polarimetric and copper tests both before and after inversion. Next 100 grammes of this glycerin were mixed with a 3 % starch solution, warmed to 40° C. and kept at that temperature for a couple of hours. After that the dissolved starch and dextrin was precipitated with alcohol, filtered, a pinch of calcium carbonate was added to the filtrate to prevent inversion by the slightly acid reaction of the filtrate, and the alcohol was evaporated off. The syrupy residue was dissolved in water, diluted to the volume of 100 cM.³ and used for the determination of the sugars by the polarimeter and Fehlings solution before and after inversion.

The original glycerin solution had contained 0.17 % of glucose both before and after inversion, while after the treatment with starch 100 grammes of the solution contained 0.60 grammes of reducing sugars before inversion and 0.67 after that operation, which shows that 0.43 grammes of glucose and 0.07 grammes of sucrose (?) have been formed from the starch by the ferment. The polarisation of the solution was + 0.9 before and + 0.4 after inversion, giving evidence, that notwithstanding the precipitation with alcohol, a small amount of starch or dextrin has still remained dissolved.

At any rate from the fact that the exclusion of oxygen prevents the saccharification of the starch in the fruit and from the negative results of the experiments on formation of sucrose by means of fresh juice and of the precipitated and re-dissolved ferments, it follows that the rapid transformation of starch into sucrose during the after-ripening of some fruits is a vital process and not a consequence of the action of some ferment contained in the fruit which, just as diastase forms maltose from starch, could be isolated to form large quantities of sucrose from any kind of starch in the laboratory.

Mathematics. — "*Congruences of twisted curves in connection with a cubic transformation.*" By Prof. JAN DE VRIES.

(Communicated in the meeting of May 30, 1908).

§ 1. If x_1, x_2, x_3, x_4 are the coordinates of a point X with respect to a tetrahedron having O_1, O_2, O_3, O_4 as vertices, then

$$x_1 x'_1 = x_2 x'_2 = x_3 x'_3 = x_4 x'_4$$

determines a cubic transformation which transforms the right line

$$x_k = \lambda a_k + \mu b_k$$

into a twisted curve ω^3, represented by

$$\varrho x'_k = \frac{1}{\lambda a_k + \mu b_k}.$$

The congruence Γ of the curves ω^3 through the five points

O_k ($k = 1, 2, 3, 4, 5$) is now transformed into a sheaf of rays having as vertex the point O'_6 conjugate to O_6.

To the bisecant b' through O'_6 of the curve σ'^3 brought arbitrarily through O_1, O_2, O_3, O_4 corresponds a β^3 through O_1, O_2, O_3, O_4, O_5, having the right line s as chord.

The following will show that the indicated transformation enables us to deduce by a simple method a number of well-known properties of systems of curves ω^3.

§ 2. Let us consider the curves ω^3 of the congruence Γ cutting the right lines l and m. They are transformed into the right lines through O'_6, resting on two curves λ'^3 and μ'^3. Now the cubic cones, projecting these curves out of O'_6 have besides the right lines $O'_6\, O_k$ ($k = 1, 2, 3, 4$) five edges more in common, which are the images of as many twisted curves belonging to Γ.

From this is evident that the curves of Γ cutting a given right line l form a *surface A^5 of order five.*

The image of A^5 is a cubic cone, projecting λ'^3 out of O'_6 and having the bisecant b' out of O'_6 as nodal edge. Therefore the curve β^3 of Γ having l as bisecant is a *nodal curve* of A^5.

If we bring the right line m through O_1 its image is a right line m' passing likewise through O_1 and having therefore with the above mentioned projecting cone of λ'^3 besides O_1 two points in common. From this we conclude that A^5 has *five threefold points* O_k.

So the section of A^5 with $O_k O_l O_m$ consists of the right lines $O_k O_l$, $O_l O_m$, $O_m O_k$ and a conic through O_k, O_l, O_m cutting $O_p O_q$ and forming with this right line a cubic curve of Γ. Consequently *eleven right lines and ten conics* lie on A^5.

§ 3. The curves ϱ^3 of Γ touching a given plane φ, are transformed by the correspondence into tangents t' through O'_6 of a cubic surface Φ'^3 having conic points in O_k ($k = 1, 2, 3, 4$). The polar surface of O'^6 passes through the four double points O, so it has as image a quadratic surface through those points. The section of the latter with φ is the image of the locus of the points, in which Φ'^3 is touched by the right lines t'. *This conic* contains therefore the points in which φ is *touched* by the curves ϱ^3.

Through O'_6 pass six principal tangents of Φ'^3, the congruence Γ contains therefore *six* curves, *osculating* φ.

The enveloping cone φ'^6 out of O'_6 to Φ'^3 has four nodal edges $O'_6 O_k$; for a plane through $O'_6 O_k$ cuts Φ'^3 according to a cubic curve with node O_k, sending but four tangents through O'_5, so that in that plane $O'_5 O_k$ replaces two edges of the cone.

So φ'^6 has with an arbitrary cubic curve through the four points O_k ten points in common lying outside O_k. By applying our transformation we find from this that the curves of Γ touching φ form *a surface* Φ^{10} *of order ten.*

A right line through O_1 cuts φ'^6 in four more points; on its image therefore rest four curves ϱ^3 touching φ. From this ensues that Φ^{10} has *five sixfold points* O_k.

The right lines $O_k O_l$ lie therefore on Φ^{10}; it can as follows become evident that they are *nodal rightlines.* A right line resting on $O_1 O_2$ and $O_3 O_4$ has six points in common with φ'^6. So its image must have on $O_1 O_2$ and $O_3 O_4$ four points in common with Φ^{10}.

The section of Φ with $O_1 O_2 O_3$ consists of the right lines $O_1 O_2$, $O_2 O_3$, $O_3 O_1$ to be counted double and a curve of order four, having nodes in O_1, O_2, O_3 and in the point of intersection of the nodal line $O_4 O_5$; thus it consists of two conics. These conics form evidently with $O_4 O_5$ two cubic curves of Γ, touching φ.

Consequently there lie on Φ^{10} ten *nodal lines and twenty conics.*

When we regard the tangential cones out of O'_5 to two quadrinodal cubic surfaces Φ'^3 it follows readily that Γ contains *twenty* curves touching two given planes.

§ 4. To determine how many curves ϱ^3 can be brought through four points O_k having the right line b as bisecant and resting on the right lines c and d, we have but to find the number of right lines r' which cut β'^3 two times and γ'^3 and δ'^3 one time, when these three curves have four points O_k in common.

Now the chords of β'^3 resting on a right line l' form a biquadratic scroll on which β'^3 is nodal curve, having thus with γ'^3 besides the four points O four more points in common. From this follows immediately that the right lines cutting β'^3 twice and γ'^3 once also form a biquadratic scroll Σ'^4. The cones which project these curves out of a point of $\beta^{3'}$ having two edges in common, not containing one of the points O, the curve β'^3 is also nodal curve on Σ'^4. With δ'^3 this scroll has besides O_k four points in common; so on γ'^3 and δ'^3 rest four chords of β'^3, and by applying the transformation we find that the curves ϱ^3 which cut b twice and c once form a *surface* Σ^4 *of order four.*

If we bring d through O_1, then its image d' has with Σ'^4 two more points in common; consequently d cuts the surface Σ^4 in two points lying outside O_1, so that O_1 is a node. Therefore the surface Σ^4 has *four double points* O_k.

Evidently b is nodal right line of Σ^4; for, b is the image of the nodal curve β'^3 lying on Σ'^4.

Through a point S of b pass two curves ϱ^3; their two points of intersection S' and S'' with b are the points which b has still in common with the surface Δ^6 determined by c, S and the points O.

As the pairs of points S and S' form a (2,2) correspondence, *four* curves ϱ^3 can be brought through four points, which *touch* a right line and *intersect* an other right line.

The section of Σ^4 with the plane $O_1 O_2 O_3$ has nodes in O_1, O_2, O_3 and in the intersection with b; so it consists of two conics. One of these conics contains also the intersection of c; it is completed to a degenerated ϱ^3 by the right line out of O_4 resting upon it and upon b. The second conic contains the intersection of the transversal drawn out of O_4 to b and c and forms with this right line a ϱ^3.

On the surface Σ^4 lie therefore *eight conics, nine simple right lines and a nodal line.*

§ 5. The number of curves ϱ^3 through O_k $(k = 1, 2, 3, 4)$ resting on the right lines a, b, c, d is evidently as large as the number of transversals of four cubic curves $\alpha^3, \beta^3, \gamma^3, \delta^3$ brought through O_k. The scroll (α, β, l), having α^3, β^3 and a right line l as directrices, is of order 14, l being fivefold and each plane through l containing nine right lines. If l_0 passes through O_1 a plane through l_0 contains but four right lines, so that the order of the scroll (α, β, l_0) amounts but to 9. From this ensues that (α, β, l) possesses four twofold points O_k.

With γ^3 the scroll (α, β, l) has 22 points in common outside O_k; so (α, β, γ) is of order 22.

On the scroll (α, β, l_0) we find that O_1 is fivefold, because a right line through O_1 cuts four generatrices; on the other hand O_2, O_3 and O_4 are threefold points, for a right line through O_2 cutting the fivefold right line l_0, meets but *one* generatrix more. With γ^3 the scroll (α, β, l_0) has still $9 \times 3 - 5 - 3 \times 3 = 13$ points in common besides the multiple points. In connection with the above follows from this that O_1 is a ninefold point on (α, β, γ).

Of the points of intersection of (α, β, γ) with δ^3 36 lie in O_k; consequently $\alpha^3, \beta^3, \gamma^3, \delta^3$ have thirty common transversals.

Therefore we can bring through *four* points *thirty* cubic curves resting on *four* given right lines.

§ 6. Let us now consider the surface ψ^{30} formed by the curves ϱ^3 resting on a, b and c. Through a point A of a and the points O

pass five ϱ^3 cutting b and c. From this is evident that a, b and c are *fivefold right lines*.

With a right line m through O_k the scroll (α, β, γ) has thirteen points lying in O_k in common, so its image m' (right line through O_k) cuts ψ^{30} likewise in 13 points lying outside O_k. We conclude from this that the four points O are seventeenfold on ψ^{30}.

So the right lines O_kO_l lie on this surface; that they are fourfold right lines can be shown in this manner.

As O_k and O_l are ninefold on (α, β, γ) the right line O_kO_l is cut outside those points by 22—18=4 transversals of the curves α, β, γ; the images of these right lines are conics through O_k and O_l resting on O_mO_n, b, c and d and forming with O_mO_n a ϱ^3 of the system.

The section of ψ^{30} with $O_1\,O_2\,O_3$ can consist outside the three fourfold right lines only of conics; these are easy to indicate. In the first place we can bring through O_1, O_2, O_3 a conic cutting b and c; it is completed to a ϱ^3 by each of the two right lines out of O_4 resting on the conic and on d. Then the sections of d and of the transversal with O_4 to b and c with O_1, O_2, O_3 determine a conic forming with the indicated transversal a ϱ^3. So we have in O_1, O_2, O_3 three double and three simple conics; with the three fourfold right lines they form a section of order 30.

On ψ^{30} lie therefore 4 seventeenfold points, 3 fivefold, 6 fourfold and 36 simple right lines, 12 double conics and 36 simple conics.

Astronomy. — "*Contributions to the determination of geographical positions on the West coast of Africa. III.*" By C. SANDERS. (Communicated by E. F. VAN DE SANDE BAKHUYZEN).

(Communicated in the meeting of May 30, 1908).

I. Introduction.

After a stay in Europe during the winter 1902—1903 I returned to Portuguese West Africa and remained there until the autumn 1906 when I again went to Europe for some time.

During this period 1903—1906 I have once more tried to contribute to the determination of geographical positions in these parts as much as time and circumstances allowed. Circumstances, however, were often unfavourable to my observations, and hence the results obtained are less than I had desired and expected at first.

The results obtained may be ranged under three heads.

1. *New determinations at Chiloango.* In November and December