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(1231)

$$p + \frac{a}{v^2} = \frac{RT \left(1 - \frac{n-1}{n}\right)}{v-b} + \frac{a \left(x - \frac{x^2}{4}\right)}{v^2}$$

Bearing in mind that $p + \frac{a}{v^2} = T \frac{dp}{dT}$, we find for small value of x :

$$T \frac{dp}{dT} = \frac{RT}{v-b} + x \left(\frac{dp}{dx} \right)_{vT}$$

according to (β)

$$T \frac{dp}{dT} = \frac{RT}{v-b} - \frac{xRT}{v} \frac{1}{n \left(\frac{Ev_k}{a} - 1 \right)}$$

or dividing by p :

$$f = s \frac{v}{v-b} - s \frac{x}{n \left(\frac{Ev_k}{a} - 1 \right)}$$

So the value of $\frac{v}{v-b}$ is found to be somewhat greater than $\frac{f}{s}$, but so little that our foregoing calculations can remain unchanged.

Geophysics. — “On tidal forces as determined by means of WIECHERT’s astatic seismograph”. By Dr. C. BRAAK. (Communicated by Dr. VAN DER STOK.)

(Communicated in the meeting of March 25, 1911).

In a previous communication the E—W component of the semi-diurnal lunar tidal motion of the ground at Batavia, as deduced from registrations of WIECHERT’s astatic seismograph during the period of July to December 1909, was stated to be:

$$0''.0114 \cos (2t - 251^\circ 53'),$$

whereas the theoretical value is:

$$0''.0155 \cos (2t - 270^\circ)$$

The registrations obtained during the following half-year have now been worked out upon the same plan and, in addition to this tide, the other principal tides have been calculated for the whole period of one year, except the semi-diurnal solar tide, which is strongly disturbed by the diurnal heat wave.

These tides, enumerated according to their importance, are:

¹⁾ These Proceedings XIII. 1910, p. 17—21.

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M_2 , semidiurnal lunar tide	E—W. component
K_1 , Sun's-moon's-declination tide	N—S. „
O, Moon's declination tide	„ „
P, Sun's declination tide	„ „
K_2 , Sun's-moon's declination tide	E—W. „
M_2 , semidiurnal lunar tide	N—S. „

As before, the semidiurnal tides were determined by first measuring off the distances between the registrations at 7^h and 10^h, 8^h and 12^h etc., and taking the differences (13—10)—(10—7) etc.

For the diurnal tides the same method was applied to the registrations at the hours 16^h and 10^h, 10^h and 4^h etc.

The sensitiveness was deduced by determining the period of vibration and the indicator-magnification, as explained in the former communication.

As the position of the instrument has not been altered, the former values :

234.3 (E—W) and 185.7 (N—S)

have been assumed for the indicator-magnification.

In the different months the following values were found for the mean times of vibration :

	E—W	N—S
1909 July	10.0 ^s	10.4
Aug.	10.1	10.3
Sept.	10.0	10.1
Oct.	9.6	9.8
Nov.	9.7	10.1
Dec.	9.6 ^s	10.0
1910 Jan.	9.8	9.8
Febr.	10.0 ^s	10.0 ^s
March	9.5	9.5
April	9.7	9.7
May	9.5	9.5
June	9.1	9.0 ^s

The equivalent pendulum-length as deduced from these data is :

E—W pendulum : 22.70 meters

N—S „ : 23.83 „

The variation of inclination corresponding with a deflection of 1 mm. in the diagram is accordingly :

for the E—W component: 0".0388

„ „ N—S component: 0".0466

As has been noticed in the previous communication the method followed of measuring off the distances mentioned above gives twice the amplitude multiplied by $1 - \cos 3a$ (a = angular velocity per hour) for the semi-diurnal tides as the argument varies in 3 hours not 90° , but $3a$.

For the diurnal tides twice the amplitude multiplied by $1 - \cos 6a$ is obtained.

The deflections have been measured off corresponding to the time signal, given 5,5 minutes before Batavia time; the arguments have been corrected accordingly.

The values of the tides mentioned above as found by means of the records, and also their theoretical values on the assumption of an absolutely rigid earth, are:

July 1, 1909, noon.

M_2 , E—W	observed value	$0''.01120 \cos (2t-58^\circ.0)^1$
	theoretical „	$0''.01544 \cos (2t-45^\circ.5)$
M_2 , N—S	observed value	$0''.00848 \cos (2t-356^\circ.9)$
	theoretical „	$0''.00167 \cos (2t-315^\circ.5)$
K_2 , E—W	observed value	$0''.00277 \cos (2t-245^\circ.8)$
	theoretical „	$0''.00229 \cos (2t-269^\circ.8)$
O , N—S	observed value	$0''.00644 \cos (t-38^\circ.2)$
	theoretical „	$0''.00700 \cos (t-312^\circ.8)$
K_1 , N—S	observed value	$0''.00449 \cos (t-313^\circ.6)$
	theoretical „	$0''.00945 \cos (t-359^\circ.8)$
P , N—S	observed value	$0''.00645 \cos (t-84^\circ.2)$
	theoretical „	$0''.00291 \cos (t-9^\circ.2)$

North and West are reckoned positive; July 1, 1909, noon has been taken as the beginning of time-reckoning.

Although the E—W components, but for a somewhat excessive value of the K_2 amplitude, closely agree with the theoretical values, the N—S components on the contrary show considerable deviations. That this disagreement is not due to errors of the instrument is proved by the following values for the N—S component of the M_2 tide, calculated for each hour.

¹⁾ Owing to an error in the calculation, the difference between the observed and the theoretical values of the argument is not the same as that given in the earlier communication.

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10 ^h	0.348	cos	(2t—170°.4)
11	0.334	cos	(2t—175°.1)
12	0.400	cos	(2t—177°.3)
1	0.346	cos	(2t—176°.5)
2	0.414	cos	(2t—178°.1)
3	0.392	cos	(2t—184°.2)
4	0.420	cos	(2t—183°.0)
5	0.367	cos	(2t—192°.4)

For the N—S component of the O-tide, we found, taking together two consecutive hours :

10 and 11 hour	0.123	cos	(t—199°.2)
12 „ 1 „	0.199	cos	(t—229°.6)
2 „ 3 „	0.399	cos	(t—235°.1)
4 „ 5 „	0.375	cos	(t—205°.4)

The differences between these values are very small especially for the M_2 -tide; for the O-tide the amplitudes show considerable fluctuations; but the agreement of the arguments clearly indicates that the differences between the experimental and theoretical values are due to an external periodical disturbance.

We shall presently see that they are caused by the watertides in the Indian Ocean and in the Java sea.

If, namely, we assume the amplitude of the undisturbed gravitation-tide M_2 to be equal to $\frac{2}{3}$ of its theoretical value — an assumption which cannot lead to an appreciable error considering the small value of this tide in proportion to the disturbing force — if, further, its argument is assumed to be equal to the theoretical argument, we find for the disturbing force :

$$0''.00772 \cos (2t-2^\circ.3)$$

If there were no retardation and, therefore, high water coincided with the moment of culmination of the fictive star, the tides would be represented for the longitude of Batavia by the expression :

$$R \cos (2t-315^\circ.5)$$

and the disturbance could be explained if the kappanumber of the ocean-tide is assumed to be :

$$\text{North of Batavia } 2^\circ.3-315^\circ.5 = 46^\circ.8$$

$$\text{South „ „ } 2^\circ.3-315^\circ.5 + 180^\circ = 226^\circ.8.$$

Applying the same reasoning to the O-tide (where, of course, the accuracy is less owing to the greater value of the amplitude if $\frac{2}{3}$ of its theoretical value is assumed) we find for the disturbing force :

$$0''.00765 \cos (t-75^\circ.7).$$

As, for $\kappa = 0$, the watertide is :

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$$R \cos(t - 312.^\circ 8)$$

the kappanumber ought to be :

North of Batavia $122.^\circ 9$

South „ „ $302.^\circ 9$.

Now the tidal constants according to VAN DER STOK¹⁾ are :

		Kappanumber	Amplitude in cm.
M ₂ -tide	S. of Batavia	225°	49.6 ²⁾
„ „	N. „ „	304°	3.6 ³⁾
O „ „	S. „ „	268°	11.7 ⁴⁾
„ „	N. „ „	129°	9.3 ⁵⁾

If, as is reasonable, the principal disturbance of the M₂ tide is ascribed to the Indian Ocean, where semidiurnal tides are paramount, and, on the contrary, the disturbance of the O-tide is ascribed to the Java sea, where diurnal tides are prevalent, the agreement between the kappanumbers, as calculated from the disturbances and as observed, is very satisfactory.

A closer inquiry into the influence of the K₁ and P tides would be useless; the period of these tides being but slightly different from that of mean solar time, meteorological influences probably cause considerable disturbances; the results for different hours therefore show large discrepancies:

K₁ tide for 10 and 11^h $0.167 \cos(t - 315.^\circ 6)$

for 12 and 1^h $0.532 \cos(t - 224.^\circ 8)$

P tide for 10 and 11^h $0.187 \cos(t - 349.^\circ 7)$

for 12 and 1^h $0.538 \cos(t - 145.^\circ 3)$

Probably annual variations in the diurnal heat wave, and the intensity of land- and seawinds have an important disturbing influence⁶⁾.

On the assumption that the disturbances acting upon the M₂ tide are wholly due to the watertide, the disturbance of the *E-W* component of this tide can be eliminated, as the disturbing force, acting on the *E-W* pendulum, in this case, has a difference of phase of about 90° and, consequently its influence on the amplitude is small.

If we look at VAN DER STOK's chart of homocumenes⁷⁾, it appears that, south of Sumatra from Padang to Vlaktehoek, the kappanumber

¹⁾ Kon. Nederl. Meteor. Instituut. Mededeelingen en Verhandelingen n^o. 8. Elementaire theorie der getijden. Getijconstanten in den Indischen Archipel, 1910.

²⁾ Amplitude at Tjilatjap, kappanumber according to chart of homocumenes.

³⁾ „ and kappanumber for Edam, Tandjong Priok and Duizendeilanden.

⁴⁾ „ „ „ for Tjilatjap.

⁵⁾ „ „ „ for Edam, Tandjong Priok and Duizendeilanden.

⁶⁾ With sudden increase of the wind, and the beginning of rainshowers, the direction of the wind can easily be traced on the seismograms; the deflections caused by these agencies amount to 1 à 2 mm. in maximo.

⁷⁾ loc. cit.

(relative to Batavia time) is about 200° , whilst its average value in Sunda Strait (Telok Betong, Java's 4th Point, Labuan) is 210° .

It is therefore reasonable to assume that over the whole region, where the disturbance exercised on the *E-W* pendulum originates, the kappanumber varies but slightly from 200° , say not more than 10° .

If we now calculate the disturbing force and the undisturbed earth-tide on the assumption that the kappanumber is 200° and that the argument of the undisturbed tidal motion of the ground is equal to its theoretical value, then we find ¹⁾:

Amplitude disturbing force	0''.0026
„ corrected earth-tide	0''.0118

For the proportion between observed and theoretical amplitudes in the case of a rigid earth this gives the value 0.76.

HECKER also found for the *E-W* component the factor 0.76, and ORLOFF ²⁾ at Jurjew gives 0.63 as deduced by means of a ZÖLLNER seismograph.

If we summarize the results given above, it is in the first place evident that great care must be taken in the choice of stations to be used for the determination of earth-tides, and that places situated in the vicinity of a sea with considerable tides are unsuitable for this purpose.

Further the unsatisfactory results obtained for the K_1 and P tides justify the conclusion that, in order to determine the constants of these tides, the instruments should be erected in a place sufficiently deep to eliminate tidal motions of meteorological origin.

It appears, further, that WIECHERT's pendulum is a very reliable and sensitive instrument, well suited for the determination of tidal motions and that, by means of its records, the diurnal as well as the semidiurnal tides may be determined without changing the erection usual for seismographic purposes.

If only the constants of the apparatus have been determined with sufficient care the material at present available from seismographs placed in favourable situations might at once be used for tidal researches without extra expense and the knowledge of the tidal problem might in this way be considerably extended.

For Batavia it proves to be possible to deduce the semidiurnal lunar tidal motion of the ground in the *E-W* direction notwithstanding the disturbance due to watertides.

Wetlevreden, February 12, 1911.

¹⁾ For $\kappa = 190^\circ$ and 210° the amplitudes of the disturbing force are respectively 0''.0025 and 0''.0028, of the corrected tide 0''.0114 and 0''.0123.

²⁾ *Astronom. Nachr.* No. 4446. Bd. 186.