Physics. — *"A new accurate formula for the computation of the self-inductance of a long coil wound with any number of layers."* By F. L. BERGANSIUS (Communicated by Prof. W. H. JULIUS).

For the accurate computation of the self-inductance of multiple layer coils, different formulae are available in the case the cross section ot the coil is a square, a circle or a rectangle. All these formulae only give results of a high degree of accuracy, when the cross section is not too large in comparison with the mean radius, and besides for the rectangular section restriction is made, that the length of the coil shall not considerably surpass this mean radius.

For the case of a *long* coil or solenoïd wound with many layers of wire, to my knowledge no formula has been derived, which, either in a closed form or in the form of a converging series, represents the value of the self-inductance with a high degree of accuracy.

LOUIS COHEN[1]) has derived for this case an approximate formula of the following form:

$$\left.\begin{aligned} L = 4\pi^2 n^2 m & \left\{\frac{2a_0^4 + a_0^2 l^2}{\sqrt{4a_0^2 + l^2}} - \frac{8a_0^3}{\pi}\right\} + \\ + 8\pi^2 n^2 & \left\{[(m-1)\,a_1^2 + (m-2)\,a_2^2 + \ldots]\left(\sqrt{a_1^2 + l^2} - \frac{7}{8}\,a_1\right) + \right. \\ & + \tfrac{1}{2}\,[m\,(m-1)\,a_1^2 + (m-1)\,(m-2)\,a_2^2 + \ldots]\left(\frac{a_1\,\delta a}{\sqrt{a_1^2 + l^2}} - \delta a\right) - \\ & \left. - \tfrac{1}{2}\,[m\,(m-1)\,a_1^2 + (m-2)\,(m-3)\,a_2^2 + \ldots]\,\frac{\delta a}{8}\right\} \end{aligned}\right\} \quad (1)$$

wherein a_0 = mean radius of coil $a_1, a_2 \ldots$ = radius of the first, second layer reckoned from the axis of the coil; δa = distance between two consecutive layers; l = length, n = number of windings per cm. m = number of layers.

COHEN says that the results obtained with this formula are accurate to within one half of one percent for a solenoïd, whose length is twice the diameter, the accuracy increasing as the length increases.

Apart from this moderate accuracy, this formula (which, moreover, contains errors in the third and fourth terms) is very laborious for numerical computations, when the number of layers m is large.

[1]) LOUIS COHEN, Bulletin of the Bureau of Standards IV, 383.

EDWARD B. ROSA [1]) describes in the same part of the above mentioned annual, a method for the accurate computation of the self-inductance of a coil of any length wound with any number of layers, which he presumes to be absolutely correct and which is used by him to check the results obtained by other formulae, especially STEFAN's. This method, though based on a correct principle, will, if applied in the manner used by ROSA, only then lead to very accurate results, when the total depth of the windings on the coil is *very* small compared with the mean radius.

In the following pages I propose to give the derivation of a new formula, which, in a simple and for numerical computation very convenient form, represents the self-inductance of multiple layer coils with a high degree of accuracy in all cases in which the formulae for short coils fail.

For the mutual inductance between two coaxial cylinders of equal length MAXWELL [2]) has derived the following expression:

$$M = 4\,\pi^2\, n^2\, a^2\, [l - 2\,A\,\alpha] \quad \ldots\ldots \quad (2)$$

wherein

$$\alpha = \frac{l-r+A}{2A} - \frac{a^2}{16A^2}\left(1-\frac{A^3}{r^3}\right) - \frac{a^4}{64A^4}\left(\frac{1}{2}+\frac{2A^5}{r^5}-\frac{5A^7}{2r^7}\right) - \\ - \frac{35\,a^6}{2048A^6}\left(\frac{1}{7}-\frac{8}{7}\frac{A^7}{r^7}+\frac{4A^9}{r^9}-\frac{3A^{11}}{r^{11}}\right) - \quad . \ (3)$$

$r = \sqrt{A^2+l^2}$, A = radius of outer cylinder, a = radius of inner cylinder, l = length, n = number of windings per cm.

Te last term of α has been added to the derivation by E. B. ROSA [3]).

Generally the self-inductance of a coil is found by integrating the expression for the mutual inductance between two 'elements of the section twice over the whole area of this section.

In order to obtain this integral we suppose the solenoïd to be formed by a very great number m of layers. Indicating by a_1 the radius of the *outer* layer and by δa the distance between two con-

1) EDWARD B. ROSA, Bull. of the Bur. of St. IV 369.

2) MAXWELL, Electricity and Magnetism, II, § 678.

3) E. B. ROSA and L. COHEN, Bull. of the Bur. of St. III 305.

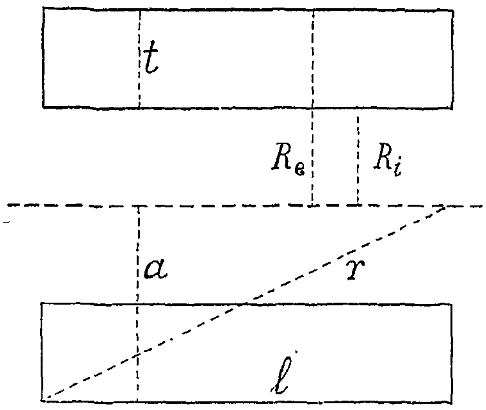

Fig. 1

secutive layers, we get for the radii of the consecutive layers:

$$a_2 = a_1 - \delta a$$
$$a_3 = a_1 - 2\delta a$$
$$\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$
$$a_m = a_1 - (m-1)\,\delta a$$

The mutual inductance between any two cylinders with radii a_p and a_q being $M_{p.q.}$, the self-inductance of the solenoïd is given by the equation:

$$L_a = 2 \sum_{p=1}^{p=q} \sum_{q=1}^{q=m} M_{p.q} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (4)$$

Substituting the value of α given by (3) in the equation (2) and taking provisionally, in order to facilitate the survey of the derivation, only the two first terms of α with omission of the term $\frac{A^3}{r^3}$, we obtain:

$$M = 4\,\pi^2 n^2 a^2 \left[\sqrt{A^2+l^2} - A + \frac{a^2}{8A}\right] \quad . \ . \ . \ . \quad (5)$$

In this expression we replace a and A by their above values, a_1, $a_1 - \delta a$ etc. and expand the terms within the square brackets according to ascending powers of δa. We neglect all terms in which δa occurs to a higher degree than the second, and for the above mentioned reason we also omit the terms with $\overline{\delta a}^2$ resulting from the expansion of the form under the radical.

Moreover putting $\sqrt{a_1^2+l^2} = r$ we find for the terms of the integral (4):

$$\left.\begin{aligned}
M_{1.1} &= 4\pi^2 n^2 a_1^2 \left\{ r - a_1 + \frac{a_1}{8} \right\} \\
M_{1.2} &= 4\pi^2 n^2 a_2^2 \left\{ r - a_1 + \frac{a_1}{8} - \frac{2\delta a}{8} + \frac{\overline{\delta a}^2}{8a_1} \right\} \\
M_{2.2} &= 4\pi^2 n^2 a_2^2 \left\{ r - \frac{a_1 \delta a}{r} - a_1 + \delta a + \frac{a_1}{8} - \frac{\delta a}{8} \right\} \\
M_{1.3} &= 4\pi^2 n^2 a_3^2 \left\{ r - a_1 + \frac{a_1}{8} - \frac{4\delta a}{8} + \frac{4\overline{\delta a}^2}{8a_1} \right\} \\
M_{2.3} &= 4\pi^2 n^2 a_3^2 \left\{ r - \frac{a_1 \delta a}{r} - a_1 + \delta a + \frac{a_1}{8} - \frac{3\delta a}{8} + \frac{\overline{\delta a}^2}{8a_1} \right\} \\
M_{3.3} &= 4\pi^2 n^2 a_3^2 \left\{ r - \frac{2a_1 \delta a}{r} - a_1 + 2\delta a + \frac{a_1}{8} - \frac{2\delta a}{8} \right\} \\
M_{1.4} &= 4\pi^2 n^2 a_4^2 \left\{ r - a_1 + \frac{a_1}{8} - \frac{6\delta a}{8} + \frac{9\overline{\delta a}^2}{8a_1} \right\} \\
M_{2.4} &= 4\pi^2 n^2 a_4^2 \left\{ r - \frac{a_1 \delta a}{r} - a_1 + \delta a + \frac{a_1}{8} - \frac{5\delta a}{8} + \frac{4\overline{\delta a}^2}{8a_1} \right\} \\
M_{3.4} &= 4\pi^2 n^2 a_4^2 \left\{ r - \frac{2a_1 \delta a}{r} - a_1 + 2\delta a + \frac{a_1}{8} - \frac{4\delta a}{8} + \frac{\overline{\delta a}^2}{8a_1} \right\} \\
M_{4.4} &= 4\pi^2 n^2 a_4^2 \left\{ r - \frac{3a_1 \delta a}{r} - a_1 + 3\delta a + \frac{a_1}{8} - \frac{3\delta a}{8} \right\} \\
M_{1.5} &= 4\pi^2 n^2 a_5^2 \left\{ r - a_1 + \frac{a_1}{8} - \frac{8\delta a}{8} + \frac{16\overline{\delta a}^2}{8a_1} \right\} \\
M_{2.5} &= 4\pi^2 n^2 a_5^2 \left\{ r - \frac{a_1 \delta a}{r} - a_1 + \delta a + \frac{a_1}{8} - \frac{7\delta a}{8} + \frac{9\overline{\delta a}^2}{8a_1} \right\} \\
M_{3.5} &= 4\pi^2 n^2 a_5^2 \left\{ r - \frac{2a_1 \delta a}{r} - a_1 + 2\delta a + \frac{a_1}{8} - \frac{6\delta a}{8} + \frac{4\overline{\delta a}^2}{8a_1} \right\} \\
M_{4.5} &= 4\pi^2 n^2 a_5^2 \left\{ r - \frac{3a_1 \delta a}{r} - a_1 + 3\delta a + \frac{a_1}{8} - \frac{5\delta a}{8} + \frac{\overline{\delta a}^2}{8a_1} \right\} \\
M_{5.5} &= 4\pi^2 n^2 a_5^2 \left\{ r - \frac{4a_1 \delta a}{r} - a_1 + 4\delta a + \frac{a_1}{8} - \frac{4\delta a}{8} \right\}
\end{aligned}\right\} \quad (6)$$

etc.

The law of succession for the numerical coefficients of the terms *within* the brackets, in each group with the same factor *before* the brackets, is very evident, so that the sum is easily found.

After adding and ranging we find:

$$\Sigma\Sigma M_{pq} = 4\pi^2 n^2 \left\{ \begin{aligned} &[a_1^2 + 2a_2^2 + 3a_3^2 + 4a_4^2 + 5a_5^2 + \ldots]\left(r - a_1 + \frac{a_1}{8}\right) + \\ &+ [\quad a_2^2 + 3a_3^2 + \ 6a_4^2 + 10a_5^2 + \ldots]\left(\delta a - \frac{a_1\delta a}{r}\right) + \\ &+ [\quad 3a_2^2 + 9a_3^2 + 18a_4^2 + 30a_5^2 + \ldots]\left(-\frac{\delta a}{8}\right) + \\ &+ [\quad a_2^2 + 5a_3^2 + 14a_4^2 + 30a_5^2 + \ldots]\frac{\delta a^2}{8a_1} \end{aligned} \right\} \quad (7)$$

The terms 2 and 3 can be combined into one, namely:

$$[a_2^2 + 3a_3^2 + 6a_4^2 + 10a_5^2 + \ldots]\left(\delta a - \frac{3\delta a}{8} - \frac{a_1\delta a}{r}\right)$$

The infinite series within the square brackets must be integrated. We replace a_2, $a_3 \ldots$ etc. by their values, $a_1 - \delta a$, $a_1 - 2\delta a \ldots$ etc. and obtain for instance for the first series:

$$\left.\begin{aligned} a_1^2 &= a_1^2 \\ 2a_2^2 &= 2a_1^2 - 4a_1\delta a + 2\overline{\delta a}^2 \\ 3a_3^2 &= 3a_1^2 - 12a_1\delta a + 12\overline{\delta a}^2 \\ 4a_4^2 &= 4a_1^2 - 24a_1\delta a + 36\overline{\delta a}^2 \\ 5a_5^2 &= 5a_1^2 - 40a_1\delta a + 80\overline{\delta a}^2 \\ &\ldots\ldots\ldots\ldots \\ ma_m^2 &= ma_1^2 - 2m(m-1)a_1\delta a + m(m-1)^2\overline{\delta a}^2 \end{aligned}\right\} \quad \ldots \quad (8)$$

The numbers in the vertical ranges, figuring as coefficients of a_1^2, $a_1\delta a$ and $\overline{\delta a}^2$ form arithmetical series of respectively first, second and third order.

The general expression for the sum of m terms of an arithmetical series of the n^{th} order is:

$$S_m = mt_1 + \frac{m(m-1)}{1.2}\Delta_1 + \frac{m(m-1)(m-2)}{1.2.3}\Delta_2 + \frac{m(m-1)(m-2)(m-3)}{1.2.3.4}\Delta_3 + \ldots + \frac{m(m-1)\ldots(m-n)}{1.2.\ldots n+1}\Delta_n \ldots \quad (9)$$

wherein t_1 = first term of the series, $\Delta_1, \Delta_2 \ldots \Delta_n$ = first terms of the series of differences, n = the order number of the series.

After adding the terms of (8) we thus obtain an expression of the following form:

$$Pa_1^2 - Qa_1\delta a + R\overline{\delta a}^2$$

wherein P, Q and R are functions of m, which are easily found

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by substituting in (9) the values of t_1, Δ_1, Δ_2 etc. obtained from the consecutive numbers in the vertical ranges of (8).

In order to determine the integral (4) the number of layers m has to be supposed $= \infty$; so it is evident, that in the functions P, Q and R, i. e. in the expression (9), we only need to retain the term with the highest exponent.

This term is:

$$\frac{m^{n+1}}{1 \,.\, 2 \ldots n+1} \Delta_n.$$

We therefore only have to find the order number of each of the series in the vertical ranges of (8), and the value of the constant difference.

For the term with $\overline{\delta a}^2$ for instance this determination gives:

$$\begin{array}{ccccc} 0 & & & & \\ & 2 & & & \\ 2 & & 8 & & \\ & 10 & & 6 & \\ 12 & & 14 & & \\ & 24 & & 6 & \\ 36 & & 20 & & \\ & 44 & & & \\ 80 & & & & \end{array}$$

$$\Delta_n = 6 \quad n = 3$$

$$R = \frac{6m^4}{1.2.3.4} = \frac{m^4}{4}$$

In the same way we find: $P = \dfrac{m^2}{2}$; $Q = \dfrac{2m^3}{3}$.

The series in the first term of (7) now becomes, if we omit the index of a_1

$$\frac{m^2}{2} a^2 - \frac{2m^3}{3} a\delta a + \frac{m^4}{4} \overline{\delta a}^2$$

Now observing that $m\delta a = R_e - R_i = t$, and reducing the fractions we obtain:

$$\frac{1}{12} [6m^2a^2 - 8m^2at + 3m^2t^2]$$

or, bringing m^2a^2 outside the brackets,

$$\frac{m^2a^2}{12} \left(6 - 8\frac{t}{a} + 3\frac{t^2}{a^2}\right)$$

Putting $\dfrac{t}{a} = \varrho$ we finally have:

$$\frac{m^2a^2}{12} [6 - 8\varrho + 3\varrho^2]. \qquad \ldots\ldots \quad (10)$$

Operating in the same way with the two other series in (7) we find for the coefficients of the terms with δa and $\overline{\delta a}^2$ successively:

$$\frac{m^2a^2}{12} \cdot \frac{1}{5}[10-15\varrho+6\varrho^2] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\frac{m^2a^2}{12} \cdot \frac{1}{15}[15-24\varrho+10\varrho^2] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Substituting the values given by (10), (11) and (12) in the equation (7) and afterwards in (4) we find:

$$L_u = \frac{2}{3}\pi^2 n^2 a^2 m^2 \left\{ [6-8\varrho+3\varrho^2]\left(r-a+\frac{a}{8}\right) + \right.$$

$$\left. + \frac{1}{5}[10-15\varrho+6\varrho^2]\left(t-\frac{3t}{8}-\frac{at}{r}\right) + \frac{1}{15}[15-24\varrho+10\varrho^2]\frac{t^2}{8a} \right\} . \quad (13)$$

Now expanding and integrating in the above described manner the other terms of the series α, it appears that each term gives a contribution to each of the terms figuring in the coëfficients of (13).

In consequence of the particular regularity of these expansions it is easy to determine the laws for the succeeding numerical coëfficients of the different series.

In the first term of (13) there appears the series:

$$S_1 : \qquad a\left(\frac{1}{8}+\frac{1}{64}+\frac{5}{1024}+\frac{35}{16384}+\ldots\right.$$

in the second term

$$S_2 : \qquad t\left(3 \cdot \frac{1}{8}+5 \cdot \frac{1}{64}+7 \cdot \frac{5}{1024}+9 \cdot \frac{35}{16384}+\ldots\right.$$

and in the third term:

$$S_3 : \qquad \frac{t^2}{a}\left(\frac{1}{8}+6 \cdot \frac{1}{64}+15 \cdot \frac{5}{1024}+28 \cdot \frac{35}{16384}+\ldots\right.$$

From the derivation of the fundamental equation (3), that can be found in the German edition of MAXWELL, edited by WEINSTEIN, it is evident, that the terms of the series S_1 are formed by the products of the equal order terms of four different series.

It is therefore very difficult to find back the law of succession in the above reduced form of these products.

The law of succession is very simple, viz.

$$\frac{u_n}{u_{n-1}} = \frac{(2n-3)(2n-1)}{2n(2n+2)},$$

which gives for the general term of the series:

$$u_n = \left[\frac{(2n-3)!}{n!(n-2)!}\right]^2 \frac{2n-1}{2^{4n-1}(n+1)}.$$

The terms with $\frac{A^3}{r^3} \cdot \frac{A^5}{r^5}$ . . etc. also contribute to each of the coëfficients of (13).

These expansions have been executed for all the mentioned terms of (3) except for the terms with $\frac{A^9}{r^9}$ and $\frac{A^{11}}{r^{11}}$, which after all would be incomplete, these two powers of $\frac{A}{r}$ reappearing in two of the succeeding terms.

After insertion of all these terms and after some simple transformations, equation (13) can be brought in the following form:

$$L_a = \frac{2}{3} \pi^2 n^2 m^2 a^3 \left\{ C_1 [\varphi_1(x) - 0.8488] + C_2 [\varphi_2(x) + 0.0848] \varrho + \right.$$
$$\left. + C_3 [\varphi_3(x) + 0.11] \varrho^2 \right\} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

wherein a = outer radius of coil, insulation included

$C_1 = 6 - 8\varrho + 3\varrho^2$ $\qquad \varrho = \frac{t}{a}$ $\qquad x = \frac{a}{r}$ $\qquad r = \sqrt{a^2 + l^2}$

$C_2 = 10 - 15\varrho + 6\varrho^2$

$C_3 = 15 - 24\varrho + 10\varrho^2$ $\qquad n$ = number of windings per cm.

m = number of layers

l = length of coil

$$\varphi_1(x) = \frac{1}{x} - \frac{1}{8} x^3 + \frac{1}{16} x^5 - \frac{15}{128} x^7 + \ldots$$

$$\varphi_2(x) = -\frac{1}{5} x + \frac{3}{20} x^3 - \ldots$$

$$\varphi_3(x) = \frac{1}{30} x - \frac{3}{40} x^3 + \ldots$$

The constants appearing in formula (14) have the following meaning:

$$0.8488 = 1 - S_1$$

$$0.0848 = \frac{1}{5} (1 - S_2)$$

$$0.11 = \frac{1}{15} S_3$$

wherein S_1, S_2, and S_3 represent the sums of the above mentioned series. The first of these constants, which has the greatest influence on the accuracy of the computed values, is accurate within a few units of the fifth decimal place.

The accurate determination of these constants is practically equivalent with including in the integration a very great number of the not mentioned terms of (3).

That in formula (14) m represents the *finite* number of layers, whereas for the integration m is supposed to be *infinite*, depends upon the fact, that the self-inductance, for the case the current is uniformly distributed over the cross section of the coil, is proportional to the square of the number of layers.

For moderate values of ϱ, which quantity in most cases is considerably smaller than 1, the mutual proportions of the coëfficients C_1, C_2 and C_3 are very nearly represented by the mutual proportions of the constants 6, 10 and 15 appearing in these coëfficients. As the terms with ϱ and ϱ^2 for long coils are always *very small* in comparison with the first term we may put approximately:

$$C_2 = \frac{5}{3} C_1 \qquad C_3 = \frac{5}{2} C_1 ;$$

substituting these values in (14), we obtain:

$$L_u = \frac{2}{3} \pi^2 n^2 m^2 a^3 C_1 \left\{ [\varphi_1(x) - 0.8488] + \frac{5}{3} [\varphi_2(x) + 0.0848] \varrho + \right.$$
$$\left. + \frac{5}{2} [\varphi_3(x) + 0.11] \varrho^2 \right\} . \quad . \quad (15)$$

Putting in this formula $\varrho = 0$ the terms with ϱ and ϱ^2 vanish and $C_1 = 6$.

We then get the formula for the self-inductance of a cylinder or single layer coil:

$$L_S = 4\pi^2 n^2 a^3 (\varphi_1(x) - 0.8488) \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The method of testing the degree of accuracy obtained in the computation of self-inductances by means of the formulae (14) and (15) is based on the same principle, as used by Rosa in his above mentioned method.

Rosa[1]) begins with the calculation of the self-inductance of a cylindrical current sheet, which has the same mean radius and length, as the solenoïd with depth of winding t. He takes the total number of windings of this cylinder equal to $\frac{l}{t}$, where l is the common length. Afterwards he considers the solenoïd of length l and depth of winding t as to be formed by one single layer of square conductor, so that the cross section of this conductor is $t \times t$ and the total number of windings is also equal to $\frac{l}{t}$.

[1]) E. B. Rosa. Bull. of the Bur. of St. IV 369.

Indicating the self-inductance of the latter by L_u and of the former by L_s, ROSA calculates the correction $\Delta_1 L$ in order to obtain L_u from L_s, so that:

$$L_u = L_s - \Delta_1 L.$$

This correction $\Delta_1 L$ consists of n times the difference of the self-inductance of one winding with square section from that of a winding on the cylinder, added to the sum of the differences of the mutual inductances of all the windings. The correction term $\Delta_1 L$ is brought in the following form:

$$\Delta_1 L = 4\pi an (A + B)$$

wherein n in the said number of windings $\frac{l}{t}$, $a =$ the mean radius. A is the part of the correction due to the difference in the self-inductance, and B the part due to the differences in the mutual inductances.

ROSA gives two tables, wherein A is given as a function of $\frac{t}{a}$ and B as a function of n.

The error in ROSA's method is concealed in this correction term B, which, as I shall show in a subsequent communication, is not only a function of n but also sensibly depends on the value of $\frac{t}{a}$, so that for this term a table with double entrance would be necessary.

I have computed for a few different values of $\frac{t}{a}$ a table for the term B, by means of which I am able — for these special values of $\frac{t}{a}$ — to get an idea about the degree of accuracy that can be obtained in calculating self-inductances by the formulae (14) and (15)

Example 1.

$l = 50$ cM. $a = 5.2$ cM. $t = 0.4$ cM. $m = 4$ $n = 10$

calculated:

by formula (14)	$L_u = 70.5976$	millihenry
„ „ (15)	$L_u = 70.5988$	„
„ by accurate correction method	$L_u = 70.5992$	„
„ by ROSA's method	$L_u = 70.544$	„
„ by formula (1) of COHEN	$L_u = 70.551$	„

For this example the correction term used by ROSA is: $B = 0.3440$ whereas the above mentioned table gives: $B = 0.3247$.

The formula of COHEN, as well as ROSA's method give too small values for the self-inductance.

An example of the extreme accuracy, obtained with the very simple formula (16) in computing the self-inductance of a cylindrical current sheet, as compared with the value calculated by the exact formula of LORENZ [1]) with elliptic integrals, may be given here:

Example 2.

$$l = 50 \text{ cm.} \qquad a = 5 \text{ cm.} \qquad n = 10$$

calculated·

by formula (16) $L_s = 4.540489$ millihenry.

„ LORENZ's formula $L_s = 4.540486$ „

Physiology. — *"On the permeability of red bloodcorpuscles in physiological conditions, more especially to Alkali and Earth-alkalimetals"*. By Prof. H. J. HAMBURGER and Dr. F. BUBANOVIĆ.

Mr. G. GRYNS has published a short article in this paper [2]), in which on the ground of some calculations he thinks it desirable to object to some of the conclusions we drew from our experiments on the subject mentioned above. (Proceedings of June 25^{th}, 1910).

We feel convinced that his remarks would not have been published, if he had waited for our more explicit communications on this subject, in the "Archives Internationales de Physiologie". As appears from a note on the first page of our paper we had promised these, and they indeed appeared shortly after [3]).

In this treatise a detailed account is given of the experimental method and moreover by way of example a lengthy report is added in an appendix, containing full particulars of *one* of the series of experiments. In these proceedings it is hardly possible to enter into details, especially when, as in this case, extensive investigations are concerned. A detailed description is better in its place in a physiological periodical.

This remark might suffice, but it is perhaps of some use that those who cannot immediately consult the "Archives Internationales" are made acquainted with the mistake of Dr. GRYNS.

1) Bull. of the Bur. of St. V, 41.

2) These Proceedings of October 29, 1910.

3) La perméabilité physiologique des globules rouges, spécialement vis-à-vis des cations. Archives Internationales de Physiologie. Vol. X. p. 1. Appeared September 24^{th} 1910.