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Geophysics. — "*On the determination of tidal constants from observations performed with horizontal pendulums.*" By Dr. J. P. VAN DER STOK.

(Communicated in the meeting of April 23, 1909).

1. When applying the theory worked out with astronomical accuracy of the analysis of tidal observations we are checked in practice by disturbing influences of meteorological nature which may be great with respect to the quantities to be calculated, making thus the accuracy of the theory as well as that of the results illusory.

These disturbing influences are greatest when we calculate the tides of long duration, where meteorological factors of irregular nature play such a preponderant part, that the definition of the constants is only then possible if it is based upon series of such long duration that those disturbances can be regarded as eliminated. Each investigation ought then to be preceded by a study of those disturbances and a taxation based upon the latter of the demand to be put as a minimum for the length of the series.

For most tides of short duration these objections do not hold, or at all events in a much less degree, they do however for those tides whose period differs but little from that of mean solar time and which are connected with it in a systematic way, namely the sidereal tides proper K_1 and K_2 and the sidereal tide improper P.

As the disturbances of meteorological nature appearing here have not only an irregular but also a regular character, the former can be eliminated by making the calculation over a great many years, but not the latter and only by means of more or less uncertain hypotheses shall we be able to come closer to our aim.

Thus e. g. the tides K_1 and P can only then be calculated out of the observations if we assume that the diurnal meteorological tide S_1 remains constant during the whole year, a supposition which is certainly inaccurate because the mean amplitudes of the motion S_1, finding its origin in the land- and seawind, must be considerably less in winter than in summer.

As lucky circumstance can be regarded that S_1 is generally small a. o. on European coasts, where however on the other hand K_1 and P too, are abnormally small, abnormally, namely, with respect to the value evaluated out of the theory of equilibrium. In tropical regions S_1 is in many places not small, and only where, as in the Java sea, K_1 and P are great, an approximative determination of K_1 is possible, whilst for that of the small tide P the circumstances will be unfavourable always and everywhere.

2. In a recent publication [1]) Prof. HECKER has made an investigation of the influence of forces generating tides on two horizontal pendulums set up for this aim in a pit so that the influence of changes in temperature was reduced to a minimum.

The result of this investigation surpasses the expectations about the possibility of such a determination of disturbances of gravity caused by celestial bodies which were until a short time ago decidedly unfavourable.

The two instruments are not of equal value; the pendulum set up in an azimuth of 222° proves to be a much better instrument than the other; the results obtained with the best pendulum are, expressed in deviations of an equivalent vertical pendulum in arc seconds

M_2	Calculated	$0''.00922 \cos (2\,t - 305°.5)$
	Observed	$0''.00622 \cos (2\,t - 285°.4)$
S_2	Calculated	$0''.00399 \cos (2\,t - 305°.5)$
	Observed	$0''.00244 \cos (2\,t - 273°.6)$

Deviations in the sense of the movement of the hand of a timepiece are reckoned positive for this instrument.

HECKER concludes from this that the earth deforms itself under the influence of the forces of the tides and that its rigidity or resistance against deformation is about equal to that of a steel sphere. This result is in accordance with the conclusions drawn from the variation of the velocity of propagation of the first forerunners of a seismic wave (longitudinal vibration) from 7 to 13 K. M. and of the second forerunners (transversal vibration) from 4 to 7 K. M. with the depth of the layers passed through and likewise with the fact that the principal periodical oscillation of the geographical latitude does not take place in a period of 306 d. (EULER) but of 427 d. (CHANDLER), from which would ensue a still greater resistance against deformation, namely about twice that of steel.

In the last chapter HECKER discusses in short the possibility to deduce from the data the influence of the partial tidal force K_1, diurnal periodical with sidereal time; here however he is checked by the above mentioned difficulty that evidently there exists in the diurnal periodic movement S_1, (Table I) not of astronomical origin, an annual variation.

[1]) O. HECKER. Beobachtungen an Horizontalpendeln über die Deformation des Erdkörpers unter dem Einfluss von Sonne und Mond. Veroffentl des Kön. Preuss. Geod. Inst. Neue Folge, no. 32, 1907.

He therefore regards an arrangement according to sidereal time not recompensatory and from a proof taken by shifting of the monthly means he concludes:

"Immerhin können wir aber aus den angeführten Werten entnehmen, dass die Amplitude einer etwa vorhandenen Sternzeitwelle nur sehr gering sein kann".

TABLE I.

Diurnal inequality in the movement of the pendulum No 1.

Commencement of time: Midday. M. E. time.

January	$0''.0021 \cos (t - 270^\circ.0)$		$+ 0''.0029 \cos (2t - 321^\circ.9$	
February	65	249.4	43	307.4
March	97	274.7	52	300.0
April	142	260.7	35	281.6
May	153	254.6	20	249.8
June	196	242.0	7	164.0
July	190	241.4	15	165.1
August	188	246.5	19	244.8
September	208	234.0	42	250.7
October	165	233.4	45	259.7
November	137	231.5	29	262.1
December	34	259.7	25	319.3
Year	$0''.01296 \cos (t - 245''.4) + 0''.00244 \cos (2t - 277^\circ 5)$			

In consequence of this discussion we can remark: 1. that for an accurate calculation of the sidereal tides an arrangement according to sidereal time, proper or improper, is unnecessary, so that little trouble is attached to it, and 2 that the influence of those sidereal tides cannot be so insignificant.

Besides the sidereal-time solar-lunar tide proper K_1 also the solar tide P appears which moves with respect to the mean solar time in the same way as sidereal time but in an opposite sense.

The angular velocity of K_1 is:

$$15^\circ + 0^\circ.04107 \text{ an hour}$$

that of P:

$$15^\circ - 0^\circ.04107 \text{ ,, ,,}$$

The theoretic coefficient of importance [1]) expressed in the mean amplitude of M_2 as unity is

for K_1: 0.58
for P: 0.19

so that yearly twice the two tides strengthen each other, namely in June and in December, when the relative amplitude is: 0.77, whilst in March and in September it comes to: 0.39.

There must therefore be, even though the meteorological tide S_1 were constant during the whole year, a considerable annual variation in the diurnal inequality.

Although during the period under treatment the lunar declination was particularly small and the circumstances therefore were unfavourable for the determination of the constants of the sidereal tides, yet the theoretical amplitude of K_1, expressed in deviations of the vertical pendulum, amounts for Potsdam to:

$$0''.0050$$

whilst the amplitude of P is in square numbers:

$$0''.0020$$

The amplitude of the annual variation in the diurnal inequality must therefore be about:

$$0''.0070$$

i. e. almost twice that in the principal solar tide S_2:

$$0''.0040.$$

From the amplitudes of the diurnal movement of Table I is evident that the amplitude of the annual variation:

$$\text{June—December} = \frac{196-34}{2} = 0''.0081$$

differs but little from the theoretical value, so that we have every reason to make an attempt for an accurate determination of the constants, which promises to lead, at all events for the tide K_1, to satisfactory results.

The value of such an investigation is not so much to be found in the determination itself as in the fact that, if the investigation is continued over several years so that the irregularities of meteorological origin have disappeared, we shall be able to correct the monthly means of the diurnal inequality for the influence of the astronomical tides, in order to obtain in this way accurate series of numbers from which the nature and the origin of the S_1 tide can be studied and deduced.

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1) Sir George Darwin. Scientific papers, Cambridge. 1907, vol. I. p 25.

3. The amplitude of a vertical pendulum under the influence of a celestial body can be represented by the expression

$$H\,p\,q\,sin\,2\,\zeta,$$

ζ = distance of zenith,

H = general coefficient or greatest amplitude which the celestial body could effect on a place situated under the equator with a declination zero:

Expressed in arc seconds H is for the moon $= \frac{3}{2} \times \frac{206265''}{2\,f k^3} = 0.''01737$,

H' for the sun $= 0.4604\,H$,

if we put:

$f = 81.4$ = quotient of mass of earth by mass of moon.

$k = 60.26$ = mean distance from moon to earth, in earthradii.

p = geographical or local coefficient, dependent on the geogr. latitude of the place of observation and the azimuth of the horizontal pendulum; for the rest different for diurnal and semidiurnal movements.

q = astronomical coefficient dependent on the inclination of the orbit and its eccentricity.

If the general expression is developed in a series of terms, behaving itself purely periodically, we find for the components in the direction of the meridian, N (north positive), and in a direction normal to it, W (west positive), as far as the terms are concerned, in the direction of sidereal time, inclusive of P, for the moon:

$$\left.\begin{aligned} N &= H\,q_m \cos 2\,\varphi \sin \gamma\,\tau \\ W &= -\,H\,q_m \sin \varphi \cos \gamma\,\tau \end{aligned}\right\} \quad \ldots\ldots \quad (1)$$

φ = geogr. latitude

$\gamma = 15° + \eta = 15°.04107$ an hour

$q_m = \sin I \cos I$ (I = inclination of orbit of moon)

and for the sun:

$$\left.\begin{aligned} N &= H' \cos 2\,\varphi\,[q_5 \sin \gamma\,\tau - q' \sin (\gamma - 2\,\eta)\,\tau] \\ W &= H' \sin \varphi\,[-\,q_5 \cos \gamma\,\tau + q' \cos (\gamma - 2\,\eta)\,\tau] \end{aligned}\right\} \quad \ldots \quad (2)$$

The deviation of a horizontal pendulum set up in an azimuth $180° + \alpha$ is:

$$-\,(N \sin \alpha + W \cos \alpha)$$

If we put:

$$-\sin \varphi \cos \alpha = p \sin \chi$$
$$-\cos 2\,\varphi \sin \alpha = p \cos \chi,$$

then for Potsdam:

$$\varphi = 52°\,23', \qquad \alpha = 42°$$
$$p = 0.6129, \qquad \chi = 286°.15, \qquad q' = 0.4127.$$

For the period treated (Dec. 1902—April 1905) holds:

$$I = 18^\circ\,33', \qquad v = 0^\circ.31$$

with which we find with the aid of the wellknown tables of Börgen for the astron. coefficient of the K_1 tide composed of (1) and (2)

$$q'' = 0.4732,$$

from which follows for the theoretical value

$$K_1 = 0''.005037, \qquad P = 0''.001869$$

4. Already 15 years ago I have pointed out in "Studiën over Getijden in den Indischen Archipel", that for the determination of the constants of the tides K_1, P and K_2 the trouble of an arrangement of the hourvalues according to the angular velocities of these tides is superfluous and that we can deduce these constants with equal accuracy and little labour directly out of the monthly means. The application of this method has furnished good results not only for the Indian tides but also for the determination of the tidal constants on the Dutch coasts where all three tides are very small.[1]) Especially if as in this case hour-observations are at hand, the calculation is exceedingly simple, for the diurnal variation can be represented in its variability in the course of the year with great approximation by the expression:

$$S_1 \cos(15t - C_1) + K_1 \cos(15t - \psi_1 + 30x) + P \cos(15t - \psi_2 - 30x), \quad . \quad (3)$$

where

$$\psi_1 = \chi_1 - V_0 \;, \quad V_0 = h_0 - v' - \frac{\pi}{2} = 204^\circ.71 \,,$$

$$\psi_2 = \chi_2 - V'_0 \;, \quad V'_0 = - h_0 + \frac{\pi}{2} = 155^\circ\,16 \,;$$

h_0 = length of the sun at the commencement of the time (epoch) (294°.84) i.e. in this case January 16; v' is a small correction, caused by the inclination of the orbit of the moon with respect to the ecliptic. It is clear that the inaccuracies committed here, namely 1 the angular velocities holding for *one* day being all taken equal instead of respectively

$$15^\circ \;, \quad 15^\circ + \eta \;, \quad 15^\circ - \eta$$

And 2 the monthly means being regarded as 12 equidistant points, can have no perceptible influence on the result of the calculation.

Omitting the first term we can bring (3) into the form:

1) Études des phénomènes de marée sur les côtes néerlandaises I. Utrecht, 1904.

$$\cos 15\,t \begin{bmatrix} (K_1 \cos \psi_1 + P \cos \psi_2) \cos 30\,x \\ + (K_1 \sin \psi_1 - P \sin \psi_2) \sin 30\,x \end{bmatrix}$$

$$+ \sin 15\,t \begin{bmatrix} (K_1 \sin \psi_1 + P \sin \psi_2) \cos 30\,x \\ - (K_1 \cos \psi_1 - P \cos \psi_2) \sin 30\,x \end{bmatrix}$$

If we put the formula for the diurnal inequality of Table I into the form:

$$A \cos 15\,t + B \sin 15\,t,$$

and subtract from the values A and B the annual means A' and B' and then again represent the differences formed in this way by:

$$\left.\begin{aligned} A - A' &= R \cos 30\,x + Q \sin 30\,x \\ B - B' &= R' \cos 30\,x + Q' \sin 30\,x \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad , \quad . \qquad (4)$$

we obtain the equations:

$$\left.\begin{aligned} K_1 \cos \psi_1 + P \cos \psi_2 &= R \\ K_1 \sin \psi_1 - P \sin \psi_2 &= Q \\ K_1 \sin \psi_1 + P \sin \psi_2 &= R' \\ -K_1 \cos \psi_1 + P \cos \psi_2 &= Q' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad , \quad . \qquad (5)$$

from which the four unknown quantities can be solved; then the amplitudes must be augmented, monthly means having been used, by multiplication by the factor:

$$R_2 = \frac{\pi}{12 \sin 15°} = 1.0115$$

and to the values ψ_1 and ψ_2 the astronomical arguments V_0 and V'_0 must be added.

In all cases in which the S_1 tide is so small that even if it is submitted to an annual variation it can only exercise an influence small with respect to the amplitudes of K_1 and P, this simple method leads to good results.

If S_1 is not small, we can start from the assumption that land- and seawind are different in winter and summer, but that they can be regarded as constant during each of the seasons. We can then eliminate out of the six summer months of the differences (4) the value $S_1 \cos (15\,t - C_1)$ and likewise out of the six winter-differences, and then we can calculate out of the equations the four unknown quantities. The combinations of the different monthly means necessary for this are of course less favourable than in the former case, but this disadvantage can be compensated by taking a great number of years together which is necessary for every method when we have to deal with disturbances of a meteorological nature.

5 Neither of the two methods, which can be used when we calculate watertides, is applicable to the case under discussion; as evidently the tide S_1 is great we may not assume that the annual variation of S_1 will be small with respect to the quantities to be found. Neither is there a reason to assume that in winter or in summer, regarded separately, S_1 will be fairly constant, as we are still quite uncertain about the nature and the origin of it.

Another favourable circumstance however, not appearing for watertides, makes in this case an approximative solution possible.

For when K_1 appears in the monthly means with a certain value K and an argument χ_1, then we can say almost with certainty that the tide P will make its appearance with an amplitude:

$$P = aK,$$

in which a is the theoretic proportion of P to K_1:

$$a = 0.371.$$

Furthermore we can state with equal probability

$$\chi_2 = \chi_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Thus out of the equations (4) two unknown quantities disappear and they can be replaced by two others characterizing S_1 more closely.

We represent this tide by the form:

$$\{S + L \cos(30\,x - m)\} \cos(15\,t - C_1)$$

and so we assume that the amplitude is submitted to an annual variation, but that C_1 remains constant, which, if this phenomenon finds its origin in the radiation of the sun, cannot be far from the truth.

Furthermore follows from (6):

$$\psi_2 = \psi_1 + V_0 - V'_0 = \psi_1 + \alpha = \psi_1 + 49°.55.$$

Instead of the formulae (5) we get the four equations

$$\left.\begin{aligned} L \cos C \cos m + K \cos \psi_1 + Ka \cos(\psi_1 + \alpha) &= R \\ L \cos C \sin m + K \sin \psi_1 - Ka \sin(\psi_1 + \alpha) &= Q \\ L \sin C \cos m + K \sin \psi_1 + Ka \sin(\psi_1 + \alpha) &= R' \\ L \sin C \sin m - K \cos \psi_1 + Ka \cos(\psi_1 + \alpha) &= Q' \end{aligned}\right\} \quad . \quad . \quad (7)$$

With

$$R = 42.22 \qquad R' = 69.94$$
$$Q = 34.98 \qquad Q' = 0.96$$
$$C = 245°.5$$

we find from this:

$$K_1 = 51.2 \qquad L = 46.3$$
$$\psi_1 = 44°.5 \qquad m = 248°.7$$

or, after applying the augmenting factor R_2 and the astronomical argument, as the result of this investigation:

$$K_1 \begin{cases} \text{Calculated} & 0''.00504 \; cos\,(t - 286^\circ.2) \\ \text{Observed} & 0''.00518 \; cos\,(t - 249^\circ.2) \end{cases}$$

$$P \begin{cases} \text{Calculated} & 0''.00187 \; cos\,(t - 286^\circ.2) \\ \text{Observed} & 0''.00192 \; cos\,(t - 249^\circ.2) \end{cases}$$

and then for the monthly means of the diurnal inequality the expression

$$0''.01296 \{1 + 0.361 \; cos\,(30\,x - 248^\circ.7)\} \; cos\,(15\,t - 245^\circ.5)$$

6. Just as for watertides the superposition of the partial tides S_2 and K_2 causes a maximum in March and September (the equinoctial tides known to every mariner) and a minimum in January and July, here too such a semi-annual variation must appear in the expressions for the semi-diurnal inequality. Indeed this variation makes its appearance clearly at first sight in the amplitudes of Table I.

If we assume that the S_2 tide is constant during the whole year, then the general expression for the monthly means is:

$$S_2 \; cos\,(30\,t - C_2) + K_2 \; cos\,(30\,t - C_{2k} + 60x)$$

where

$$C_{2k} = \chi - V_0 \quad , \quad V_0 = 2\,h_0 - 2\,v''.$$

We have then to do nothing but to analyse the expressions of Table I, after subtraction of the mean for the whole year, into the components:

$$A \; cos\,30\,t + B \; sin\,30\,t$$
$$A = cos\,(60\,x - C_{2k})$$
$$B = - sin\,(60\,x - C_{2k})$$

If we bring through the differences A and B doubly-periodic curves, we obtain for each of the quantities to be found:

$$K_2 \; sin\,C_{2k} \text{ and } K_2 \; cos\,C_{2k} \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

two values which must be about equal and from which we deduce, after applying the augmenting factor

$$R_4 = \frac{\pi}{6 \; sin\,30^\circ} = 1.0472$$

and the astronomical argument

$$V_0 = 2\,h_0 - 2\,v'' = 229^\circ.68 - 0^\circ.25,$$

the quantities K_2 and χ.

The astronomical coefficient calculated according to the tables of Börgen is:

$$q = 0.0878$$

and if we put

$$-\sin\varphi\cos\alpha = p\sin\chi$$
$$\sin\varphi\cos\varphi\sin\alpha = p\cos\chi$$

we then find for the theoretical value:

$$K_2 = 0''.00085 \quad , \quad \chi = 305^\circ.5.$$

From the data of Table I we find, however, for the values (8) quantities, which are equal in sign but which differ, for the rest pretty much, namely:

$$K_2 \cos C_{2k} = \begin{cases} 3.97 = R \\ 10.77 = Q \end{cases}$$
$$K_2 \sin C_{2k} = \begin{cases} 4.10 = R' \\ 16.34 = Q' \end{cases}$$

So here too we must assume that S_2 is not constant and that there is a semi-annual variation in the semi-diurnal inequality which indeed is immediately evident from the amplitudes of Table I on account of the inequality of the two maxima and minima.

If, as above, we represent the S_2 tide by

$$\{S_2 + L\cos(60\,x - m)\}\cos(30\,t - C)$$

we then find out of the four equations

$$L\cos C\cos m + K_2\cos C_{2k} = R$$
$$L\cos C\sin m + K_2\sin C_{2k} = Q$$
$$L\sin C\cos m + K_2\sin C_{2k} = R'$$
$$L\sin C\sin m - K_2\cos C_{2k} = Q'$$

as the result of the investigation:

$$K_2 \begin{cases} \text{Calculated} & 0''.00085\cos(2\,t - 305^\circ.5) \\ \text{Observed} & 0''.00070\cos(2\,t - 260^\circ.6) \end{cases}$$

and for the semi-diurnal inequality:

$$0''.00244\{1 + 0.574\cos(60\,x - 158^\circ.6)\}\cos(2\,t - 277^\circ.7)$$

This result justifies the expectation that if monthly means of the diurnal variation are available calculated over a greater number of years and by preference over years in which the declination of the moon is great, also the calculation of the small tide K_2 can be made with all the looked for accuracy. For, at the greatest declination of the moon the amplitude of K_2 is almost twice greater than at the smallest.

Out of the monthly means of the diurnal variation for the second horizontal pendulum set up at Potsdam these sidereal tides do not admit of a deduction. The continual displacements of the zero point, considerable for both instruments, surpass for this instrument all the small regular movements entirely.

A determination of the siderial tides might thus serve as a criterion for the quality of seismic instruments.