Huygens Institute - Royal Netherlands Academy of Arts and Sciences (KNAW)

**Mathematics.** — *"Investigation of the functions which can be built up by means of infinitesimal iteration."* By Mr. M. J. VAN UVEN. (Communicated by Prof. W. KAPTEYN).

(Communicated in the meeting of October 30, 1909).

In the paper presented by me in the preceding meeting I have roused the impression by an ambiguous mode of expression as if the method to obtain standardforms followed by me were the most serviceable. This is in nowise the case. I have exclusively intended to give a summary of what can be reached in this direction by differentiation. The last observation lying at hand and moreover already made by SCHROEDER according to which out of each function $y = \varphi(x)$ constructible by means of infinitesimal iteration a whole series of suchlike functions can be deduced all of the form $y = h_{-1}[\varphi\{h(x)\}]$ may have been made too much incidentally to weaken that impression properly. For, this last principle furnishes, as is immediately evident, an inexhaustible wealth of standardforms. With the aid of this principle we can i. a. just as well deduce the standardform $(D)$. We have but to show that

$$y = \frac{ax + \beta}{\gamma x + \delta}$$

follows directly out of the formula of ABEL or SCHROEDER. By applying the standardform of SCHROEDER:

$$g(y) = m\, g(x)$$

we find out of

$$y = g_{-1}\{m\, g(x)\} = \frac{ax + \beta}{\gamma x + \delta},$$

$$g(x) = \frac{ax + b}{cx + d},$$

where $m, a, b, c$ and $d$ must satisfy

$$\frac{ad}{m\,a - \delta} = \frac{bc}{a - m\delta} = \frac{bd}{(1+m)\beta} = \frac{ac}{-(1+m)\gamma},$$

so that $m$ is a root of

$$m^2 - \frac{a^2 + \delta^2 + 2\beta\gamma}{a\delta - \beta\gamma}\, m + 1 = 0.$$

We then find without difficulty the form $(d)$ for $\varphi_n(x)$. In the parabolic case, $a\delta - \beta\gamma = 0$, this calculation does not hold; we have then but to call in the aid of the equation of ABEL. Let us put

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$$y = g_{-2} \{ g(x) + 1 \} = \frac{ax + \beta}{\gamma x + \delta},$$

then follows out of it for $g(x)$

$$g(x) = \frac{ax + b}{cx + d},$$

where $a, b, c$ and $d$ must satisfy

$$\frac{ad - bc}{a + \delta} = \frac{cd}{a - \delta} = \frac{d^2}{2\beta} = \frac{c^2}{-2\gamma}.$$

In this case too the form $(d')$ of $\varphi_n(x)$ can be determined without any difficulty.

The necessity of this supplement did not become quite clear to me until I received a communication of Dr. L. E. J. BROUWER, who was so kind as to draw my attention to this lack and who likewise brought out the immediate connection between the standardform $(D)$ and that of SCHROEDER. I set great store by the fact of thanking him kindly for his observations.


**Physics.** — *"Remarks on the Experiments of* WILSON *and* MARTYN *on the Velocity of Rotation of the Electric Discharge in Gases in a Radial Magnetic Field."* By Dr. J. A. VOLLGRAFF. (Communicated by Prof H. A. LORENTZ).

(Communicated in the meeting of October 30, 1909).

WILSON and MARTYN [1]) have proposed a theory on the rotation of the electric discharge in gases at low pressures in a radial magnetic field, agreeing satisfactorily with their experiments. The discharge consists in a motion of positive and negative ions.

They find for the value of the velocity of rotation

$$U = k_1 k_2 H X, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$H$ being the magnetic and $X$ the electric intensity at the place of the discharge, and $k_1$ and $k_2$ the velocities which unit electric intensity acting on the gas gives to a positive and a negative ion resp.

The apparatus was composed of two vertical coaxial cylinders of glass $C_1$ and $C_2$ along the axis of which an electromagnet was so placed that one of its poles was situated in the middle of the magnet and at the same time in the middle of the axis. The magnetic field

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1) H. A. WILSON and G H. MARTYN. "On the Velocity of Rotation of the Electric Discharge in Gases at Low Pressures in a Radial Magnetic Field". (Proc. Roy. Soc, Ser. A. Vol. 79, N⁰ A 532, 2 Aug. 1907).