, 3, 4, 5$ of the five lines a_1, a_2, a_3, a_4, a_5 forming the starting element is $1, 5, 10, 0, 5, 1$; in other words any of the 27 lines cutting three of the five lines a_i at least cuts four of them. By this rational explanation of the mentioned hiatus the three other projections are also explained. If we take the projection of fig. 2^b in the reverse sense, we find that each of the ten lines $(b_1\, b_2\, c_{01}\, c_{02} \ldots c_{25})$ cuts three, each of the two lines $(a_1\, a_2)$ cuts five of the lines of the starting element $(b_0\, b_3\, b_4\, b_5\, c_{12})$. The third of the four cases, that of the projection $(6, 15, 6)$ of fig. 2^f, can be explained in the same way. Finally we have still to remark that in the last case the displacement of the starting element, the tenline, over *one* segment to the right of the origin, is no mistake; it corresponds to this that the ten lines do not cross each other altogether, but that each of them intersects *one* of the nine others.

8. So the subject proper of this communication is exhausted. However we will finally move the question if it may be possible that considerations analogous to those mentioned above lead from

other known configurations of lines to unknown polydimensional polytopes with a certain degree of regularity and reversely newly discovered polytopes of this character to unknown configurations of lines. According to our opinion there can be no doubt about the answer to this question.

But instead of entering into this new subject just now we will only point out the configuration of the 16 lines crossing one of the 27 lines, to which corresponds the fivedimensional polytope with the 16 vertices $(5a_i, b_0, 10c_{ik})$. As is known this configuration presents itself on the quartic surfaces with double conic On the other hand the 16 points $(5a_i, b_0, 10c_{ik})$ are the vertices of the half measure polytope $\frac{1}{2}[1\,1\,1\,1\,1]$ of the fivedimensional space $x_6 = -\frac{1}{3}\sqrt{3}$. So we have here before us a second example of a correspondence as the one treated above. In order to enable the reader to study this correspondence we have repeated in the fig. 5 and 6 the part of the fig 2 and 4 which relates to these systems of 16 objects

LIST I.

Vertices.

$a_0 - 5a_1, b_0, 10c_{12} - 5b_1, 5c_{01}.$

Edges.

$5a_0a_1, a_0b_0, 10a_0c_{12} - 10a_1a_2, 30a_1c_{23}, 10b_0c_{12}, 30c_{12}c_{13} -$
$5a_1b_1, 20a_1c_{02}, 5b_0b_1, 30b_1c_{23}, 20c_{01}c_{12} - 10b_1b_2, 20b_1c_{02}, 10c_{01}c_{02}.$

Faces.

$10a_0a_1a_2, 30a_0a_1a_{23}, 10a_0b_0c_{12}, 30a_0c_{12}c_{13} -$
$10a_1a_2a_3, 30a_1a_2c_{34}, 60a_1c_{23}c_{24}, 30b_0c_{12}c_{13}, 20c_{12}c_{13}c_{14}, 10c_{12}c_{13}c_{23} -$
$30a_1a_2c_{03}, 30a_1b_1c_{23}, 60a_1c_{02}c_{23}, 30b_0b_1c_{23}, 60b_1c_{23}c_{24}, 30c_{01}c_{12}c_{13} -$
$20a_1b_1c_{02}, 30a_1c_{02}c_{03}, 10b_0b_1b_2, 3\ b_1b_2c_{34}, 60b_1c_{02}c_{23}, 10c_{01}c_{02}c_{12} -$
$10b_1b_2b_3, 30b_1b_2c_{03}, 30b_1c_{02}c_{03}. 10c_{01}c_{02}c_{03}.$

Tetrahedra.

$10a_0a_1a_2a_3, 30a_0a_1a_2c_{34}, 60a_0a_1c_{23}c_{24}, 30a_0b_0c_{12}c_{13}, 20a_0c_{12}c_{13}c_{14},$
$10a_0c_{12}c_{13}c_{23} - 5a_1a_2a_3a_4, 10a_1a_2a_3c_{45}, 30a_1a_2c_{34}c_{35}, 20a_1c_{23}c_{24}c_{25},$
$20a_1c_{23}c_{24}c_{34}, 20b_0c_{12}c_{13}c_{14}, 10b_0c_{12}c_{13}c_{23}, 5c_{12}c_{13}c_{14}c_{15} -$
$20a_1a_2a_3c_{04}, 60a_1a_2c_{03}c_{34}, 60a_1b_1c_{23}c_{24}, 60a_1c_{02}c_{23}c_{24},$
$60b_0b_1c_{23}c_{24}, 20b_1c_{23}c_{24}c_{25}, 20b_1c_{23}c_{24}c_{34}, 20c_{01}c_{12}c_{13}c_{14} -$
$30a_1a_2c_{03}c_{04}, 60a_1b_1c_{02}c_{23}, 31a_1c_{02}c_{03}c_{23}, 30b_0b_1b_2c_{34}, 30b_1b_2c_{34}c_{35},$
$60b_1c_{02}c_{23}c_{24} - 30a_1b_1c_{02}c_{03}, 20a_1c_{02}c_{03}c_{04}, 10b_0b_1b_2b_3, 10b_1b_2b_3c_{45},$
$60b_1b_2c_{03}c_{34}, 30b_1c_{02}c_{03}c_{23} - 5b_1b_2b_3b_4, 20b_1b_2b_3c_{04}, 30b_1b_2c_{03}c_{04},$
$20b_1c_{02}c_{03}c_{04}, 5c_{01}c_{02}c_{03}c_{04}.$

Fivecells common to $S(6)$ and $Cr(10)$.

$5a_0a_1a_2a_3a_4, 30a_0a_1a_2c_{34}c_{35}, 20a_0a_1c_{23}c_{24}c_{34}, 20a_0b_0c_{12}c_{13}c_{14},$
$5a_0c_{12}c_{13}c_{14}c_{15} - a_1a_2a_3a_4a_5, 10a_1a_2c_{34}c_{35}c_{45}, 5b_0c_{12}c_{13}c_{14}c_{15} -$

$20a_1a_2a_3c_{04}c_{15}$, $20a_1b_1c_{23}c_{24}c_{25}$, $20a_1c_{02}c_{23}c_{24}c_{25}$, $20b_0b_1c_{23}c_{24}c_{34}$ —
$10a_1a_2a_3c_{04}c_{05}$, $30a_1a_2c_{03}c_{04}c_{14}$, $60a_1b_1c_{02}c_{23}c_{24}$, $30b_0b_1b_2c_{34}c_{35}$,
$10b_1b_2c_{34}c_{35}c_{45}$, $20b_1c_{02}c_{23}c_{24}c_{25}$ — $20a_1b_1c_{02}c_{03}c_{04}$, $5a_1c_{02}c_{03}c_{04}c_{05}$,
$5b_0b_1b_2b_3b_4$, $20b_1b_2b_3c_{04}c_{45}$, $30b_1b_2c_{03}c_{04}c_{34}$, — $b_1b_2b_3b_4b_5$,
$10b_1b_2b_3c_{04}c_{05}$, $5b_1c_{02}c_{03}c_{04}c_{05}$.

Fivecells common to two $Cr(10)$.

$10a_0a_1a_2a_3c_{45}$, $20a_0a_1c_{23}c_{24}c_{25}$, $10a_0b_0c_{12}c_{13}c_{23}$—$5a_1a_2a_3a_4c_{05}$,
$30a_1a_2c_{03}c_{34}c_{35}$ $20a_1b_1c_{23}c_{24}c_{34}$, $20a_1c_{02}c_{23}c_{24}c_{25}$, $5c_{01}c_{12}c_{13}c_{14}c_{15}$ —
$10b_0b_1b_2b_3c_{45}$, $30b_1b_2c_{02}c_{34}c_{35}$, $10a_1a_2c_{03}c_{04}c_{55}$, $30a_1b_1c_{02}c_{03}c_{23}$ —
$5b_1b_2b_3b_4c_{05}$, $10b_1b_2c_{03}c_{04}c_{05}$. $c_{01}c_{02}c_{03}c_{04}c_{05}$.

Simplexes $S(6)$.

$a_0a_1a_2a_3a_4a_5$, $10a_0a_1a_2c_{34}c_{35}c_{45}$, $5a_0b_0c_{12}c_{13}c_{14}c_{15}$ — $10a_1a_2a_3c_{04}c_{05}c_{45}$,
$20a_1b_1c_{02}c_{23}c_{24}c_{25}$, $10b_0b_1b_2c_{34}c_{35}c_{45}$ — $5a_1b_1c_{02}c_{03}c_{04}c_{05}$,
$b_0b_1b_2b_3b_4b_5$, $10b_1b_2b_3c_{04}c_{05}c_{45}$.

Cross polytopes $Cr(10)$

$5a_0a_1b_0b_1c_{23}c_{24}c_{25}c_{34}c_{35}c_{45}$, $5a_0a_1a_2a_3a_4c_{05}c_{15}c_{25}c_{35}c_{45}$ —
$a_1a_2a_3a_4a_5c_{01}c_{02}c_{03}c_{04}c_{05}$, $10a_1a_2b_1b_2c_{03}c_{04}c_{05}c_{34}c_{35}c_{45}$.
$5b_0b_1b_2b_3b_4c_{05}c_{15}c_{25}c_{35}c_{45}$ — $b_1b_2b_3b_4b_5c_{01}c_{02}c_{03}c_{04}c_{05}$.

LIST II

Vertices.

$6a_1$, $6b_1$, $15c_{12}$.

Edges.

$$\begin{array}{l|l|l|l} 15a_1a_2 & 6a_1b_1 & 60a_1c_{23} & 60c_{12}c_{13} \\ 15b_1b_2 & & 60b_1c_{23} & \end{array}$$

Faces.

$$\begin{array}{l|l|l|l|l} 20a_1a_2a_3 & 90a_1a_2c_{34} & 180a_1c_{23}c_{24} & 60c_{12}c_{13}c_{14} & 20c_{12}c_{13}c_{23} \\ 20b_1b_2b_3 & 90b_1b_2c_{34} & 180b_1c_{23}c_{24} & & \end{array}$$

Tetrahedra.

$$\begin{array}{l|l|l|l|} 15a_1a_2a_3a_4 & 60a_1a_2a_3c_{45} & 180a_1a_2c_{34}c_{35} & 120a_1c_{23}c_{24}c_{25} \\ 15b_1b_2b_3b_4 & 60b_1b_2b_3c_{45} & 180b_1b_2c_{34}c_{35} & 120b_1c_{23}c_{24}c_{25} \end{array}$$

$$\begin{array}{l|l|l} 60a_1c_{23}c_{24}c_{34} & 180a_1b_1c_{23}c_{24} & 30c_{12}c_{13}c_{14}c_{15}. \\ 60b_1c_{23}c_{24}c_{34} & & \end{array}$$

Fivecells common to $S(6)$ and $Cr(10)$.

$$\begin{array}{l|l|l|l|l} 6a_1a_2a_3a_4a_5 & 60a_1a_2a_3c_{45}c_{46} & 60a_1a_2c_{34}c_{35}c_{45} & 30a_1c_{23}c_{24}c_{25}c_{26} & 120a_1b_1c_{23}c_{24}c_{15}. \\ 6b_1b_2b_3b_4b_5 & 60b_1b_2b_3c_{45}c_{46} & 60b_1b_2c_{34}c_{35}c_{45} & 30b_1c_{23}c_{24}c_{25}c_{26} & \end{array}$$

Fivecells common to two $Cr(10)$.

$$\begin{array}{l|l|l|l} 15a_1a_2a_3a_4c_{56} & 60a_1a_2c_{34}c_{35}c_{36} & 60a_1b_1c_{23}c_{24}c_{34} & 6c_{12}c_{13}c_{14}c_{15}c_{16}. \\ 15b_1b_2b_3b_4c_{56} & 60b_1b_2c_{34}c_{35}c_{36} & & \end{array}$$

Simplexes $S(6)$.

$$\begin{array}{l|l|l} a_1a_2a_3a_4a_5a_6 & 20a_1a_2a_3c_{45}c_{46}c_{56} & 30a_1b_1c_{23}c_{24}c_{25}c_{26}. \\ b_1b_2b_3b_4b_5b_6 & 20b_1b_2b_3c_{45}c_{46}c_{56} & \end{array}$$

Cross polytopes $Cr(10)$.

$$\begin{array}{l|l} 6a_1a_2a_3a_4a_5c_{16}c_{26}c_{36}c_{46}c_{56} & 15a_1a_2b_1b_2c_{34}c_{35}c_{36}c_{45}c_{46}c_{56}. \\ 6b_1b_2b_3b_4b_5c_{16}c_{26}c_{36}c_{46}c_{56} & \end{array}$$