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Mathematics. — "*Reciprocity in connexion with semiregular polytopes and nets.*" By Mrs A. BOOLE STOTT and Prof. P. H. SCHOUTE.

1. The tables added to the memoir "Geometrical deduction of semiregular from regular polytopes and space fillings", recently published by this Academy (*Verhandelingen*, eerste sectie, deel XI, N°. 1) [1]), show that the same semiregular polytope or net may sometimes be derived from different regular polytopes or nets by different operations. It was stated there (p. 13) that this is due to the "reciprocity of the figures". We propose to examine here the influence of this reciprocity on the mutual relationship between the results of the different operations of expansion explained there. Before doing so it will be well to give a definition of what is understood here by reciprocity of two polytopes in space S_n, where these polytopes have either a finite or an infinite number of limiting elements l_{n-1}, the first case referring to two polytopes in S_n and the second to two nets of polytopes in S_{n-1} considered as two polytopes in S_n.

2. *Definition of reciprocity.* Two regular polytopes in S_n are reciprocal to one another if the number of limiting elements l_p passing through a limiting element l_q of the one is equal to the number of limiting elements l_{n-p-1} lying in a limiting element l_{n-q-1} of the other, where $p > q$.

We illustrate this by some examples which we divide into two groups, the first *a*) dealing with *pairs* of polytopes of different forms, the second *b*) with *self reciprocal* polytopes.

a. In S_3 we have two pairs of reciprocal regular polyhedra, C and O, I and D, in S_4 we have once more two pairs of regular polytopes, C_8 and C_{16}, C_{120} and C_{600}, and one pair of regular nets, NC_{16} and NC_{24}. So for $n=4$ the number 3 of faces passing through an edge in C_8 (see the "Table of incidences" in the memoir quoted) is equal to the number of edges lying in a face in C_{16}. So for $n=5$ the number 8 of faces passing through an edge in NC_{16} (see the same table) is equal to the number of faces lying in a limiting body in NC_{24}, while the number 12 of limiting bodies passing through an edge in NC_{16} is equal to the number of edges lying in a limiting body in NC_{24}, etc.

b. In S_3 we have only one self reciprocal regular body, T, and one self reciprocal regular net, the net of cubes, in S_4 we have the

[1]) The figures alluded to in the following pages will be found in the memoir quoted.

two self reciprocal regular polytopes C_5, C_{24} and one self reciprocal regular net, the net NC_8. In passing we may remark that in space S_n the net NM_n of measure polytopes M_n is self reciprocal.

3. By the application of the operation of expansion e_{n-1} to a regular polytope A in S_n each vertex, each edge, each face, etc. is replaced by a limiting polytope of $n-1$ dimensions, filling up the *gaps* caused by the expansion; these polytopes will be indicated respectively by the symbols g_0, g_1, g_2, etc., the last one g_{n-1} being the *original* limiting $n-1$-dimensional polytope itself in an other position. The subscripts $0, 1, \ldots, n-1$ of these symbols $g_0, g_1 \ldots, g_{n-1}$ represent the *import* of the limiting polytopes Now, if we apply the operation e_{n-1} to two polarly related polytopes A and A' of S_n, the gaps $g_0, g_1, \ldots g_{n-1}$ of $e_{n-1} A$ are respectively equal, in form — and in number as long as this remains finite —, to the gaps g_{n-1}, $g_{n-2}, \ldots g_0$ of $e_{n-1} A'$, in other words the polytopes $e_{n-1} A$ and $e_{n-1} A'$ have their gaps of *reciprocal import* equal (p. 9 of the memoir quoted). We will try to make this clear by a few examples [1].

In the simple case of C and O in S_3 the e_2 expansion applied to both gives an RCO (fig. 3^a and 3^b), where the gaps g_0, g_1, g_2 of the one are equal to the gaps g_2, g_1, g_0 of the other. In the case of the cells C_8 and C_{16} in S_4 the e_3 expansion leads up to the same form (fig. 6^a and 6^b); here we have $g_0\, e_3\, C_8 = g_3\, e_3\, C_{16} = T$, $g_1\, e_3\, C_8 = g_2\, e_3\, C_{16} = P_3$, $g_2\, e_3\, C_8 = g_1\, e_3\, C_{16} = P_4$, $g_3\, e_3\, C_8 = g_0\, e_3\, C_{16} = C$ (see the numbers indicated in the diagrams). In the case of the nets NC_{16} and NC_{24} (fivedimensional reciprocal polytopes) the two polytopes $e_4 NC_{16}$ and $e_4 NC_{24}$ (fig. 26) show the relations $g_0 e_4 NC_{16} = g_4 e_4 NC_{24} = C_{24}$, $g_1\, e_4\, NC_{16} = g_3\, e_4\, NC_{24} = Po$, $g_2\, e_4\, NC_{16} = g_2\, e_4\, NC_{24} = (3\ ;\ 3)$, $g_3\, e_4\, NC_{16} = g_1\, e_4\, NC_{24} = P_T$, $g_4\, e_4\, NC_{16} = g_0\, e_4\, NC_{24} = C_{16}$.

4. We have shown above that the application of the operation e_{n-1} (with the highest subscript) to two reciprocal polytopes A and A' in S_n produces the same form with reciprocal imports. If any second operation e_k be applied to $e_{n-1} A$, will it be possible to find an operation $e_{k'}$ by which $e_{n-1} A'$ may be transformed so as to make $e_{k'}\, e_{n-1} A' = e_k\, e_{n-1} A$?

The answer to this question is very simple: in order to obtain the same result in both cases we have only to take care that the two operations e_k and $e_{k'}$ act upon the *same subject*. Now the limiting

[1]) An analytical proof of this theorem and the following one will be published later on.

polytopes of k import in the first are the same as those of $n-k-1$ import in the second; so k' has to be equal to $n-k-1$, i.e. we have $k+k'=n-1$. So we get

$$e_k\, e_{n-1}\, A = e_{n-k-1}\, e_{n-1}\, A',$$

i.e.: If we apply respectively to $e_{n-1} A$ and $e_{n-1} A'$ any two *reciprocal* operations e_k and e_{n-k-1} the result is the same but the imports are reciprocal.

This simple general theorem accounts for the equality of all the pairs of polytopes (and nets) indicated in the tables added to the memoir quoted.

Hever, Kent, England. September, 1910.

Physics. — *"An improved semicircular electromagnet."* II. By Prof. H. E. J. G. du Bois. (Communication from the Bosscha-Laboratory.)

Recently I described a new type of semicircular electromagnet together with some results obtained with it.[1]) In the present paper I beg to communicate a few more measurements; and also its adaptation to special purposes, which lately have come to prominent notice.

Influence of polar windings. The reproduction given previously exhibited the windings as split into two divisions by a rectangular flange: *a.* polar windings, which are in the neighbourhood of the pole-pieces, the efficiency of which can be increased by supplementary loose polar coils; *b.* circuital windings round the other parts of the magnetic circuit. A second instrument was wound and connected in a somewhat different way; the field was determined again under different circumstances by means of a ballistic moving-coil galvanometer; this was standardised by means of a normal solenoid, and the proportionality of the readings ascertained. A small test-coil was made with a diameter of 3 m.m. and a thickness of 0,3 m.m.; the thickness of the bare copper wire used was 0,025 m.m., silk-covered 0,07 m.m.; it was wound in collodium. The equivalent area of the 45 windings was 1,544 cm^2., determined by comparison with a slightly smaller normal coil of 1,530 cm^2., measured geometrically. The results are given in the subjoined table:

[1]) H. du Bois. These Proc. 18 p. 189, 1909.